SYNCHRONOUS MOTORS

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SYNCHRONOUS MOTORS AND CONVERTERS

THEORY AND METHODS OF CALCULATION AND TESTING

BY

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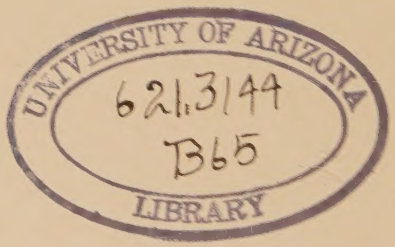

TRANSLATOR'S PREFACE

PROF. A. BLONDEL was the first writer to publish a systematic, comprehensive work on Synchronous Motors; and his book, although it has now been before the public several years, still remains the leading work on that important subject. Owing to the non-existence of an English edition, it is not, however, as well known and as much appreciated as it deserves to be, by English-speaking readers.

The translation of this celebrated, one might almost say classical, work into English, was undertaken at the suggestion of teachers and others who were desirious of making more extensive use of the work than is possible if the French text alone is available.

The author and the publisher both accepted the suggestion that the scope and the usefulness of the book might be increased materially by including in it some reference to "Rotary Converters." Excellent material for this purpose was already available in the form of a paper presented by Prof. Blondel, at the Electrical Congress in Paris, in 1900. Two other papers presented by him at the Electrical Congress at St. Louis, in 1904, also were of sufficient interest in this connection to make their reproduction desirable.

It was decided to separate the contents of the book into three distinct parts. Part I, relating to Synchronous Motors, corresponds to the original French work on Synchronous Motors. The author himself corrected the French text, and he also supplemented it with much new matter while the translation was in process. The proofs of the English text were submitted to several persons who were well qualified to criticise the text and suggest improvements therein. The French text of Part I is also supplemented by an additional chapter contributed by Prof. C. A. Adams, of Harvard University. Part I may therefore be considered fairly well brought up to date. Part II, relating to Synchronous or Rotary Converters, is made up of old and new matter. The old matter (Chapters I and II), constitutes a translation of Prof. Blondel's Paris-Congress

paper of 1900. The new matter consists of three chapters by Prof. Blondel, and a chapter contributed by Prof. C. A. Adams. Part III contains the reprints of the two papers presented by Prof. Blondel at the St. Louis Electrical Congress in 1904, relating to the application of his " two-reaction " method to alternators.

A very few modifications and additions have been made in the text by the translator. His own strong objections to the terms " wattless " and " watted " made him very willing to eliminate them; and the recent action of the Standards Committee of the American Institute of Electrical Engineers in sanctioning and recommending " reactive " and " active " as substitutes, furnished the incentive and the pretext for doing this, even after a considerable portion of the type had been set. This book will, therefore, be the first book in which " wattless " and " watted " are replaced by " reactive " and " active " respectively.

The translator desires to express his gratitude and sincere thanks to all who have encouraged and assisted him in the preparation of this book for the press. Special acknowledgment should be made of the very valuable services rendered by Prof. C. A. Adams. He was the first to suggest the publication of this book, and he has made many excellent suggestions, besides contributing two new chapters, and also reading and correcting the proofs of the entire book. Special thanks are also due to Prof. E. J. Berg, of the University of Illinois, for his very thorough reading of the proofs of Part I, and for very excellent suggestions for changes and additions made by him, some of which are incorported in notes inserted in the text, and identified by the initials " E.J.B." Thanks for reading the proofs of Part I, and making corrections or suggestions are due to the following:

Prof. D. C. Jackson, of Massachusetts Institute of Technology; Profs. W. I. Slichter and Morton Arendt, of Columbia University; Prof. H. H. Norris, of Cornell University; Mr. A. L. Jones, of the General Electrical Company.

Thanks are due to Mr. C. W. Stone, of the General Electric Company, and Mr. W. S. Rugg, of the Westinghouse Electric and Manufacturing Co., for the data relating to American Synchronous Motors, given in Appendix B; and also to the Edison Illuminating Company of Detroit, Mich., and to Mr. A. A. Meyer of its engineering department, for the information relative to the practical use of synchronous condensers contained in Appendix C.

The Translator desires to make special acknowledgment of the great courtesy and kindness of the distinguished author, Prof. André Blondel himself. The preparation of this book for the press has been a labor of love which has occupied very pleasantly and profitably a portion of the leisure moments of the translator. He considers himself remunerated amply, for the work involved, by the great privilege which has been one of the perquisites incidental to the task, namely, that of closer personal contact and acquaintance with the author; and he is very glad to have had the opportunity, through this translation, to help make the work and the talents of the author better known, as they deserve to be; for, unquestionably, Prof. Blondel is one of the great productive workers of our time in pure and applied electrical science. His work, great as it is in itself, becomes really wonderful and phenomenal, when the circumstances under which it has been done are realized and appeciated. Though handicapped most unfortunately, by protracted serious ill health and physical suffering, he has, nevertheless, kept well in the front rank with his more fortunate contemporaries and colleagues in the entire world; and he has achieved fame and renown by great mental powers, by wonderful originality and versatility, not only as a scientist, a teacher, and an author, but also as an inventor, an engineer, and an expert. The great respect which is inspired by the prodigious quantity and the superior quality of Prof. Blondel's work turn to absolute wonder and to profound admiration, before the wonderful activity and the untiring energy of his highly gifted, well-trained mind. No tribute of praise is too great for the work of this man, who is at the same time a genius and a hero, with an innate love of science and a devotion to scientific progress which uphold and uplift him, and urge him onward, *quand même*, in spite of ill-health and physical suffering, to new researches and new achievements.

THE TRANSLATOR.

NEW YORK, December, 1912.

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INTRODUCTION

Classification of Singlephase and Polyphase A.C. Motors. An alternating current motor comprises, like any dynamo-electric machine, an *inducing magnetic field* and its *induced* circuit, the one turning with respect to the other. But, while the field of D.C. motors is always constant, that of A.C. motors may be either *constant*, *alternating*, or *revolving*, according to whether it be produced by a direct current, an alternating current, or a system of polyphase alternating currents serving to excite windings suitably interlaced. Therefore, as was proposed quite logically, in 1891, by E. Hospitalier,[1] A.C. singlephase and polyphase motors can be classified according to the nature of their magnetic fields, into three classes:

(1) Constant field motors.

(2) Alternating field motors.

(3) Revolving field motors.

The first class constitutes the subject-matter of the present book, the second and third classes being reserved for another volume. As will be seen, these three classes each contain singlephase and polyphase motors.

Constant field motors can also be characterized by the fact that the armature-rotation can be maintained at a single speed only, which is *synchronous* with the alternations of the currents employed. We will therefore call them, more frequently, in accordance with common custom, *synchronous motors*, in contradistinction to the two other classes, which may be characterized as *asynchronous or non-synchronous*. For the sake of greater precision, we will apply the latter qualification to motors of the last class only; and since alternating-field motors (with one single exception, which is of little importance) are characterized by the use of a commutator similar to that used with D.C.

[1] Bulletin de la Société francaise de physique, 17. Juillet, 1891.

machines, we will give them the more distinctive name of *Commutator Motors.*

Synchronous Motors. A synchronous motor can be defined as being, merely, *an alternator used as a motor.* The transmission of power between an A.C. generator and an A.C. motor is, therefore, nothing more than a particular case of the coupling of two alternators in synchronous operation. Indeed, it is precisely through the study of the features of the coupling of alternators in parallel that the occasion presented itself of noting the phenomenon of the *reversibility of alternators,* that is to say, the possibility of using the same machine both as a motor and a generator, provided that it shall have been previously brought to a speed absolutely equal to that of the generator which supplies it with current.

We can easily understand the possibility of operating such a motor by comparing it to a motor with commutated current. It is known that if the current of a shuttle armature of the Siemens ("H") type is commutated at each half revolution, the motor-couple is always in the same direction when the machine is supplied by direct current. In an A.C. system, the same result is obtained without a commutator, because the direction of the supply-current changes at each half revolution, and this effect occurs only when the motion of the motor is synchronous, that is to say when the armature advances the distance of one pole during one alternation of the supply-current.

Although this property was noted as early as 1869 by Wilde,[1] it passed unnoticed during more than ten years, and it has really been known only since the experiments of J. Hopkinson and Grylls-Adams, at the South Foreland Lighthouse, in 1883. The Memoir of Hopkinson[2] (in which, without knowing the work of Wilde, he gives the explanation to which reference will be made later), was epoch-making in the history of alternating currents.

In the South Foreland experiments, the alternators used were three similar de Meritens singlephase alternating current machines, all belt-driven from a common source of power.

[1] Wilde, *On a Property of the Magneto-electric Current to Control and Render Synchronous the Rotations of the Armature of a Number of Electromagnetic Induction Machines,* Philosophical Magazine, January, 1869.

[2] Hopkinson, *On the Theory of Alternative Currents, Particularly in Reference to Two Alternate Current Machines Connected to the Same Circuit,* Journal of the Society of Telegraph Engineers, 1884, p. 496, Vol. XIII. See also the paper on *The Alternate Current Machine as a Motor,* by Grylls-Adams, presented at the same meeting.

These machines could be easily coupled in parallel, as generators, by bringing them to the same speed before coupling them. The belt being then removed from one of them, it was observed that it continued to run synchronously by the action of the current of its neighbors, and that it could even develop a considerable amount of power, as measured by a friction brake, before losing its synchronism. These experiments were repeated a few years later by Mordey, on a much larger scale, with machines of low inductance presenting a much greater stability of operation and driven by independent prime movers. He was thus able to demonstrate the synchronizing power of the alternators on the motors or engines driving them, and even to cause one of the latter, with the power shut off, to be dragged by one of the alternators which it was driving. This gives the key to the principles involved in parallel working. He also showed, later, the possibility of accomplishing this coupling with machines connected by means of long lines of high resistance.

Synchronous singlephase motors have two great disadvantages: they are not self-exciting, and they cannot start alone, even without load. Zipernowsky was the first to overcome this difficulty by the expedient of adding a commutator to his motors, which enables them to be started with alternating current, and, after they have attained synchronous speed, to be excited by a portion of the alternating current which they consume.

These motors, manufactured by the firm of Ganz & Co., had a certain vogue, in consequence of the tests made of them at Frankfort, in 1899, by a Technical Commission. The efficiency was satisfactory, being 77 per cent for motors of 15 H.P., and 86 per cent for motors of 30 H.P.[1] This system is no longer used at the present time, except for small powers (1 to 5 H.P.).

When the invention of *polyphase currents* became known, it led naturally to the idea of utilizing them for the transmission of power between two synchronous machines of the same type. Bradley, in America, and Haselwander, in Germany, took out patents, as early as 1887, the former on a two-phase synchronous motor, and the latter on a three-phase synchronous motor. In both cases the motor was produced by making taps on a Gramme ring and connecting these with insulated rings mounted on the armature-shaft. Non-synchronous motors were only invented in the following year, by Ferraris and by Tesla.

[1] La Lumière Electrique, Vol. XXXII (1889), p. 328.

It was in 1891, at the Frankfort Exhibition, that synchronous polyphase motors, with flat ring or Gramme ring armatures, constructed by the firms of Schuckert and of Lahmeyer, and of sizes as large as 50 H.P., were seen, for the first time, alongside the first non-synchronous motors of Dolivo-Dobrowolski and of Brown.

Since that time the principle of synchronous motor operation has been extended to ordinary polyphase alternators, with any winding whatever, stationary or movable, with poles alternating or not; and the only improvements that have been made have been in the means of their excitation or of starting them.

In 1890–1 Swinburne had had the idea of producing, by means of over-excited synchronous motors, the relatively considerable magnetizing current consumed by his "hedgehog" transformers. Under these conditions the motor played the same role as a condenser. This very interesting property was utilized industrially in 1893 in the Bulach-Oerlikon power-transmission installation, at the suggestion of Dolivo-Dobrowolski; and it was also used in the distributing system at Bockenheim, by Lahmeyer, to compensate for the wattless current of the non-synchronous motors, and even for raising the voltage of the generators. This method has come into extensive use at the present time, especially in the United States.

It constitutes an advantage in favor of synchronous motors, and it has prevented them from disappearing from the commercial field, after the development of motors of revolving field type, which are quite superior to them from other standpoints, notably in regard to starting power. It is desirable to utilize, wherever possible, both these types of motors in distributions of mechanical power. The synchronous polyphase motors are especially useful. They start readily without load, are self-exciting, and have an efficiency equal to that of alternators.

The principal objection to synchronous motors (which is, however, in certain cases, an advantage) is the impossibility of modifying their speed of rotation (without modifying that of the generators). Revolving field motors are superior to them in that respect, but this advantage is obtained at the expense of efficiency. Commutator motors, of which no mention will be made here, are alone capable of running at all speeds, the same as direct current motors; and for this reason they present a certain amount of interest.

PART I

SYNCHRONOUS MOTORS

CHAPTER I

GENERAL PRINCIPLES OF SYNCHRONOUS MOTORS

Construction. Synchronous motors have the same construction as alternators. The few special features relative to the production of the direct current necessary for their excitation will be treated separately, later. It will be assumed that the reader is already familiar with the general details of construction of alternators.

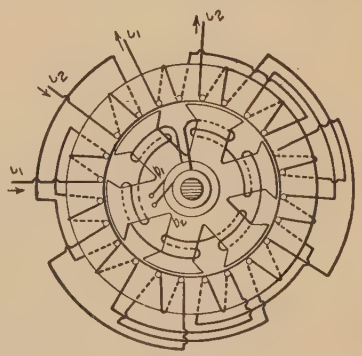


FIG. 1.

There are motors having movable armatures and stationary fields, or vice versa, and also motors with revolving iron masses in which all the windings are stationary. These machines are similar to the generators of the same types; for example, Fig. 1 indicates, diagrammatically, the principle of construction of a two-phase synchronous motor, with a ring armature and movable fields, receiving an exciting current through the brushes b_1 and b_2.

These motors are designed like generators, the essential condition to be fulfilled being to have a low armature-reaction and powerful inducing fields, in order to obtain good stability.

Although it is more difficult to increase the number of poles for small powers than for large powers, the construction of small

synchronous motors for ordinary frequencies (40 to 60 cycles) presents no special difficulties, if the speeds corresponding to these frequencies are not objectionable, because these speeds are perfectly allowable so far as centrifugal force is concerned.

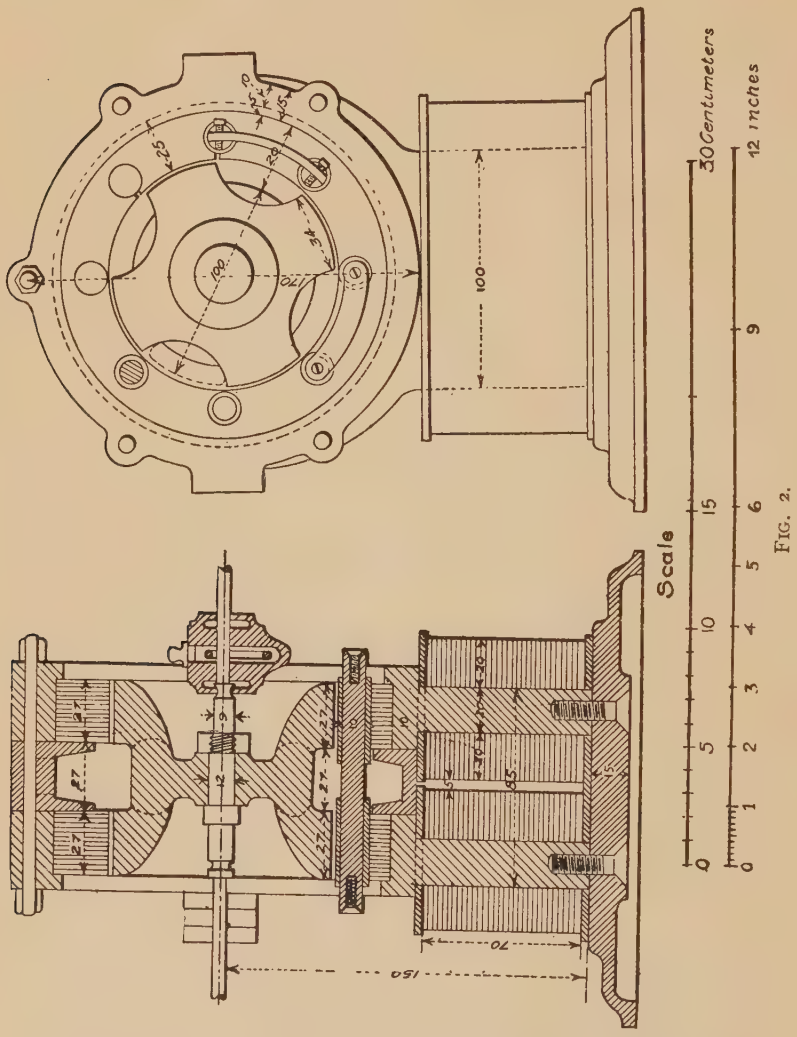


FIG. 2.

On the other hand, in the construction of small synchronous motors to run at low angular velocities, it is extremely difficult to find space for the numerous conductors and for the exciting or field coils, which

must produce as many ampere-turns as in the case of large motors. For this reason non-synchronous motors are more convenient for low rotative speeds.

The author has been able, however, to produce motors of low power (a few hundred watts) which have moving iron and have a very high number of poles (as many as 50 for example), by utilizing induction-type excitation, the magnetic circuit being closed exteriorly, as shown in Fig. 2, in such a way as to allow all the space needed for the exciting coils.

These coils can then be replaced by permanent magnets, thus producing motors which run without excitation, at speeds sufficiently

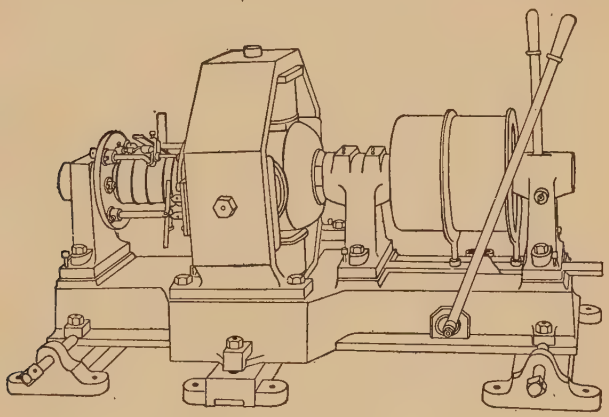


FIG. 3.

low to be synchronized by hand, and which can render useful service, in certain applications, such as for oscillographs. For this purpose the author preferably employs a small horseshoe magnet that is made to revolve around a stationary armature having a number of poles which is a multiple of 6. It is possible, in this way, to obtain very stable synchronous rotation of a revolving mirror without expending more than $1\frac{1}{2}$ to 2 watts.

Several firms made a specialty of synchronous motors, at an early date, among which we may mention La Société l'Eclairage Electrique in France, and the Fort Wayne Company in America.

One form of motor constructed in France by the Société l'Eclairage Electrique (Figs. 3 and 4), is constructed for polyphase currents or for single-phase currents, for powers ranging from 1 to 130 H.P. The table on page 5 gives the principal data referring to these matters.

The efficiencies of the three-phase motors are a little higher than those given for the single-phase motors. The horse-powers given in this table correspond to a frequency of 42 periods, but these motors can be also used at frequencies between 40 and 60 periods, and their power then increases with the frequency.

As the table shows, types Nos. 14 to 30 are made with 4 poles, self-exciting. For higher powers, the number of poles increases, and the excitation is obtained by means of a small direct current exciter mounted on the same base. Above type 90 the armature is stationary and the fields turn inside. The fields are of mild cast steel, the armatures being slotted.

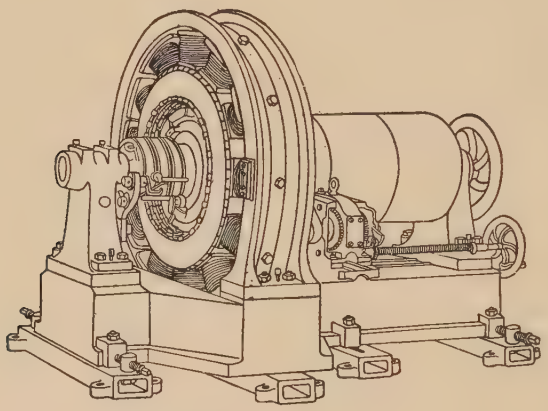


FIG. 4.

As an example of these large motors may be cited several from 50 to 100 H.P., giving the best of results on the power-transmission system around Grenoble, notably at Voiron, a distance of 30 kilometers from the generating station. Their efficiency is from 90 to 92 per cent. One of these motors even works in parallel with a steam-engine of the same power, and it compensates for the variation of angular velocity of the engine as it passes the dead centers.

All these motors are provided with a clutch and with an idle pulley for starting, as will be explained later. When running, they can undergo considerable variations of load without falling out of step.

Attention should also be called to another interesting type of synchronous motor, the Maurice Leblanc type, which is characterized by the addition of closed circuits in the pole-pieces to insure a perfect damping of oscillations, as will be seen later.

SINGLE-PHASE AND THREE-PHASE SYNCHRONOUS MOTORS OF THE "SOCIETE L'ECLAIRAGE ELECTRIQUE"

Type.	No. of Poles.	Revolutions per Minute for 42 Periods.	Pulley.		Approximate Weight. (Kilogs.)	Single Phase.				Three Phase.		
			Diameter, Cm.	Face, Cm.		H.P. Developed at Pulley for 42 Periods.	Efficiency.	Amperes with 110 Volts.	Kw. of Transformer.	H.P. Developed at Pulley for 42 Periods.	Amperes with 110 Volts.	Corresponding Transformers, Kw.
14	4	1260	14	6.5	125	1	72	9	1	1½	9	1½
16	4	1260	16	7.5	150	1½	75	14	1½	2½	14	2½
18	4	1260	18	8.5	210	2½	78	23	2½	3½	20	4
20	4	1260	20	9.5	270	3½	80	32	4	5	28	5
22	4	1260	22	10.5	375	5	85	45	5	7	37	7½
24	4	1260	24	11	500	7	87	65	7½	9	48	10
26	4	1260	26	12	625	9	88	80	10	12	65	12½
28	4	1260	28	13	775	12	89	110	12½	16	85	15
30	4	1260	30	14	900	16	90	145	15	22	110	25
42	6	840	42	17	1350	22	90	200	25	30	155	30
56	8	630	56	22	1750	30	90	270	30	40	210	40
70	10	504	70	29	2500	40	91	360	40	50	260	50
75	10	504	75	33	3200	50	91	450	50	65	340	60
90	12	420	90	24	3800	60	91	550	60	80	410	75
90a	12	420	90	29	5200	75	91	675	75	100	500	100
120	16	316	120	38	6800	100	91	900	100	130	650	125

Experimental Properties. As already stated, it is an experimental fact that synchronous motors can only be run after they have first been brought to synchronous speed by some external means. As will be seen later the motors themselves can run indifferently in either direction, but the direction of rotation selected in bringing them to synchronism should be that which is suitable for the brushes of the motor or of the exciter.

Case of Equal Electromotive Forces. Let us suppose that the electromotive forces of the generator and motor are equal, and, to simplify matters, let us take, as generator and motor, two machines whose excitations are regulated to approximately the same value. Let the two machines be driven by belts (Fig. 5); and, when they have attained the same speed, let them be coupled together (experiment of Hopkinson and Grylls-Adams). Let us, moreover, make use of an apparatus of the kind described in Chapter VII, whereby the difference of phase between the two machines may be determained.

It will be noted, in the first place, that as soon as the two machines are brought to the same speed, the current which passes from the one

to the other practically disappears. Moreover, the "phases are identical," i.e., the poles of like polarity pass at the same time in front of the corresponding portions of the two armatures.

The *induced* E.M.F.'s between the corresponding terminals *a*, *b*, and *A*, *B*, are therefore in unison. If we measure them, on the contrary, in the direction in which they appear, by following the circuit *ab*, *BA*, it will be found that they are exactly opposed to each other.

Let us now suppose the belt of one of the two machines to be removed. This machine will continue to turn at the same speed, but it gives indication of a certain very slight delay or falling behind, technically termed "lag," with respect to the other machine. Moreover, the current in the circuit now becomes appreciable.

If a brake be placed on the pulley and if the load be gradually increased in such a way as to increase the mechanical power produced

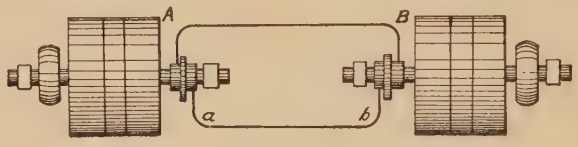


FIG. 5.

by the motor, the "lag" of the motor will be seen to increase at the same time as the current.

When this lag approaches a quarter of a period, i.e., half an interpolar space, the machine slows up all at once and stops as if held fast by the brake. We then say that it is "stalled," or "out of synchronism," or "out of step." The current in the circuit rises to a very high value as soon as the machine falls out of synchronism; and it becomes approximately equal to the short-circuit current in the circuit when the machine is stopped. In order to avoid accidents, it is necessary to introduce fuses in the circuit, or to provide some automatic disconnecting device, which will prevent the excessive load.

It is seen that what characterizes the synchronous motor is the increase of phase-lag with the load and the "stalling" of the motor or its falling out of step beyond a certain maximum load.

In a good motor, the limiting load should amount to at least 1.5 times, or, better, to twice the normal load. This limit is guaranteed by most makers of synchronous motors.

On the other hand, if the motor is run by a belt in such a way as to give it a "lead in phase," with respect to the machine or the circuit

which supplies it with current, it can be found, by wattmeter measurements, that this power changes in *sign*, i.e., the motor acts as a brake and returns energy to the circuit instead of receiving it therefrom.

The phenomena become more complicated still on varying the E.M.F. of the motor or of the generator.

Case of Unequal Electromotive Forces. An interesting and characteristic property of synchronous alternating current motors, and which distinguishes them absolutely from direct current motors or from alternating current motors having commutators, is that they can be excited so as to give a voltage greater than that of the supply-circuit. For example, it is possible to feed, from a 110-volt circuit, a motor which, driven by belt at the same speed, produces an E.M.F. of 120 to 150 volts at its terminals. But, if the E.M.F.'s are thus unequal, the current passing between the generator and the motor, when the latter is running without load, can, instead of being inappreciable, attain a considerable value. Likewise, when the motor is running with load, the current is greater than that which corresponds to the work to be done. The same effects are produced when, instead of giving to the motor an excessive excitation, it is given an insufficient induced E.M.F. It is then observed, if the machines are alike, that the potential difference at the terminals assumes a third value, which is the mean of the two E.M.F.'s involved.

In both cases, the greater the inequality between the two E.M.F.'s the more the current measured will increase, by the change of excitation. If we plot a diagram, taking, as abscissæ, the values of the excitation of one of the machines, and, as ordinates, the current passing through the circuit, the curve of variation of the latter, as a function of the former, has the form of a *V* more or less rounded at the bottom (Fig. 6). This form persists, although it may be less marked, when a constant load is placed on the brake. At the same time that the current increases, by reason of an inequality of the E.M.F.'s, it can be noted, by means of an apparatus for indicating phase-difference, that the current undergoes a change of phase, either forward or backward, with respect to the E.M.F. of the motor. This can be expressed in another way by saying that the machine consumes or produces *wattless* current,[1] i.e., current which is "out of phase," being $\frac{\pi}{2}$ behind or ahead of the E.M.F. This "wattless" current, which has the effect of increasing the "apparent" current, is thus named because it produces no work, the load on the brake remaining constant, by hypothesis.

[1] See note at bottom of page 42.

The effect of this wattless current is, therefore, to produce, in the motor, a supplemental positive or negative E.M.F., which adds itself to its own induced E.M.F., in such a way as to produce, at the terminals, a difference of potential equal to that of the generator. We can conclude from this, without further argument, that when the motor generates an E.M.F. which is too low, the current of the generator tends to over-excite it and that, in the contrary case, it tends to under-excite it.

The action of the current on the generator itself produces inverse effects.

The effects are more complicated still when resistances or inductances are added in the circuit between the machines, with the general

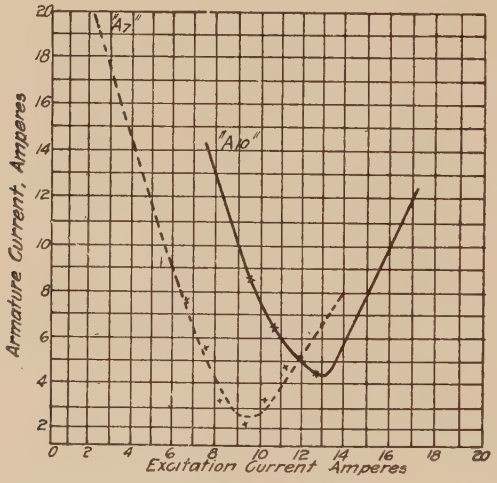


FIG. 6.

effect of lowering the voltage. Synchronous operation remains possible, nevertheless, even when the resistance attains high values.[1]

When the circuit includes reactance it is observed that, by over-exciting the motor, the voltage will be raised at its terminals, and even at the terminals of the generator, so as to attain values which are higher than the E.M.F. of the generator, measured with open circuit. On the contrary, by under-exciting the motor, it is possible to produce increasing and rapidly exaggerated voltage-drop along the whole line.

[1] When the resistance is high, say 75 per cent, it may be necessary to add some external reactance, so as to cause the current to lag more behind the cross E.M.F.—(E. J. B.)

An over-excited synchronous motor, connected to the terminals of an alternator having excessive armature-reaction, can even replace the excitation of the latter. It is observed, indeed, that on suppressing this excitation, the generator continues to run the motor, and furnishes the normal voltage at its terminals; but it can develop only little power. An over-excited motor thus produces an indirect self-excitation which is equivalent to that obtainable from a condenser. There is, in other respects, a complete analogy of effects between the two forms of apparatus.

These experimental results are much too complex to be studied more in detail here. They can be discussed more satisfactorily later, in connection with the theory of these motors and their applications.

Elementary Explanation of Polyphase Synchronous Motors. If we turn our attention, first, to polyphase synchronous motors, the explanation of the phenomena just described is made easy by the consideration of revolving magnetic fields.

For the sake of brevity we will adopt the terms "rotor" and "stator" to designate the movable and fixed portions, respectively, of the motor, in accordance with the terminology of Professor S. P. Thompson.

Let us take, as an example, a motor having two pairs of pole-pieces, in which the inductive circuit is of movable form (rotor) and the induced is circuit of stationary form (stator).

In the ordinary form of polyphase alternators, the rotor will consist of a crown of iron cores, with protruding poles excited by coils receiving direct current from a separate exciter. The stator, on the other hand, will consist of a circular core of laminated iron having some induced windings disposed in notches or slots in such a manner that the wires in the successive slots shall have alternating currents of different phase passing through them. If we suppose, for example, that we have a winding for four poles and for six phases (three slots per pole) such as is shown in Fig. 7, the wires in the six slots which cover two poles of the stator, as we follow along the periphery of the latter, will have, passing through them, six currents [1] which are out of phase with respect to each other by $\frac{1}{6}$ of a period, and which can be represented by the equations

$$i_1 = I_0 \sin \omega t;$$

$$i_2 = I_0 \sin \left(\omega t - \frac{2\pi}{6}\right);$$

[1] In reality, the six phases are supplied by three-phase currents only; the windings which are of exactly opposite phases being connected in series, with reversed connections.

$$i_3 = I_0 \sin\left(\omega t - 2\frac{2\pi}{6}\right);$$

$$i_4 = I_0 \sin\left(\omega t - 3\frac{2\pi}{6}\right);$$

$$i_5 = I_0 \sin\left(\omega t - 4\frac{2\pi}{6}\right);$$

$$i_6 = I_0 \sin\left(\omega t - 5\frac{2\pi}{6}\right);$$

in which $\omega = \frac{2\pi}{T}$.

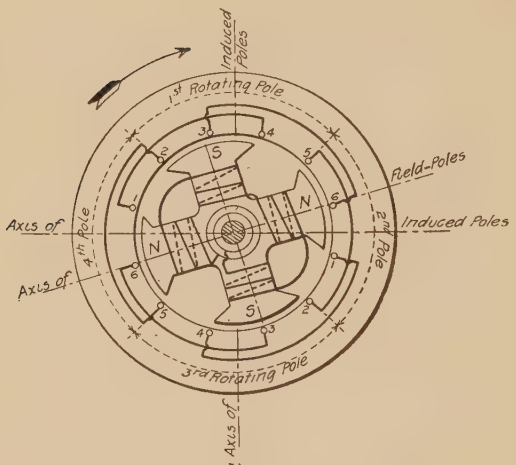


FIG. 7.

T being the common duration of the period of the alternating currents considered, I_0 their common amplitude (i.e., maximum value), and i_1, i_2, i_3, i_4, i_5, i_6, being the currents in the slots 1, 2, 3, 4, 5, 6.

It will be seen that, at every one-sixth of a period, the currents in the stator resume the same values, but the latter are displaced one-sixth of the width of a double field (2 poles) in the direction in which the currents succeed each other along the stator. Therefore, the axes radiating from the magnetic fields produced by the windings of the stator displace themselves around this stator with an angular velocity;

$$\alpha = \frac{\omega}{p} = \frac{2\pi}{pT},$$

corresponding to a number of turns $= \frac{60}{pT}$ per minute.

The magnetic strength of these fields can be considered practically constant inasmuch as it also resumes the same value at every one-sixth of a period (although it may, in the intervals, undergo slight variations which are dampened by the hysteresis and the eddy currents produced in the pole-pieces of the rotor). Therefore, even though the armature (stator) may be stationary, the result is the same as if it had revolving poles which attract or repel the poles of the field (rotor) and we can, henceforth, reason as if we were dealing with the attractions of two systems of magnets presenting the same number of poles which are alternately north and south in polarity. (Fig. 8.) The poles of unlike polarities, of the two systems, attract each other; the others repel each other. Therefore, when at rest, the poles of unlike polarity will always face each other. If the external magnets begin to rotate slowly, starting from rest, they will drag with them the stationary magnets, whose poles tend to remain opposite the poles of unlike polarity. (This result may be obtained by supplying the motor with current obtained from a generator which is started from rest and, consequently, gives polyphase currents of increasing frequency.)

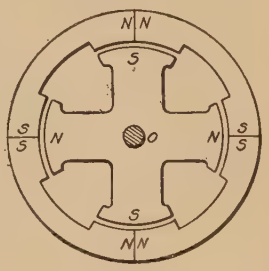


FIG. 8.

The attractions can only be concordant and continuous when the two systems turn at the same speed; which explains the necessity of *synchronism*. Otherwise, there would only be successive attractions and repulsions which would neutralize each other.

The *stability* of synchronous operation is also easily demonstrated by considering the moment of the motor-couple (i.e., the torque). If the poles of the rotor remain opposite the revolving poles of the stator, the attractions produced are directed radially and consequently they produce a motor-couple or torque which is equal to zero. If, on the contrary, for any reason whatever, the rotor loses or gains speed, some tangential attractions or repulsions will appear, whose resultants tend to bring back the opposite poles of the rotor into coincidence with the poles of unlike polarity of the stator, so long as the poles of the rotor remain near these, because the attractions of unlike poles and the repulsions of like poles act in the same direction; but if the difference in phase amounts to one interpolar space, the poles of like sign of the rotor and stator will be opposite each other, the motor-couple or torque

will become zero, and will then change sign if the difference in phase increases. By reason of the symmetrical construction of the motor the torque will have points of maximum and minimum value at equal distances between the points of zero-value, i.e., in the positions where the poles of the rotor are midway between the poles of the stator.

To sum up, taking as abscissæ the *difference* of phase l of the poles of the rotor, expressed in terms of the interpolar space L, and taking as ordinates the *torque* C, the representative curve will take the form shown herewith (Fig. 9), the magnetic strength at the armature-poles being supposed constant, i.e., assuming the currents that produce this magnetic flux to be constant.

The machine will have stable operation for the difference of phase comprised between the two maximum points B and C (the positive

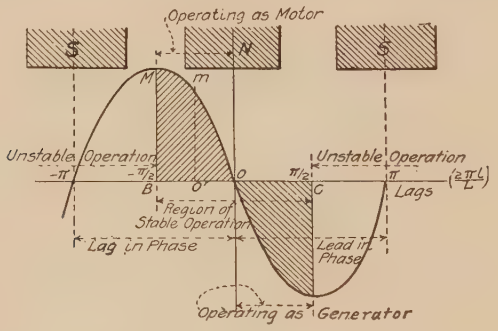


Fig. 9.

maximum being due to a *lag*, and the negative maximum being due to a *lead*), because every accidental advance (or lag) is corrected of itself by a contrary variation of the torque. If the rotor lags, for example, in consequence of a passive mechanical resistance, the increase in torque compensates for this resistance.

When the motor is running without load, its condition corresponds to the position O, at which there is no phase-difference. When the motor is loaded, i.e., whenever mechanical resistance is applied to the shaft, the position of the poles of the rotor changes in phase and comes to a point O', such that the couple $O'm$ shall balance the resisting couple. If the resisting couple is greater than the maximum torque, the machine can no longer run; and even for positions of m which are a little below M, the machine will fall out of step, in consequence of unavoidable oscillations.

On the other hand, to obtain a difference of phase *ahead*, between O and C, it is necessary to apply to the shaft an effort in the direction of rotation, i.e., it is necessary to apply to the shaft a certain amount of propelling power which must, evidently, be transformed into electrical energy.

The armature current has been supposed constant. In practice, it is the *voltage* of the supply-circuit which is constant, at its terminals; and the question is thus complicated by the spontateous variation of the current with the variation in phase-difference. This variation, itself, depends on the ratio of the induced E.M.F. of the motor to the voltage applied at its terminals.

In fact, as they displace themselves before the armature at the synchronous speed, the poles of the inducing field induce in the windings counter E.M.F.'s. which are of the same order and magnitude as the voltage at the terminals. If, mentally, we locate the E.M.F.'s. in the wires which are placed in the slots, we perceive readily that the E.M.F. in each slot varies periodically and passes through a maximum at the moment when the middle of a revolving pole comes in line with the slot. The axes of maximum *values* of the induced E.M.F.'s therefore coincide with the axes of the inducing poles, and revolve with them. If the currents were in phase with the E.M.F.'s, they would give rise to revolving fields whose axes would be retarded in phase by an amount equal to half the width of a pole, since each conductor forms a coil with a conductor similarly placed, but in the contrary direction, under the next pole.

But we must take into account the voltage at the terminals, with which the induced E.M.F. combines, and also the self-induction of the machine, which throws the current out of phase by a quarter of a period, i.e., half an interpolar space. Therefore the question can only be treated with precision by calculation, as will be seen later. From the *qualitative* point of view, the result differs but slightly from the preceding result. The form of the curve of torque remains analogous to that of Fig. 9, but it is no longer so symmetrical, and the lags OC and OB, which determine the limits of stability, take a value, γ, which is a little lower than $\frac{\pi}{2}$, and which is defined by the relation

$$\tan \gamma = \frac{\omega L}{R};$$

ωL and R being, respectively, the reactance and the resistance of the armature circuit.

It will be observed that, when the current is in phase with the induced E.M.F., the magneto-motive force of the armature-reaction has no action on the inducing field, and can only produce a transverse distortion of the field; while, on the other hand, when the current is out of phase one-fourth of a period in advance, or behind, the M.M.F. is directly opposed to, or coincident with, that of the field. With regard to the sign, it can be easily seen that a phase-difference of $\frac{\pi}{2}$ in advance of the induced E.M.F. produces a magnetizing reaction which is the same as in a generator, and that a phase-difference of $\frac{\pi}{2}$ behind produces a demagnetizing reaction. It must not be forgotten, however, that the internal E.M.F. is opposed to the external E.M.F. and that the lag and lead are therefore transposed if they are referred to the latter.

Elementary Explanation of Single-Phase Synchronous Motors. The phenomena are more complicated in single-phase motors. The same explanation may nevertheless be retained by means of a simple artifice of reasoning.

The coils of the armature-winding, being excited by a single alternating current (Fig. 10), produce poles which no longer revolve, but are stationary. These poles are alternately positive and negative, and have a magnetic flux which varies periodically like the current that produces it. There is, therefore, no tendency to rotation; and the motor can only be put in operation by external means, as already seen. But we may suppose it brought previously to synchronism.

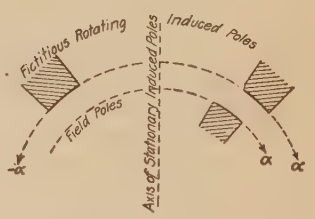


FIG. 10.

M. Maurice Leblanc has enunciated a theorem which is an electrical analogue of the following well-known optical theorem: A vibration of luminiferous ether polarized rectilinearly may be replaced by two circularly polarized vibrations of contrary sign having the same frequency and having amplitudes equal to the half amplitude of the rectilinear vibration.

According to M. Leblanc's theorem, an alternating stationary magnetic field may be replaced by two fields revolving in contrary directions, each having a flux half as large, and having equal velocities, such that they advance a distance equal to that of two poles during a

single period. The fields turning in the same direction as the inducing poles will have the same angular velocity α, as the latter, and will drag them in very much the same way as in polyphase motors. On the other hand, the fields which turn in the opposite direction will have a relative velocity, 2α, which is contrary to and double that of the field-poles, so that their attracting or repelling actions, since they succeed each other in inverse directions, will produce no resisting torque. These reversed revolving fields will give rise only to supplemental losses by hysteresis and by eddy currents.

By this simple analysis (which is, in reality, only approximate) the operation of single-phase motors can, it is seen, be discussed and explained in the same way as that of polyphase motors.

It has been supposed, in what precedes, that the armature is stationary and the field movable. In the contrary case the explanation is the same if we consider the relative velocities of the two portions, but the fields displace themselves only with respect to the armature and therefore remain stationary in space the same as the field-poles.

Equations of Synchronous Motors. Analytical Theory. We have just examined the phenomena of synchronous motors from a physical point of view. We shall now represent them analytically, according to the theory first expounded by Dr. J. Hopkinson, but with a few modifications in form. We shall suppose with him that the E.M.F.'s and currents follow the sinusoidal law, and that the reactances of the machine are constant.

Let us suppose, then, a single-phase A.C. generator and motor, defined by their induced E.M.F.'s, their resistances, and their mean inductances, which are all supposed constant.

Let T = the duration of the period;

$\omega = \frac{2\pi}{T}$ = the speed of pulsation of the currents;

e_1 and e_2 = the instantaneous values of the generator and motor E.M.F.'s respectively, at the instant t;

E_1 and E_2 = the effective values equal to the amplitudes of the sine-functions, i.e., the maximum value, divided by $\sqrt{2}$;

$\frac{t_0}{T}$ = the phase-difference between e_1 and e_2;

θ = the angle of lag (phase-difference) corresponding to $\theta = 2\pi\frac{t_0}{T}$;

R and L = the resistance and inductance, respectively, of the *total circuit*, of the two machines;

i = the instantaneous value of the current;

I = the effective value of the current, equal to the maximum value divided by $\sqrt{2}$.

Let us suppose the conditions of stability to be unknown and let us seek to ascertain how two alternators connected in series will operate.

The two sine-functions of the E.M.F. represented by the curves e_1 and e_2 in Fig. 11, may be formulated by the equations,

$$e_1 = E_1\sqrt{2}\,\sin\left(\omega t + \frac{\theta}{2}\right);$$

$$e_2 = E_2\sqrt{2}\,\sin\left(\omega t - \frac{\theta}{2}\right);$$

in which θ designates the angular distance between the actual position of e_1 and the position of opposition of e_2.

The E.M.F. which is acting in the circuit is equal to the algebraical sum of the opposing E.M.F.'s.

$$\varepsilon = e_1 + e_2 = E_1\sqrt{2}\,\sin\left(\omega t + \frac{\theta}{2}\right) - E_2\sqrt{2}\,\sin\left(\omega t - \frac{\theta}{2}\right).$$

From this the current, i, may be deduced, by the well-known differential equation,

$$Ri + L\frac{di}{dt} = \varepsilon = E_1\sqrt{2}\,\sin\left(\omega t + \frac{\theta}{2}\right) - E_2\sqrt{2}\,\sin\left(\omega t - \frac{\theta}{2}\right). \qquad (A)$$

In this equation let $i = X\,\sin\,\omega t + Y\,\cos\,\omega t$.

If this value be substituted in the equation, the values of X and Y can be determined by making the coefficients of the sine-terms and of the cosine-terms successively equal to zero. We can then obtain, by differentiation, substitution, etc.,[1] the following value for i:

$$i = \frac{R}{R^2+\omega^2L^2}\left\{\begin{array}{l} E_1\sqrt{2}\left[\sin\left(\omega t + \frac{\theta}{2}\right) - \frac{\omega L}{R}\cos\left(\omega t + \frac{\theta}{2}\right)\right] \\ -E_2\sqrt{2}\left[\sin\left(\omega t - \frac{\theta}{2}\right) - \frac{\omega L}{R}\cos\left(\omega t - \frac{\theta}{2}\right)\right] \end{array}\right\}. \qquad (B)$$

[1] See Appendix *A*.

This may also be written,

$$i=\sqrt{\frac{E_1^2+E_2^2-2E_1E_2\cos\theta}{R^2+\omega^2L^2}}\sin(\omega t-\beta-\gamma);$$

wherein

$$\tan\beta=\frac{E_2-E_1}{E_2+E_1}\tan\theta;$$

$$\tan\gamma=\frac{\omega L}{R}.$$

In this equation β = the phase-angle of the resultant E.M.F. and γ = the supplemental phase-difference of the current measured from this E.M.F.

In the simple particular case where $E_1=E_2$ this expression reduces to

$$i=2E\sqrt{2}\frac{\sin\frac{\theta}{2}}{R^2+\omega^2L^2}[R\cos\omega t+\omega L\sin\omega t];$$

or, since

$$\frac{R}{\sqrt{R^2+\omega^2L^2}}=\cos\gamma;\quad\frac{\omega L}{\sqrt{R+\omega^2L^2}}=\sin\gamma;$$

we will have

$$i=\frac{2E\sqrt{2}\sin\frac{\theta}{2}\cos(\omega t-\gamma)}{\sqrt{R^2+\omega^2L^2}};$$

i.e., the current will have the effective value

$$I_{\text{eff.}}=2E\sqrt{2}\sin\left(\frac{\theta}{2}\right)\frac{1}{\sqrt{R^2+\omega^2L^2}};$$

and will be out of phase by the angle γ with respect to the resultant E.M.F., which is itself out of phase $\frac{\pi}{2}$ with respect to the mean of e_1 and e_2. This result is easily interpreted in Fig. 11, by drawing the resultant curve e_1+e_2, obtained by taking the difference of the ordinates of the first two curves. It will be seen that the curve has actually a phase-difference equal to $\frac{\pi}{2}$ with regard to the mean of e_1 and e_2, and

that it increases with the phase-difference of e_2 with respect to e_1. In consequence of the lag, γ, of the current, measured with respect to this resultant E.M.F. $\left(\text{when } \gamma \text{ is near } \frac{\pi}{2} \text{ in value}\right)$, it will be seen that the current is approximately in phase with this mean value; it would be completely so if there were no resistance-losses.

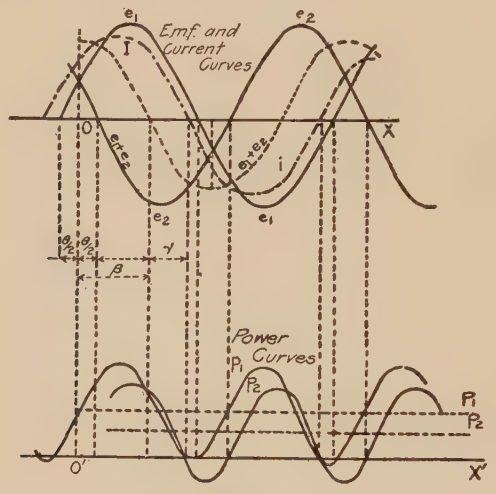


FIG. 11.

The power-outputs of the two machines will be obtained by multiplying the instantaneous current i by the E.M.F.'s. e_1 and e_2. For example, in the case where $E_1 = E_2$, we have

$$p_1 = \frac{4E^2}{\sqrt{R^2+\omega^2L^2}} \sin\frac{\theta}{2} \cos(\omega t - \gamma) \sin\left(\omega t + \frac{\theta}{2}\right)$$

$$= \frac{2E^2}{\sqrt{R^2+\omega^2L^2}} \sin\frac{\theta}{2}\left[\sin\left(\gamma + \frac{\theta}{2}\right) + \sin\left(2\omega t - \gamma + \frac{\theta}{2}\right)\right].$$

Likewise, on multiplying by $-e_2$, we will have

$$p_2 = \frac{+2E^2}{\sqrt{R^2+\omega^2L^2}} \sin\frac{\theta}{2}\left[\sin\left(\gamma - \frac{\theta}{2}\right) + \sin\left(2\omega t - \gamma - \frac{\theta}{2}\right)\right].$$

These equations show that the power is not constant, in either case, but pulsating, i.e., it presents variations of frequency $= 2T$, as represented in Fig. 11.

These variations constitute sine-functions having pulsations twice as rapid as those of the current, which have for their axes the horizontal lines (P_1, P_2,) corresponding to the *mean powers* given by the first terms within the brackets in the following equations:

$$P_1 = \frac{2E^2}{\sqrt{R^2+\omega^2L^2}} \sin\frac{\theta}{2} \sin\left(\gamma+\frac{\theta}{2}\right)$$

$$= \frac{E^2}{\sqrt{R^2+\omega^2L^2}}[-\cos(\gamma+\theta)+\cos\gamma];$$

$$P_2 = \frac{2E^2}{\sqrt{R^2+\omega^2L^2}} \sin\frac{\theta}{2} \sin\left(\gamma-\frac{\theta}{2}\right)$$

$$= \frac{E^2}{\sqrt{R^2+\omega^2L^2}}[-\cos\gamma+\cos(\gamma-\theta)].$$

The very small difference between P_1 and P_2 represents the loss by resistance (Joule effect). The axis of the curve P_1 is therefore a little more *above* the axis of zero power than the axis of symmetry of the curve P_2 is *below* it.

The torque could be obtained, in each case, by dividing the power by the angular velocity. These expressions show that the current increases with θ until θ equals π, but the torques, which equal zero so long as the lag θ=zero, increase with θ only until the value $\theta=\gamma$; and they then decrease.

Stability will, therefore, exist only with $\theta<\gamma$ having for its axis the exact opposition of E.M.F.'s.

The solution, in the case where E_1 is different from E_2, will be obtained in an analogous manner, by forming the products e_1i-e_2i; and it would still give pulsating values for p_1 and p_2; but, since these caculations are uselessly complicated, we will pass them by and turn to more simple methods.

Case of Symmetrical Polyphase Motors. In the case of polyphase motors the same considerations and equations remain applicable to each of the symmetrical circuits, if only care be taken to include, in the self-induction of these circuits, their mutual induction effects. By reason of symmetry itself, the result is obtained by a simple increase of the coefficient, L, in a ratio which depends on the number of phases in the machine.

The currents are, in general, approximately equal in effective values in the different circuits, if there are no defects in construction. It

is sufficient, therefore, to apply the reasoning to one of the circuits only, and to multiply the power by the number of circuits.

It should be noted, moreover, that, in these motors, the pulsations of the power derived from the different circuits, occurring at different intervals, compensate each other, from the standpoint of the total power, which becomes constant. This is easily shown by taking the sum of the powers. For example, in a three-phase motor, the pulsations at the instant $z\omega t$ will have the form

$$A \sin\left(2\omega t - \gamma - \frac{\theta}{2}\right);$$

$$A \sin\left(2\omega t - \gamma - \frac{\theta}{2} - \frac{2\pi}{3}\right);$$

$$A \sin\left(2\omega t - \gamma - \frac{\theta}{2} - \frac{4\pi}{3}\right).$$

It is known that the sum of the sines of three angles differing from each other by 120° is identically equal to zero. Therefore the "resultant" pulsation is equal to zero. The same thing would be true for any number whatever of equidistant and symmetrical phases.

The result is that the torque is constant (to a sufficient degree of approximation for this theory, which neglects the higher harmonics of the field-distortions), whereas, in a single-phase motor, it undergoes heavy periodical variations. In the latter case, the inertia of the armature plays the rôle of a flywheel storing and restoring energy twice during each period; but if the inertia is insufficient, the velocity of the armature will experience slight variations which will greatly interfere with the stability of operation. Polyphase motors are, in this respect, superior to single-phase motors; they are also superior to them in being lighter for a given output (less weight per kilo-volt-ampere) and also in having higher efficiency.

Graphical Representation of Operative Conditions. Blakesley's Method. Mr. Blakesley was the first to apply Fresnel's method of vectors to the study of alternating currents.[1]

The following principles constitute the basis of Fresnel's method:

1. Any sinusoidal function can be represented in magnitude and in phase by a vector, or a segment of a right line, whose length is proportional to the amplitude of the function and whose phase is measured by an angle reckoned from some other vector serving as a point of origin.

[1] Blakesley, *Alternating Currents of Electricity*, 1885

2. The addition or subtraction of sinusoidal functions may be made on the graph by a geometrical addition or subtraction of the vectors of these functions.

3. The mean product of two functions is equal to the work done by one of the vectors on the other, i.e. it is equal to the area of the triangle constructed on vectors which are equal to the effective values of the variables, or to half this area when the triangle is constructed with the amplitudes of the sinusoidal functions.

This method, with which the reader is supposed to be familiar, has been applied by Blakesley to the problem of synchronous motors in the following manner:

Let us represent the amplitudes of the E.M.F.'s. of the generator and of the motor, $E_1\sqrt{2}$, and $E_2\sqrt{2}$, respectively, by means of two vectors, OA and OB, (Fig. 12), having between them the angle $\pi+\theta$. These E.M.F.'s are approximately opposed to each other, as we have seen in what precedes.

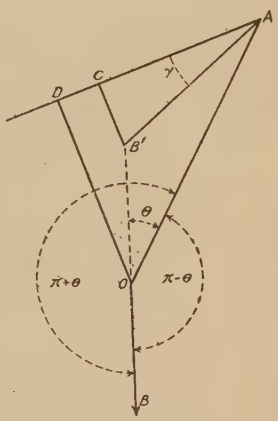


FIG. 12.

Let OB' be equal but opposed to OB. The E.M.F. $\varepsilon\sqrt{2}$, which is the resultant of $E_1\sqrt{2}$ and $E_2\sqrt{2}$, will be represented by the vector $B'A$ which is equal to the resultant of OA and OB.

Let us again designate by γ the phase-angle between the current and the resultant E.M.F. defined by the relation

$$\tan\gamma=\frac{\omega L}{R},$$

and let this angle be drawn with respect to the point A. The direction CA will represent the current which is in phase. Let CA be the projection of $B'A$. We will have

$$\overline{CA}=\overline{B'A}\cos\gamma=\varepsilon\sqrt{2}\cos\gamma=\frac{\varepsilon\sqrt{2}}{\sqrt{R^2+\omega^2L^2}}\times R,$$

or

$$I\sqrt{2}=\frac{\overline{CA}}{R}.$$

The length of CA is therefore proportional to I.

The power supplied by the generator to the motor will be

$$P_1 = E_1 I \cos OAD$$
$$= \frac{\overline{DA} \times \overline{CA}}{2R}.$$

In like manner, the electric power consumed by the motor will be

$$P_2 = \frac{\overline{DC} \times \overline{CA}}{2R}.$$

The power lost by ohmic resistance (Joule effect) is equal to the difference, or to

$$\frac{CA \times CA}{2R}.$$

The diagram therefore determines, by the measurement of simple lines or areas, the current strength and the power of each of the two machines, for each value of the phase-angle.

Let us suppose that the vector OA (Fig. 13) is stationary, and that

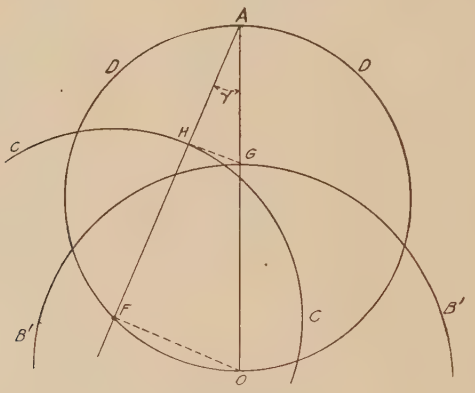


FIG. 13.

the phase-angle, θ, varies. The point B' will describe a circle of radius OB'; the point D describes a circle constructed on OA as a diameter; finally, it is easily seen that the point C describes a third circle having for its center and for its radius the projections of O and of OG on a right line, AF, making the angle γ with OA. These three circles having been drawn, the conditions of operation can be followed on the diagram. Mr. Blakesley has thus determined the limits of stability and the maximum electrical efficiency.

But even for easy questions the use of this diagram is too complicated; the same thing is true of the diagrams devised by Kapp,[1] after Blakesley, and based on the same principles. The author has also given[2] a more convenient method of representation of power by circles which cut off segments directly from the lines OA and OB.

In all that follows, we will give preference to another graphical method, still more simple, which will be explained in Chapter II.

Equation of the Synchronous Motor by the Method of Complex Variables. Finally, to complete the review of the different methods of study proposed or used for the representation and study of the phenomena, attention should be given to the equations which translate the preceding diagram by the method of complex variables, already applied to alternating current problems, in the exponential form, by Oberbeck, Cornu, Chaperon, etc., and in the linear form by Kennelly Steinmetz, Guilbert, etc. It is merely an application of the ordinary geometrical representation of imaginary quantities.

We will employ the notation of Steinmetz modified by M. Guilbert (*La Lumière Electrique*, Vol. L, p. 451, and *l'Eclairage Electrique*, Vol. XIV, p. 69), i.e., by expressing the impedance as $(r+si)$ instead of $(r-si)$, the latter expression being scarcely logical, since the reactance acts in the same direction as a *resistance* to *reduce* the current. It thus becomes possible to retain the ordinary axes.[3]

(1) Let OX, OY (Fig. 14) represent two rectangular co-ordinate axes. Let OA be a vector representing the sinusoidal function whose projections are x and y. Geometrically, the vector A is defined by the two projections, and analytically it is represented by a single imaginary value

$$x+yj=(A\cos\theta)+(A\sin\theta)j,$$

in which j is the imaginary symbol $\sqrt{-1}$.

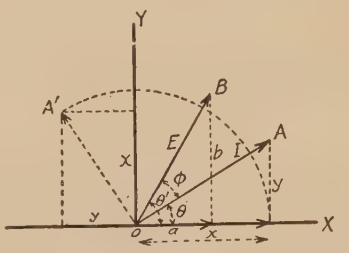


FIG. 14.

All the harmonic functions may be also represented by imaginary quantities.

[1] *Electrical Transmission of Energy.* London, Whittaker, 1892.

[2] *La Lumière Electrique*, Vol. XLV, 1892, p. 415, and *Bulletin de la Société des Electriciens*, 1893.

[3] Compare with C. P. Steinmetz, *Theory of the Synchronous Motor* (Transactions of the American Institute of Electrical Engineers, Oct. 17, 1894)

(2) These quantities are added or subtracted the same as real quantities; the resultant quantities indicate immediately, by their real and imaginary portions, the magnitude and the phase of the resultant vector.

(3) The multiplication of a complex quantity by a real quantity only changes the dimension of the vector, without changing its phase. On the contrary, the multiplication by an imaginary portion, such as jb, modifies not only the magnitude of the vector, which thereby becomes multiplied by b, but it also modifies its phase, which is then made to advance by the amount $\frac{\pi}{2}$. In fact, we have

$$j(x+jy)=-y+jx,$$

i.e., the vector OA is then replaced by the vector OA'.

(4) From this, it follows that the E.M.F. absorbed by an impedance, z, composed of a resistance, r, and of a reactance, s, in series, through which passes a current

$$i=x+jy,$$

is obtained, in magnitude and phase, by forming the product

$$zi=(r+js)(x+jy).$$

(5) The work done by an E.M.F. $\overline{OB}$ having the imaginary value

$$a+jb,$$

and a current $\overline{OA}$ having the imaginary value

$$x+jy,$$

is easily obtained by decomposing it into the work of each of the components a and b of this E.M.F. The first component, a, does no work on the component y of the current with which it is "*in quadrature*," but it is "*in phase*" with the component x; likewise b is in quadrature with x but in phase with y. The *power* required is therefore *the sum of the products* of the analogous *coefficients* of E and of I.[1] We will have

$$ax+by.$$

[1] Use can also be made of other equivalent rules. A rule is given by M. Janet (*Eclairage Electrique*, 18. Dec., 1897, p. 530) under the following form: To find the power, j is changed into $-j$ in either the expression for the current strength

Let us apply this notation to the generator and motor already considered. Let us direct the E.M.F. E_1 of the generator along the axis OX. E_2, lagging in phase by the angle θ, measured from the opposite direction, will have the form

$$E_2 (\cos \theta - j \sin \theta).$$

The imaginary impedance is

$$R + \omega L j.$$

The resultant E.M.F. being

$$E_1 - (E_2 \cos \theta - j E_2 \sin \theta),$$

the current will be obtained by taking the quotient

$$I = \frac{E_1 - E_2 \cos \theta + j E_2 \sin \theta}{R + \omega L i} = (E_1 - E_2 \cos \theta + j E_2 \sin \theta) \frac{R - \omega L j}{R^2 + \omega^2 L^2}.$$

Separating the real and imaginary portions, we have

$$I = \frac{1}{R^2 + \omega^2 L^2} \left\{ \begin{array}{c} R(E_1 - E_2 \cos \theta) + \omega L E_2 \sin \theta \\ + j[R E_2 \sin \theta - \omega L (E_1 - E_2 \cos \theta)] \end{array} \right\}.$$

This equation, when transformed into finite values, and taking z to represent the impedance, gives

$$Z^2 I^2 = [R(E_1 - E_2 \cos \theta) + \omega L E_2 \sin \theta]^2 + [R E_2 \sin \theta - \omega L (E_1 - E_2 \cos \theta)]^2$$

This equation defines the relation between I, E_1, E_2, and θ in a synchronous motor.

The mean power, P_1, is obtained by simply multiplying the real portion by E_1 thus:

$$P_1 = \frac{E_1}{R^2 + \omega^2 L^2} [R E_1 - E_2 (R \cos \theta - \omega L \sin \theta)]$$

$$= \frac{E_1 [E_1 \cos \gamma - E_2 \cos (\theta + \gamma)}{\sqrt{R^2 + \omega^2 L^2}}.$$

or the expression for the E.M.F. The product is calculated, and then only the real portion thereof is taken. M. Guilbert has also given the following rule (E. E., 10 Mar., 1900, p. 361): Change the sign of the variable which is out of phase and the real portion gives the power, with its sign.

As for P_2, it can, according to the rule already stated, be expressed as follows:

$$P_2=\frac{1}{R^2+\omega^2L^2}\left\{\begin{array}{c} E_2\cos\theta[R(E_1-E_2\cos\theta)+\omega LE_2\sin\theta] \\ -E_2\sin\theta[RE_2\sin\theta-\omega L(E_1-E_2\cos\theta)] \end{array}\right\}$$

$$=\frac{E_2}{\sqrt{R^2+\omega^2L^2}}[E_1\cos(\gamma-\theta)-E_2\cos\gamma].$$

This expression could have been written directly, by symmetry.

Such is the method of imaginaries for establishing the fundamental equations. It will be seen that the calculations thereby made are more simple than with the analytical method of Hopkinson, because they give, immediately, the general solution, and do not require any integration. In reality, however, complex variables only constitute an artifice for writing down the results of the graphical method. In place of *detailed reasonings* which are rendered more precise by means of diagrams, they substitute algebraical operations, which are effected mechanically, without benefit in helping the mind to understand the physical phenomena. In all that follows we will therefore adopt the graphical method.

Excitation of Synchronous Motors. Synchronous motors may be excited in three ways:

(1) As in the case of alternators, it is possible to use the current produced by a small exciter-dynamo mounted on the same shaft. The machine is put in operation by one of the methods indicated later, or else by running the exciter-dynamo as a motor, by means of current from a storage battery. This method of excitation is advantageously used for large motors.

(2) To avoid the complication of the exciter-dynamo, especially for small motors, the machine is often made self-exciting by means of commutated currents, by sending through the field-circuit a portion of the current obtained from the mains. The oldest method of commutating this current consisted in using a simple shell-commutator by which the current was reversed at each change of polarity. Such was the Ganz commutator shown diagrammatically in Fig. 15 for a six-pole motor. The inducing circuit connects with three segments p, p', p'' connected in parallel with one pole, and the other end of the circuit connects with three other segments n, n', n'', which are connected in parallel with the other pole. The alternating current to be commutated is

brought by two double brushes bb', b_1b', which can be shifted to a suitable extent by means of the collar supporting them, the object being to reduce sparking to a minimum. The use of double brushes

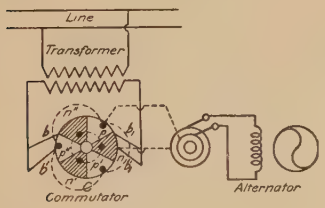


FIG. 15.

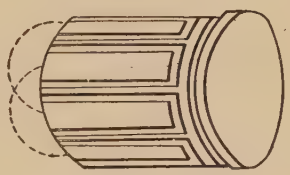


FIG. 16.

has for its object to reduce the sparking by short-circuiting the field windings at the time when the induced current is reversing; the extra current which is then produced in the exciting coils prolongs the current in the field-circuits and diminishes the undulations of the inducing magnetic flux. Nevertheless, these fluctuations are quite perceptible and interfere very much with the efficiency and the stability of the machine. These commutating arrangements can only be used for low voltages. For this reason, when the voltage at the terminals of the motor exceeds 110 volts, the commutator is supplied by the secondary winding of a transformer which is connected in multiple with the mains or else by a special low-voltage circuit wound on the same armature, as in the case of the old Westinghouse alternators (Fig. 17).

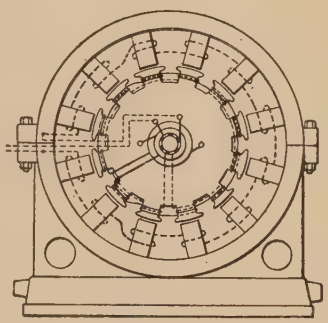


FIG. 17.

Commutated currents can, theoretically, be used for excitation either in series or in shunt (even compound) as indicated in the diagram of Figs. 18 and 19. The advantages and disadvantages of these various systems are the following: [1]

Series-excitation allows the use of large wires for the field-coils, and insures coicidence in phase between the currents of the inducing

[1] A more detailed study of these "commutated" excitations will be found in an article on synchronous motors by the author, in *La Lumière Electrique*, 1892, Vol. III, pp. 465, 466, etc.

and induced circuits during the operation of starting, which operation is thereby facilitated. It has, however, the great objection of making the induced E.M.F. vary with the load, whereas, according to theory, the most advantageous E.M.F. is fixed *a priori*, and should remain practically independent of the load. With series-excitation, strong wattless currents occur at very light and very heavy loads (which may, it is true, be reduced by shunting the field-coils, as shown in Fig. 19), and, besides, there is a lack of stability of operation at light loads

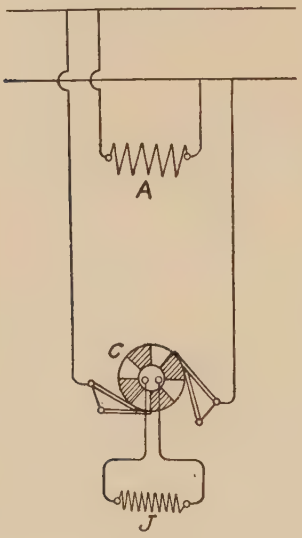


FIG. 18.

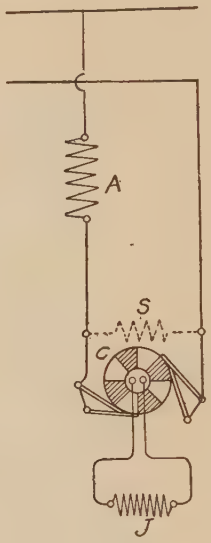


FIG. 19.

because the field is then too weak. Morevoer series-excitation causes very bad sparking at the brushes.

Shunt-excitation has the advantage of being constant and easily regulated; but it has another objection, that of necessitating change of position of the diameter of commutation with respect to the phase of the current to be commutated. With normal load, the position of the brushes can be easily regulated in such a way that the commutation may occur exactly when the alternating current to be commutated is passing through the zero-point; but if the load increases, the lagging of the armature behind the field increases, without being followed by the phase of the current to be commutated. The same thing occurs during any accidental oscillation of the speed of the armature (rotor).

Now, as Fig. 20 shows, the mean strength of the commutated current is proportional to the algebraical sum of two areas, one of which (S_1) is positive, and the other (S_2) is negative. The former diminishes and the latter increases, when the lag increases, the effect being a rapid decrease in the excitation. If we let t_0 = time at which the commutation occurs; if T = the period, and $2\pi\frac{t_0}{T}$ = the lag in question, the exciting ampere-turns will be given by the following equation:

$$Ni_{av.} = \frac{2N}{T}\int_{t_0}^{\frac{T}{2}+t_0} i_0 \sin\left(2\pi\frac{t}{T} - B\right)dt = \frac{2}{\pi}Ni_0 \cos 2\pi\frac{t_0}{T},$$

i.e., they are reduced in the proportion indicated by the factor $\cos\frac{2\pi t_0}{T}$ as compared with their value when the commutation occurs at the

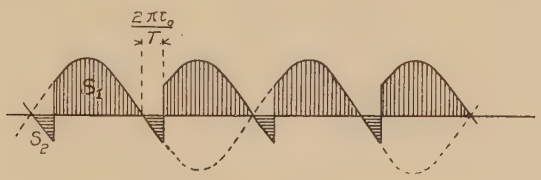


FIG 20.

proper moment of passing through zero (at which time the factor $\cos\frac{2\pi t_0}{T}$ equals unity). It is necessary, therefore, to vary the position of the brushes with the load. This cannot be done, however, during the fluctuations of load, and it is conceivable that this decrease of excitation under the influence of the lag may then greatly reduce the stability of operation. It appears, therefore, that the method of excitation by commutated currents is a process open to criticism.

Advantage might be taken of the use of the transformer in Fig. 21, as the author showed in 1892 (*Lumière Electrique*, loc. cit.), to compound the excitation in such a way as to cause the inducing flux to increase with the load, and thus produce an increase in stability. It would be sufficient, to this end, to wind on the same transformer a second primary coil with a large conductor through which the armature or rotor current passes.

(3) Another method of commutating the current, which is preferable to the preceding one, consists in combining with the armature-

winding a continuous-current commutator from which the exciting current is collected by means of two brushes. Such is the system of the Fort Wayne Company in America, and of the Société l'Eclairage Electrique (Labour system) in France. The commutator need be only of small dimensions, proportional to the current which it delivers, unless it is also used in starting the motor. When the voltage of the armature is too high or when it is desirable to simplify the connections of the main circuit, it is sufficient to wind, on the armature, a small exciter winding connecting with a small commutator, which serves solely for the purpose of excitation.

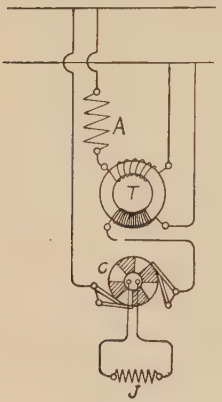


FIG. 21.

This method of excitation, which is the simplest of all, unfortunately still presents several objections. It is applicable only to motors having continuous current windings and a rotating armature; it cannot be adopted in the case of high voltage machines or alternators having a stationary induced winding; it does not lend itself to automatic regulation of the inducing field; finally, (and this is the most serious objection), when there are wattless demagnetizing currents in the motor, from any cause, the E.M.F. at the brushes and the excitation are both weakened in consequence. Now, that is precisely what occurs when the motor experiences a reduction in speed in consequence of an overload. The stability of operation, which decreases with the excitation, will, therefore, be diminished when the load increases. The oscillations due to improper operation are therefore exaggerated by this reaction.

The self-excitation of synchronous motors must, therefore, be studied with care, and its application is limited to small motors. For larger machines the use of a separate exciter is the most satisfactory plan.

CHAPTER II

DETAILED STUDY OF OPERATION WITH NORMAL LOAD

I. PRINCIPLES OF THE ELEMENTARY THEORY

Notation. Let

R = the resistance of the total circuit of the motor (including the motor, the line, and the armature of the generator, in the case of an electric transmission system. (The armature resistance of the alternators is supposed to be increased from 50 to 100 per cent in such a way as to take into consideration the eddy currents produced by the armature in the pole-pieces);

L = the mean inductance of the motor;

l = that of the external circuit (including line and generator);

T = the period; $\omega = \frac{2\pi}{T}$ = the velocity of pulsation;

X = the reactance of the total circuit, including the motor; $X = \omega(L+l)$;

Z = the impedance of the total circuit, including the motor;

$$Z = \sqrt{R^2+X^2};$$

E_1 = the effective external E.M.F., i.e., the E.M.F. obtained from the line (or induced in the generator, in the case of an ordinary simple transmission system). This E.M.F. is supposed *constant*, unless otherwise stated, in all that follows;

E_2 = the effective internal E.M.F. induced in the synchronous motor, such as it would be measured on open circuit with the same excitation. (E_2 should not be confounded with the resultant induced or effective E.M.F. produced by the resultant field of the field ampere-turns and of the armature-reaction. The latter will be designated, later, by the symbol ε_2.)

ε = the resultant E.M.F. of the two E.M.F's. E_1 and E_2;

I = the current in the armature of the motor;

P_1 = the electric power produced by the E.M.F. E_1;

P_2 = the electric power generated in the armature by the current, and counted positively in the same direction as the mechanical power obtained from the motor;

θ = the phase-angle between the E.M.F's. E_1 and E_2;

ϕ = the phase-angle between the current, I, and the E.M.F. E_2, of the motor;

ψ = the phase-angle between the current, I, and the external E.M.F., E_1.

γ = an auxiliary angle representing the phase-angle existing between the current and the resultant E.M.F., ε, which produces it in the circuit. It is defined by the known relation,

$$(1)\quad \tan\gamma=\frac{\omega(L+l)}{R}=X; \qquad \text{whence} \quad \begin{cases} \sin\gamma=\dfrac{X}{Z}, \\ \cos\gamma=\dfrac{R}{Z}. \end{cases}$$

δ = an analogous angle, obtained by neglecting the reactance of the motor. It is defined by the relation,

$$(2)\quad \tan\delta=\frac{\omega l}{R}.$$

The standard diagram which has already been referred to is not easy to apply, partly because a complicated construction is necessary for each load to give the current-strength in magnitude and in phase, and partly because the vectors of the current-strength become too small, inasmuch as the E.M.F. may attain thousands of volts while the currents are of a few amperes only. The geometrical constructions deduced from this diagram by other authors are extremely complicated.

The author devised, some years ago,[1] for the study of problems of this kind, a so-called " bipolar " diagram, which avoids these two objections and at the same time gives the theory an almost childlike simplicity, and renders graphical calculation sufficiently accurate.

Principle of Bipolar Diagrams. The principle of this method consists in taking different axes of reference and different scales for the currents and for the E.M.F's., instead of taking the same axis of reference and the same scale for both, as is usually done. The axes and scales may then be selected for the current in such manner that the

[1] *Théorie des moteurs synchrones.* Lahure, publ. 1895. *L'Industrie Electrique,* Feb., 1895.

current can be represented in magnitude and in phase by the same vector as the resultant E.M.F. of the circuit.

As regards the magnitude, since we have

$$\varepsilon = ZI,$$

it suffices to take, for the current I, a scale of amperes Z times greater than the scale of volts used for the E.M.F. values.

As to the axis of reference, its choice depends on the axis selected for the E.M.F. values.

The bipolar diagram may therefore be established in two distinctly different manners, according to whether it be the vector of the generator E.M.F., E_1 or that of the motor, E_2, which is taken as the fixed line.

We will indicate, successively, these two types of diagrams which, in many cases, do not serve exactly the same purpose, as will be seen later.

Bipolar Diagram of the First Kind. Motor-Vector E_2 taken as Fixed Axis. Let us suppose the effective values E_1 and E_2 of the E.M.F's. taken with the signs which they have when one follows the circuit formed by the motor, the line, and the generator. It is known, by experience, that the E.M.F's. are very nearly in opposition. The two corresponding vectors can, therefore, be represented by two right lines OA_2 and OA_1, drawn in different directions (as indicated by the arrow-heads) and making, a very small angle, θ, with each other (Fig. 22). (This angle indicates the phase-difference between the two E.M.F's., as measured at the terminals of the two machines when coupled in parallel.)

When under load, the generator tends to lead with respect to the motor. The vector E_1 will, therefore, turn in the positive direction which is conventional in mechanics, i.e., in the direction contrary to the hands of a watch.

The resultant E.M.F. ε, acting in the circuit, is represented by the vector A_1A_2, which is the resultant of E_1 and E_2. By projecting A_1A_2 on a right line, A_2F, which makes the angle γ backward from A_1A_2, we obtain the vector $A_2F = RI$, representing the component that is in phase with the E.M.F. E_1. The angle DA_2A_1, therefore measures the phase-difference of the current with respect to the E.M.F. E_2 with. reversed sign, i.e., measured at the terminals, the same as the E.M.F. of the generator. In order to be able to take A_1A_2 as equal in

magnitude and in phase to the vector of the current, it suffices, therefore, to attribute to it, as an axis of reference for phases, a right line $Y'A_2Y$, which makes, with OX, the axis of reference for the E.M.F's., an angle equal to γ, and having A_2Y' for its positive direction. We thus obtain the diagram Fig. 22.

This diagram is bipolar because the axis to which the E.M.F. E_2 is referred is the horizontal line OX, and the vector E_1 turns around O

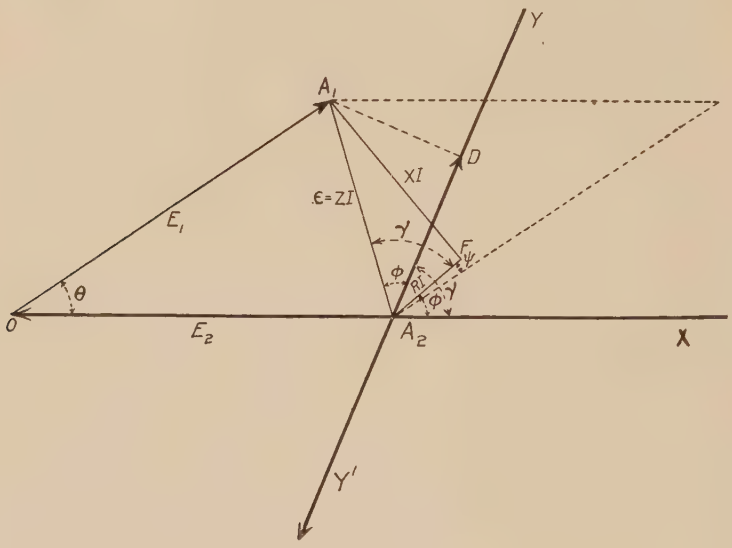


Fig. 22.

as a center; whereas the current vector A_1A_2 turns around A_2 and is referred to the axis YA_2Y'.

The diagram of Fig. 22 shows that the current is more or less directly opposed to the E.M.F. E_2. It can be decomposed into two components, one directly opposed, the other having a phase-difference of $\frac{\pi}{2}$. This decomposition is easily made on the diagram (Fig. 23) by projecting the vector A_2A_1 on the line A_2Y and on a line (A_1D) perpendicular thereto. We thus obtain:

(1) The component having the same phase as E_2 which, following the expression of Dolivo-Dobrowolsky, we will call the "watted" current,

$$I_w = A_2D.$$

(2) The component having a phase-difference of $\frac{\pi}{2}$,

$$I_d = A_1 D.$$

The arrows indicate the direction of the vectors. Taking into consideration the direction of YY' (in the case of Fig. 22), it is seen that every wattless current A_1D, drawn to the left of the line of reference A_2Y, is a current which lags with respect to the E.M.F. E_2 but

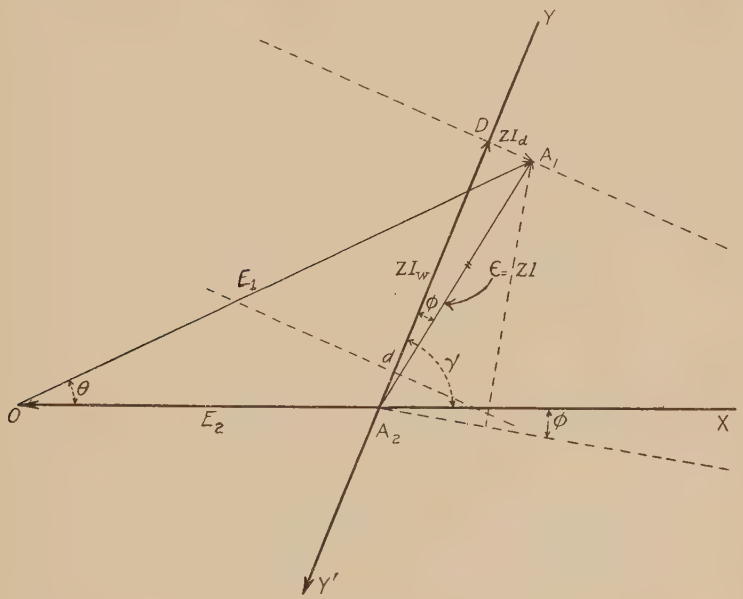


FIG. 23.

which leads with respect to the E.M.F. E_1. If the point A_1 should come to the right of A_2Y_1 (Fig. 23), the wattless current would lead with respect to E_2 but would lag with respect to E_1.

Applications of the Diagram of the First Kind. The diagram has for its object to enable the different working conditions of the motor to be grasped at a glance. It lends itself especially well to this when the motor E.M.F. E_2 is maintained constant, because only the point A_1 is then displaced, as the result of a variation either of the phase-angle θ, or else of the generator E.M.F. E_1. In both cases the outputs are defined by the curve which A_1 describes and which we will call the "locus" of A_1. This locus is a circle when the excita-

tion of the generator is also constant. Each position of the point A_1 defines the vector of the E.M.F. $E_1 = OA_1$ and the vector of the current $ZI = A_2A_1$. Knowledge of any two whatever of the elements E_1, θ, I, ϕ leads to that of the two others.

The diagram can be employed when E_2 varies, but the point O then varies at the same time as A_1, so that the locus of A_1 no longer suffices to define the conditions of operation. There is then some advantage, if E_1 is constant, in having recourse to the second kind of diagram.

Line of Equal Power Occurring with Constant Excitation. The electric power of the motor, being equal to the product of the E.M.F. E_2 by the watted current I_w, will, when the excitation is constant, be proportional to the segment A_2D. When the motor-excitation remains constant, all outputs corresponding to equal watted currents are represented by points, A_1, situated on a single right line A_1D, perpendicular to the axis A_2Y. This right line A_1D is, therefore, a line of constant power for the motor, whatever may be the E.M.F. of the generator, which may be variable. The same thing is true for all parallel right lines. It is possible to represent, on the diagram, a series of such right lines, each bearing the indication of the corresponding power E_2I_w under constant excitation; and then, for each position of the point A_1, the corresponding power will be known.

The mechanical power is equal to the electric power less the losses due to eddy currents and hysteresis. (The heat-losses are not included in the electric power E_2I_w). The effect of these losses is itself equivalent to a certain appreciable watted current, j_0, and to a wattless current which can be practically neglected. Therefore it will only be necessary to draw on A_2Y a vector, $A_2d = Zj_0$, which represents the "lost" current; and the power that is really useful will be proportional to the segment dD. Although the loss under consideration varies slightly with the output, it can, without material error in constructing the diagram, be considered as constant for a given excitation, and as equal to the power necessary to operate the motor without load. The electric power supplied by the generator is deduced immediately from the electric power of the motor by adding the resistance-losses (I^2R) in the circuit.

Lines of Equal Phase. Since the phase ϕ of the current is measured by the angle A_1A_2D, all the points representing outputs A_1, located on the same right line issuing from A_2, correspond to the same angle of lag ϕ. Any line issuing from A_2 therefore constitutes a line of equal phase.

Limit-Circle of Current. Let I_m be the maximum value of the current which the armature can withstand. If, around A_2 as center, a circle be drawn with a radius equal to I_m according to the scale of amperes (or equal to ZI_m according to the scale of volts), this circle will constitute the boundary of a space having certain characteristics. All load-points inside that space correspond to loads which can be maintained indefinitely without the current, I, exceeding that limit. All load-points situated outside this space represent loads which cannot be maintained indefinitely. This limit-circle of current is, therefore, also a limit-circle of stability of operation, of the motor.

Algebraical Relations Deduced from the Diagram. To facilitate graphical calculation it is always useful to have the algebraical expressions for the variables. What we are interested in knowing is the current strength, its phase-angle, the power of the motor as a function of the E.M.F.'s E_2 and E_1, of the constants of the circuit, and of the angle of lag θ, or conversely, any of these as a function of the power, etc.

The solution of the triangles represented in Fig. 23 gives immediately the relation sought. In the triangle A_1DA_2 we have

$$I_d = I \sin \phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$I_w = I \cos \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Again, the projection ZI is equal to the projection of the broken line A_1DA_2:

$$ZI \cos (\gamma - \phi) = ZI_d \cos \gamma + ZI_d \sin \gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$= RI_w + XI_d.$$

The triangle OA_2A_1 enables E_1 or I to be expressed in terms of other quantities, thus:

$$E_1^2 = E_2^2 + (ZI)^2 + 2E_2ZI \cos (\gamma - \phi), \quad . \quad . \quad . \quad . \quad (4)$$

$$(I)^2 = E_1^2 + E_2^2 - 2E_2E_1 \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Finally, if the triangle OA_2A_1 be projected on the line A_2D and on a line DA_1 perpendicular thereto, we will have I_w and I_d as a function of E_1, E_2 and θ:

$$ZI_w = E_1 \cos (\gamma - \theta) - E_2 \cos \gamma, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$ZI_d = E_1 \sin (\gamma - \theta) - E_2 \sin \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The power of the motor and that of the generator will be easily obtained by noting that the phase-angle, which is ϕ with respect to E_1, becomes $\phi+\theta$. We will therefore have

$$P_2=E_2I\cos\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$P_1=E_1I\cos\psi=E_1I\cos(\phi+\theta); \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or, replacing $I\cos\phi$ and $I\sin\phi$ by their values, I_w and I_d, we have

$$\left.\begin{aligned} P_2&=\frac{E_2}{Z}[E_1\cos(\gamma-\theta)-E_2\cos\gamma] \\ P_1&=\frac{E_1}{Z}\left\{\begin{aligned}&\cos\theta[E_1\cos(\gamma-\theta)-E_2\cos\gamma]\\ &-\sin\theta[E_1\sin(\gamma-\theta)-E_2\sin\gamma]\end{aligned}\right.\end{aligned}\right\}; \quad (10)$$

which can also be written

$$P_1=\frac{E_1}{Z}[E_1\cos\gamma-E_2\cos(\theta+\gamma)]. \quad . \quad . \quad . \quad (11)$$

Numerical Example. Let us consider the case of a power-transmission to a distance of 10 kilometers by two Mordey alternators of 37 kilovolt-amperes (2000 volts and 18.5 amperes), having a frequency of 100 cycles, the resistance being 3 ohms and the reactance 43 ohms.[1] Let us assume that the loss-allowance in the line is 200 volts at 20 amperes. The line will therefore have a resistance $r'=10$ ohms. If the conductors have a resistivity (ρ) of 2 microhms per centimeter-cube, the sectional area of each conductor is

$$s=\frac{2\rho l}{r'}=\frac{2\times10^{-6}\times20\times10^{5}}{10}=0.40 \text{ sq.cm.}$$
$$=0.062 \text{ sq.in.},$$

and the corresponding diameter will be 7.15 mm. (0.28 inch). Let us assume that the distance between the wires is 60 cm. (23.6 inches). The tables for line-inductances[2] give, for the linear inductance, in millihenrys per kilometer, the following values:

$$2(0.8188+0.1173)=1.872.$$

[1] This reactance was determined directly from the characteristic curves given for these machines by Mr. Mordey. The value usually given for the inductance of this machine is too low.

[2] See an article by the author in *l'Eclairage Electrique*, Oct.–Dec., 1894.

For a line of 10 kilometers and a frequency of 100 cycles, the reactance (assuming the capacity to be negligible) will therefore be

$$k' = \frac{10 \times 628 \times 1.872}{1000} = 11.8 \text{ ohms},$$

or approximately 12 ohms.

Under these conditions, the constants of the circuit are, in round numbers, as follows:

$R = 16$ ohms, $K = 98$ ohms; $Z = 100$ ohms; and we have

$$\tan \gamma = \frac{98}{16} = 6.12, \qquad \cos \gamma = \frac{16}{100} = 0.16.$$

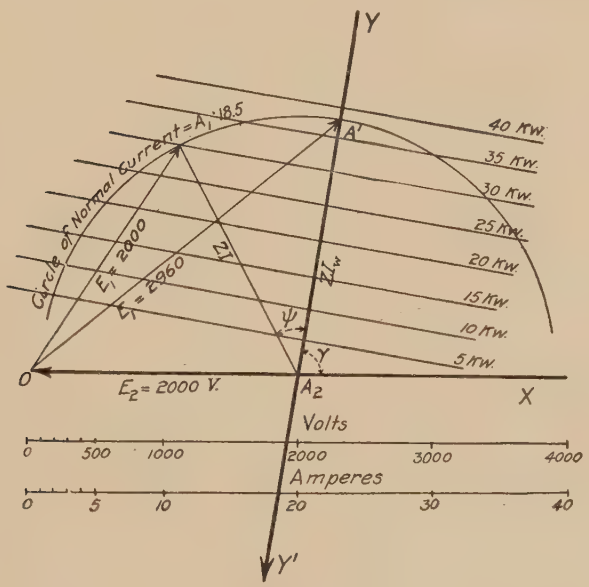


FIG. 24.

The ratio of the two scales will be 100. Therefore a distance corresponding to 2000 volts on one scale will correspond to 20 amperes on the other scale.

Suppose, now, the E.M.F. of the motor to be $E_2 = 2000$ volts, and let it be represented by the stationary vector A_2O (Fig. 24) drawn on the axis of OX in the negative direction. The right line YA_2Y', making the angle γ with OX, backward, will be the origin of the vectors representing currents. The lines of equal electric power for the

motor (not including resistance-losses in the armature) will be right lines perpendicular to YA_2Y'. In the diagram, several of these right lines have been drawn, corresponding to powers increasing each time by 5 k.w. Let us complete the diagram by describing, from A_2 as a center, with the radius $ZI = 100 \times 18.5$ A, the circle of normal current. The point A_1, which depends on the load, can be displaced anywhere in the whole space inside the circle of normal current-value; but it cannot be displaced outside this space, except for a short time, owing to the heating of the machine. For each position of the load-point A_1, its distance from the point O, measured by the scale of volts, represents the corresponding E.M.F. of the generator, and the distance A_1A_2, divided by Z, represents the value of the corresponding current.

If we suppose the generator to be exactly like the motor, i.e., if its E.M.F. is 2000 volts, the diagram shows immediately that the maximum power practicably obtainable cannot exceed 30 k.w. To attain 37.5 k.w. without exceeding the normal current, it would be necessary to raise the E.M.F. of the generator, E_1, to 2960 volts.

Diagram of the Second Kind. The Vector of the Generator E.M.F., E_1, as a Fixed Axis. Let the vectors corresponding to the generator and motor E.M.F.'s be represented by two right lines, OA_1 and OA_2, respectively (both being taken with the signs they present when the closed circuit is followed around). The angle A_1OA_2 (Fig. 25), which we will still call θ, now measures the lag of E_2 behind E_1.

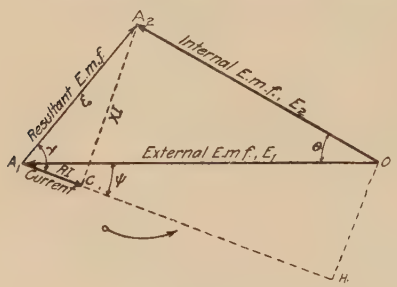


FIG. 25.

The current I is equal to the resultant E.M.F. A_1A_2 divided by the impedance,

$$I = \frac{\varepsilon}{Z}, \quad \ldots \ldots \ldots \quad (12)$$

and it lags by the angle γ behind A_1A_2. Its vector $\overline{A_1C} = RI$ is therefore the projection of $\overline{A_1A_2}$ over that angle, and the phase-difference ψ between the current and the active E.M.F. E_1 is the angle OA_1C. In the case shown in the diagram, the current lags behind A_1O, i.e., behind the E.M.F. E_1 measured at the terminals of the generator.

The E.M.F. $\overline{OA_1}$ can be decomposed into two components, of

which one, $\overline{A_1H}$, is in phase, and the other, $\overline{OH}$, is in quadrature, with the current I. The effect of the motor is therefore equivalent to an apparent reactance, which is here positive,

$$X_{\text{app.}}=\frac{\overline{OH}}{\overline{A_2C}}X,$$

combined with an apparent resistance

$$R_{\text{app.}}=\frac{\overline{A_1H}}{\overline{A_1C}}R.$$

Finally, the electric power developed by each of the two E.M.F.'s E_1 and E_2 will be

$$\left.\begin{aligned} P_1&=E_1I\cos(\angle E_1, I)=\frac{\overline{A_1C}}{R}\times\overline{A_1H}\\ \text{and}\qquad P_2&=E_2I\cos(\angle E_2, I)=\frac{\overline{A_1C}}{R}\times\overline{CH}\end{aligned}\right\}\quad\ldots\quad(13)$$

where $(\angle E_1, I)$ and $(\angle E_2, I)$ mean the *angle of lag between* E_1 and I, and *between* E_2 and I, respectively.

The ohmic loss in the circuit (I^2R) is equal to the difference P_1-P_2. Let us now transform the diagram, as above, in such manner that A_1A_2 shall represent the current, in magnitude and in phase. To obtain the magnitude, it will be sufficient to use, for measuring the amperes, a scale Z times greater than the scale of volts, Z being the numerical value of the impedance, expressed in ohms.

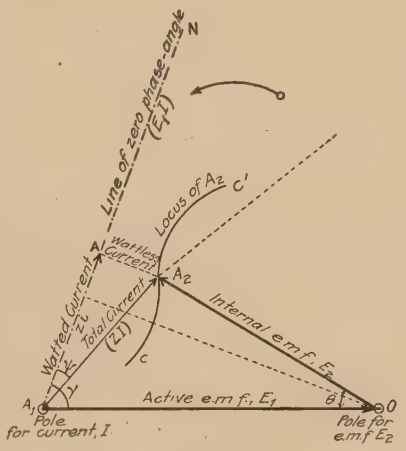


FIG. 26.

To obtain the lag of the current thus represented, with respect to the generator E.M.F. E_1, it will be sufficient to draw (Fig. 26) a *stationary* right line $\overline{A_1N}$ making the angle γ in advance of OA_1. It will be seen immediately, on comparing Figs. 25 and 26, that the angle NA_1A_2 formed by the vector A_1A_2, with the line NA_1 is equal to the angle OA_1C previously referred to, and, consequently, represents the apparent phase-angle ϕ.

[To make the diagram complete we should draw *two* similar diagrams, turned in different directions, one corresponding to the volts, the other to the amperes, each with its own special scale. However, this would be of no advantage, and we will, therefore, retain a single diagram with the two scales, remembering that the origin of phases is OA_1 for the E.M.F.'s, and A_1N for the currents. We shall indicate, on that diagram, the expressions for the segments of lines, as if they were all E.M.F.'s; and it will be sufficient to divide them all by Z to have their expressions for currents.

For example, the segments $\overline{OA_1}$ and $\overline{OA_2}$ will represent, respectively, by the scale of amperes, the two currents

$$\left.\begin{aligned} J_1 &= \frac{E_1}{Z} \\ J_2 &= \frac{E_2}{Z} \end{aligned}\right\}, \quad \ldots\ldots\ldots\ldots \quad (14)$$

which each one of the alternators would produce, respectively, in the circuit, if the other were stopped.]

The line A_1N will therefore be the origin of phases for the current with respect to the E.M.F. E_1.

If we project A_1A_2, on the line A_1N and on the line AA_2 perpendicular thereto, we will obtain, by the scale of amperes, two current-components:

(*a*) The watted current $i=\overline{A_1A}$, or the current which is in phase with the E.M.F.

(*b*) The wattless current $j=\overline{A_2A}$, or the current which is in quadrature with the E.M.F.[1]

The phases of the two currents are both referred to A_1N as origin.

The work of Dobrowolsky has shown that this decomposition of the current is very interesting from a practical standpoint; it is more so, in general, than the decomposition of the E.M.F. already indicated (page 41).

The power furnished by the active E.M.F. may be deduced from the watted current, thus:

$$P_1 = E_1 \times i.$$

The power actually utilized may be deduced from this, in the following manner:

$$P_2 = P_1 - r_2 I^2,$$

in which r_2=the armature-resistance of the motor.

[1] These terms, "watted" and "wattless," are adopted here to conform to the international language of manufacturers, although they are objectionable.

Power-Values as Function of the Lag-Angle θ. To express the power-values P_1 and P_2 in terms of the angle θ, it is only necessary to substitute, in the two equations (13), other values for $I\cos(\angle E_1, I)$ and for $I\cos(\angle E_2, I)$.

$I\cos(\angle E_1, I)$ is nothing more than the watted current i for the case represented in Fig. 23.

Let $J_1=\frac{E_1}{Z}$ and $J_2=\frac{E_2}{Z}$. (See eq. 14, p. 42.)

The diagram gives, readily,

$$i=I\cos(\angle E_1, I)=J_1\cos\gamma-J_2\cos(\theta+\gamma). \quad . \quad (15)$$

The current $I\cos(\angle E_2, I)$ will be obtained by analogy, by projecting the diagram of Fig. 26 upon a right line making with OA_2 an angle that is again equal to the angle γ. From this we obtain the absolute value

$$I\cos(\angle E_2, I)=J_1\cos(\gamma-\theta)-J_2\cos\gamma. \quad . \quad . \quad . \quad . \quad (16)$$

Replacing J_1, J_2 by their values, $\frac{E_1}{Z}$, $\frac{E_2}{Z}$, we will therefore have

$$P_1=\frac{E_1}{Z}[E_1\cos\gamma-E_2\cos(\theta-\gamma)]. \quad . \quad . \quad . \quad . \quad (17)$$

$$P_2=\frac{E_2}{Z}[E_2\cos(\gamma-\theta)-E_2\cos\gamma]. \quad . \quad . \quad . \quad . \quad (18)$$

These are expressions wherein only two variables, E_2 and θ, enter. The utility of these expressions will be shown later.

Use of this Diagram for the Study of Different Loads. These loads cannot be determined without imposing some condition which makes the variables (i.e. E.M.F. current, power, lag, etc.,)vary according to some definite law. In that case, to each value of the lag there will correspond a single (or several) quite definite values of the E.M.F. E_2, and vice versa. Consequently, on the diagram, the point A_2, instead of being able to range over the whole surface, can be *displaced only on a single curve* such as CC', for example (Fig. 26), whose equation, in polar co-ordinates, with respect to the point O as a pole, is nothing more than the relation imposed between E_2 and θ.

For each point of this curve, which may be called the locus of A_2, there correspond two radial vectors issuing from two fixed poles.

One of these vectors, OA_2, is drawn from the point O as a pole; and it indicates the E.M.F. E_2, in magnitude and in phase (starting from OA_1); the other, A_1A_2, is drawn from the point A_1 as a pole;

and it indicates the corresponding value of the current, in magnitude and in phase (starting from A_1N).

Knowledge of any two of the elements E_2, θ, I, enables all the others to be known, and also the lag ϕ and the reactive and active currents.

When E_1 is variable, this diagram becomes inconvenient, and preference should be given to the diagram of the first kind.

Curves of Constant Electric Power of the Motor when the Generator has Constant Excitation. These curves, which *indicate*

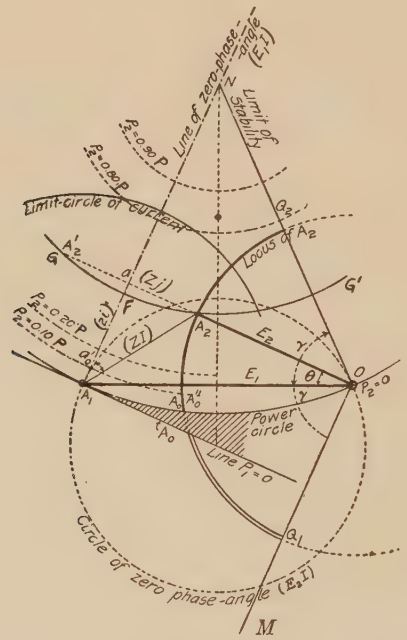


FIG. 27.

the power developed by the motor for each position of A_2, in the diagram, are represented by Eq. (18), in polar co-ordinates referred to the pole O (Fig. 27) (P_2 being supposed constant), and they now become circles instead of straight lines. Let us take two rectangular axes passing through O, the axis of x being directed along the line ON, which makes with OA_1 the angle γ. We can then write

$$\begin{cases} E_2 \cos(\gamma-\theta)=x, \\ E_2^2=x^2+y^2. \end{cases}$$

By substitution, we have

$$x^2+y^2-\frac{E_1 x}{\cos\gamma}=-\frac{P_2 R}{\cos^2\gamma},$$

or

$$\left(x-\frac{E_1}{2\cos\gamma}\right)^2+y^2=\frac{\frac{1}{4}E_1{}^2-P_2R}{\cos^2\gamma},$$

from which it can be seen that the curve is a circle whose center has the following co-ordinates:

$$x=\frac{E_1}{2\cos\gamma}; \qquad y=0.$$

Each circle has for its center the point of intersection, N, of the right lines ON and OA_1, both making the angle γ with OA_1; and the radius of this circle is

$$\rho=\frac{1}{\cos\gamma}\sqrt{\left(\frac{E_1}{2}\right)^2-RP_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This result is also easily obtained by geometry.

As for the lines of constant electric-power, P_1 (applied at the terminals), they are evidently straight lines perpendicular to A_1N (none of which has been drawn on the diagram except the line $P_1=0$); because they are defined by the simple condition that the current which is active with respect to E_1, (i.e., i), is constant.

The expression (19) can be calculated algebraically, for various values of P_2, or it can even be determined graphically by means of rectangular triangles. It is convenient to express the power P_2 as a function of the maximum power which can be obtained by making E_2 vary, E_1 being supposed to remain constant. As will be seen later this power, which will be designated by the letter **P**, has the following value:

$$\mathbf{P}=\frac{E_1{}^2}{4R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

It is easy to construct a series of equipotential lines corresponding to the given values of P_2, and thus to predetermine the variations of power resulting from the displacements of the point A_2.

The radius ρ can then be written,

$$\rho=\frac{E_1}{2\cos\gamma}\sqrt{1-\frac{P_2}{\mathbf{P}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

It is therefore sufficient to divide the segment $\overline{A_1N}$ in the proportion of $\sqrt{1-\frac{P_2}{\mathbf{P}}}$ to obtain the power-lines shown in Fig. 27.

If, for example, we take the series of values,

$$\frac{P_2}{\mathbf{P}}=0.10,\ 0.20,\ 0.30,\ 0.40,\ 0.50,\ 0.60,\ 0.70,\ 0.80,\ 0.90,\ 1.00,$$

we will find

$$\frac{\rho}{A_1N}=0.949,\ 0.894,\ 0.837,\ 0.774,\ 0.707,\ 0.632,\ 0.548,\ 0.447,\ 0.316.$$

In practice, these lines should be graduated in kilowatts, in the usual way.

It is often unnecessary to draw the lines for the high powers, since the power $\mathbf{P}$ is much greater than that which the armature can withstand without dangerous heating, in continuous operation.

Since $\cos\gamma$ is very small (usually <0.10), in ordinary practical cases, the center N of the circles will often be outside the diagram, but it will be always easy to draw the circles by points, from their equation in rectangular co-ordinates.

Moreover, in that case, the circles are of very large radius, and are quite sufficiently determined by two tangents. Now for each circle, such as GFG', the directions of the two tangents at F and G' are known, because they are, respectively, perpendicular to A_1N and ON. Again, the position of the points F and G' corresponding to a power P_2, is given by the obvious equation:

$$\overline{A_1F}=\overline{OG'}=\frac{E_1}{2\cos\gamma}-\rho=\frac{E_1}{2\cos\gamma}\left(1-\sqrt{1-\frac{P_2}{\mathbf{P}}}\right).$$

It is very easy, therefore, to construct these two tangents and then to draw approximately the circular arc which they determine.

It will be noted that the "P_2" power-values, in Fig. 27, are positive above the point A_1, and negative below it. This shows that the alternator will operate as a motor in a certain portion of the plane, contained inside the power-circle $P_2=0$, and that it will operate as a generator beyond that circle. In the cross-hatched space comprised between the lines $P_2=0$ and $P_1=0$ it cannot operate either in one way or in the other, because the current which would then pass through the machine would cause a loss of energy in the armature greater than the available power.

Current-limit Circle. It is known that the armature cannot indefinitely withstand a current exceeding a certain limiting value I_m. If we draw, around the pole A_1 as a center, a circle of radius precisely equal to I_m (by the scale of amperes), all the points A_2 situated outside that circle will give values $I = \overline{A_1 A_2} > I_m$; whereas, inside that circle the value of I will be $< I_m$. The circle drawn is therefore a limit beyond which the load-point must not go, for continuous operation.

Lines of Equal Phase. In the same way that lines of equal power have been drawn, lines of equal current-phase can be easily drawn, such that, at the loads represented by their various points, the phase-angle between the E.M.F. and the current will always be the same.

If the phase of the current I is measured from the E.M.F. E_1, these lines are evidently straight lines issuing from the point A_1. The right line $A_1 N$ corresponds to "zero" phase-angle. All the loads to the right thereof correspond to a lag in phase, and all the loads to the left correspond to a lead in phase. (Fig. 27.)

To obtain lines corresponding to currents of equal phase with respect to the E.M.F. E_2, we note that all the points for any current value whatever, among these lines, should present a constant angle OA_2A_1. These lines are therefore circles having OA_1 as a chord. If, for example, we wish the line of phase corresponding to the lag α between E_2 and I, we construct, on OA_1 as a chord, a circle admitting of the angle $\alpha - \gamma$ (since the true phase of the current lags by the angle γ with respect to the line $\overline{A_1 A_2}$).

Only the circle corresponding to an angle $\alpha = \pi$ has been drawn, in the example represented in the diagram. This circle represents the exact opposition in phase between the current and the internal E.M.F. E_2. It is easily seen that this circle is tangent to the two lines ON and $A_1 N$.

For all loads corresponding to points situated inside this circle, the phase is $> \pi$, outside the circle, on the contrary, it is $< \pi$.

[It will be noted, in passing, that the circle of zero power $P_2 = 0$, passing through the point A_1 is nothing more than the circle of phase $\frac{\pi}{2}$, because the power is zero every time that the current is in quadrature with the E.M.F.]

Numerical Example. Let us return to the example of power-transmission by means of two Mordey alternators, for which the numerical data have already been given (page 38), and let us apply the second diagram to this case.

The maximum theoretical power is

$$P=\frac{1}{16}\left(\frac{2000}{2}\right)^2=62{,}500 \text{ watts.}$$

The radii of the power-circles are given by eq. (21); whence we have

$$\rho=6250\sqrt{1-16\frac{P_2}{10^6}};$$

in which P_2 is expressed in watts, ρ being measured by the scale of volts. In this manner we find the radii for the circles

of	5	10	20	30	40	and	50 k.w.
to be	6000,	5700,	5130,	4520,	3750,	and	2800 volts.

These simple calculations give all the elements necessary for the diagram in Fig. 28. In this case the current-circle corresponding to

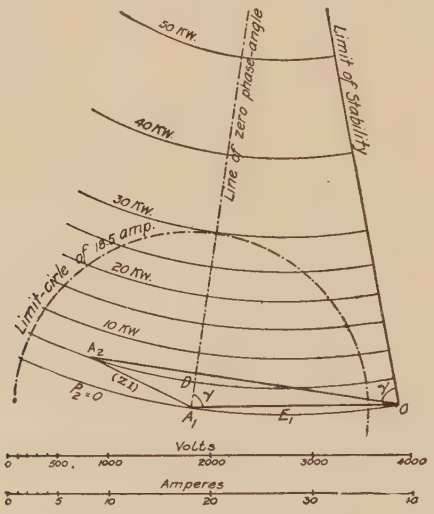


FIG. 28.

the normal current of 18.5 amperes has also been drawn, in addition, to show the limit of loads. Let A_2 be any position whatever of the load-point, within this circle; it is only necessary to measure the various lines, according to the proper scales, to see that the corresponding voltage E_2 is 3000 volts, the current strength is 11 amperes, the reactive current (leading) is 10.5 amperes, and the power is 5 k.w. The

minimum current corresponding to the same power would be $OD =$ 2.8 amperes, approximately.

II. OPERATION OF A MOTOR WITH CONSTANT EXCITATION, SUPPLIED AT CONSTANT E.M.F.

The two diagrams lend themselves equally well to the discussion of this case, which is usually the most frequent one in practice. In the first diagram (Fig. 29) the E.M.F. vector of the motor will remain stationary and that of the generator will turn round the pole O, when

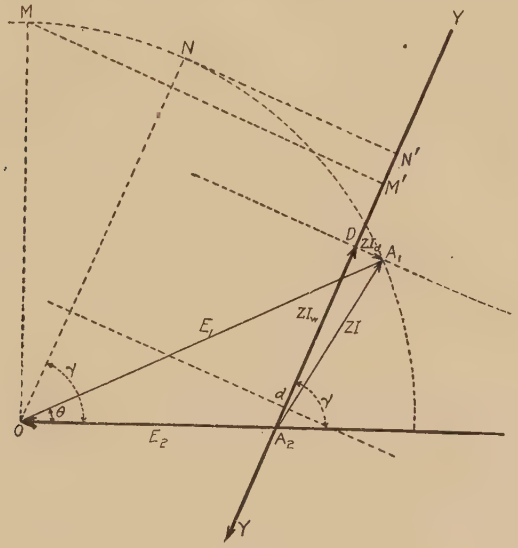


Fig. 29.

the load varies. The extremity A_1 describes a circle having O for its center and E_1 for its radius. If the lines of equal load have been drawn, their points of intersection with the circle will define completely the conditions (currents and lags) for each load. The useful active currents will be obtained by subtracting the loss-current $\frac{A_2 d}{Z}$ from the active currents themselves.

It will be noted that the power is zero when A_1 is on the power line passing through d. If, in consequence of overload, the motor-

speed diminishes, θ increases and the power developed also increases. The condition is therefore stable for low powers.

But the power-output is limited. Its maximum corresponds to the power-line drawn tangentially to the circle. The corresponding output, N, is the limit of possible outputs, because the effect of an overload, by increasing further the lag θ, would be to bring the vector OA_1 into a position such as OM, where the power-output would be diminished, and, consequently, the motor would stop.

The line ON, drawn through O parallel to A_2Y, therefore defines theoretically the limiting line of stability; and the limiting lag between the E.M.F.'s. is equal to γ. The maximum electric power applied to the motor is equal to the product of the E.M.F. E_2 by the corresponding active current $\frac{A_2N'}{Z}$.

Likewise, on the second diagram (Fig. 27), if the excitation is supposed to be constant and if the load is made to vary, the end of the E.M.F. vector E_2 describes a circle around O as a center. Its point of intersection, A_0, with the circle of zero-power ($P_2=0$), gives, theoretically, the condition of operation without load; but, practically, the power necessary to overcome friction and other losses not being equal to zero, the unloaded condition will correspond to a higher point, such as A''_0, situated on the power-circle $P_2=0.10$ **P**.

When the load applied to the motor-shaft increases, the point of output A_2 is displaced on its circle in the direction of increasing power-outputs, i.e., toward the top, and the angle of lag θ and the current I increase correspondingly.

The power can increase up to the point Q_2, where the characteristic is tangent to a power-circle (Fig. 27); but it could only decrease if the angle of lag were to be further increased. The direction ON therefore marks the theoretical limit of stability of operation. When the angle of lag is increased further the motor falls out of step and comes to a stop.

It may be seen in the same manner that the alternator operates as a generator below the point A'_0, and that the necessity of stability of operation limits the angle of lag to the direction OM, which is perpendicular to the right lines of equal power P_1, and symmetrical with ON, with respect to OA_1.

Maximum Power. The maximum power P_2, corresponding to the limit of stability just determined, can be easily calculated by making $\theta=\gamma$ in the general power equation (18).

From this we get

$$\left.\begin{aligned} P_{2\max.} &= E_2\frac{E_1-E_2\cos\gamma}{Z} \\ &= E_2\frac{\cos\gamma\,(E_1-E_2\cos\gamma)}{R} \\ &= \frac{E_2}{Z}\left(E_1-E_2\frac{R}{Z}\right) \\ &= \frac{E_2}{R}\frac{R}{Z}\left(E_1-E_2\frac{R}{Z}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

The power-value will be higher in proportion as the impedance Z and the resistance R are smaller $\left(\text{provided that } \cos\gamma < \frac{1}{2}\frac{E_1}{E_2}\right)$.

The formula (22), can also be written directly from the diagram (Fig. 29) corresponding to the load which is at the limit of stability, because, we have, from this diagram,

$$ZI_w = E_1 - E_2\cos\gamma,$$

and also $$P_2 = E_2 I_w.$$

In well-constructed modern motors (having low resistance and low impedance), there is usually no tendency of the machine to fall out of step from gradual overloading, when it is supplied from a constant potential circuit, because, as Fig. 28 shows clearly, the current becomes excessive before the power-outputs approach the limit of stability just indicated. For example, the Labour synchronous motors, which can be cited as excellent existing examples of this class of apparatus, are only limited, in regard to load, by the heating of the armature, and not by the tendency to fall out of synchronism. It is the circle of maximum current, drawn as we have already seen, which limits *long-continued loads*.

But loss of synchronism may result from anything which can increase the impedance and diminish the reactance-factor, and, consequently the tendency to fall out of step is increased by the resistance of the line in long-distance transmission.

It is therefore necessary to be able to foresee, under certain circumstances, whether the operation will be stable or not.

Means of Determining the Practical Stability of Synchronous Motors. In practice, owing to oscillations of speed, the load is restricted to a value which is less than the maximum power just determined. It can be conceded, in all cases, that, during these speed-oscillations,

the excitation of the motor (even if it is compounded) has not time to vary. It can, therefore, always be assumed that the motor has a constant induced E.M.F., and the results of experiments made with such motors can always be used as the starting point.

On this basis, the stability of operation can be calculated in a sufficiently precise manner by simply comparing the effective load of the motor at the maximum power, P_m, which it might develop with its E.M.F. E_2, and the power, $P_{2\max.}$, which is given by eq. (22).

[This value can be determined experimentally by the method set forth in the author's pamphlet on the "Coupling of Alternators", Paris, 1894, p. 4.]

Experience shows that it is possible to have stable operation so long as the power P_2 does not exceed about two-thirds of P_m, when the apparatus driven by the motor constitutes a very constant load, or about the half of P_m, when the load is somewhat irregular. [It is assumed, of course, that the apparatus can withstand the corresponding current without overheating.]

These figures have been verified by various experiments and they are in accordance with the results published by other experimenters. On analyzing the power transmission at Cassel (see *La Lumière Electrique*, Vol. XLV, p. 616) where the motors furnish power for charging batteries, we find that the ratio of practical to theoretical maximum power is as high as $\frac{P_2}{P_m}=\frac{4}{5}$, but this is exceptional. It is safer to assume $\frac{P_2}{P_m}=\frac{1}{2}$.

Variations of Stability with Operating Conditions. It is very evident that, all things being equal, the stability will decrease with the load. To see how it varies with the conditions of the circuit and of the motor-excitation, when the power P_2 and the voltage E_1 are constant, it is only necessary to study the variations of the power P_m, by means of equations (22). With equal resistance it presents a maximum of maximums

$$\mathrm{P}=\frac{{E_1}^2}{4R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

whenever E_1, E_2, Z, and γ satisfy the condition

$$E_2=\frac{E_1}{2\cos\gamma}=\frac{E_1 Z}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

(1) Suppose a given motor-load, and suppose the E.M.F.'s. E_1 and E_2 to be nearly equal. The stability will first increase with the reactance until we have

$$\frac{Z}{R}=\frac{2E_2}{E_1},$$

i.e., until Z approaches $2R$; and it will then decrease if $\frac{Z}{R}$ is greater. Inasmuch as $\frac{Z}{R}$ is always greater than 2 in ordinary alternators, any increase of inductance is objectionable in a motor which is connected directly to the source of current. It could only be useful if the motor was connected to the generator by a very long line of exceptionally high resistance.

(2) Suppose a given motor-load and suppose the external E.M.F. E_1 to be constant. The stability of operation will always vary inversely with R; but the result will depend upon whether $\cos\gamma$ be greater or smaller than $\frac{1}{2}$, i.e., whether the reactance-factor $\left(\tan\gamma=\frac{X}{R}\right)$ will be higher or lower than $\sqrt{3}$. When $\cos\gamma > \frac{1}{2}$ an increase of the internal E.M.F. E_2 until it exceeds the external voltage E_1 will cause an increase of stability, whereas, when $\cos\gamma < \frac{1}{2}$ the increase will cause a reduction of stability. This observation explains some apparent contradictions of experience. When a motor is connected on a constant potential line or on a line of low resistance, it is more difficult to make it fall out of step when it is over-excited. If, on the other hand, the same motor is used for power-transmission over a line of high resistance, it will be observed that increasing the excitation will also increase its tendency to fall out of step.

In practice, the first case is the more frequent, and it is advisable, in order to improve the stability, to increase the excitation with the load either by hand or automatically.

At the same time, it is seen that, in the most frequent practical cases, it is necessary, in order to have good synchronous motors, to diminish both the resistance and the reactance of the armature. This means that the field must be powerful and the armature-reaction must be low.

If we were to make Z constant by hypothesis, we would be led to reduce as much as possible the resistance and to increase the reactance; but this case does not occur in practice.

Numerical Example. Let us take, for example, two Mordey alternators of 37.5 kilowatts electrical output [1] coupled for electrical trans-

[1] The power available at the shaft must be reduced in proportion to the efficiency (exclusive of the excitation-energy).

mission, and let $E_1 = 2200$ volts be the generator E.M.F. The internal resistance of each machine is 3 ohms and its reactance at 100 cycles is 43 ohms, as previously stated.

(1) Let us first suppose the line to have no resistance and no reactance. The total circuit will have the following constants—resistance 6 ohms, reactance 86 ohms and impedance 86.20 ohms.

The maximum power

$$\mathbf{P} = \frac{(2200)^2}{4 \times 6} = 201{,}660 \text{ watts},$$

could only be obtained by giving to the motor E.M.F. the value

$$E_2 = \frac{1100}{0.0695} = 15{,}800 \text{ volts},$$

and it could only be obtained with a current

$$I = \frac{2200}{2 \times 6} = 183 \text{ amperes}.$$

It is therefore unattainable from all points of view.

If we limit the E.M.F.'s to $E_2 = E_1 = 2200$ volts, the possible maximum will be

$$(P_2)_{\max} = \frac{(2200)^2}{86.20}(1 - 0.0695) = 52{,}000 \text{ watts};$$

i.e. about 3/2 times the normal output. The stability will therefore be sufficient provided the machines are not subjected to variations of load.

Every increase in voltage will here produce an increase of $(P_2)_{\max}$. If, for example, we take $E_2 = 3000$ volts, 69 k.w. instead of 52 is found to be the maximum output possible without causing the machine to fall out of step. The stability is then excellent at the normal load of 37.5 k.w. If, on the contrary, the voltage is reduced, so high a load could no longer be attained.

(2) Suppose now the resistance of a line to be interposed between the two alternators. Stability will evidently become impossible as soon as the maximum power $\mathbf{P}$ approaches the effective power P_2. If we take, for the coefficient of load, the figure 3/2 already mentioned, the maximum power $\mathbf{P}$ should not be less than $3/2 \times 37.5 = 56$ k.w., which corresponds to a line-resistance of about 16 ohms.

In his experiments Mordey (Proc. Inst. of Electr. Eng., 1894) was able to increase the resistance to very much higher values (140 ohms),

because he made his machines run *without load.* This proves nothing, however, in regard to their operation under load. Moreover, these conditions were absolutely exaggerated, since lines of such high resistance would never be used in practice.

Finally, when we attain a resistance of approximately $R=\frac{86}{\sqrt{3}}=50$ ohms, we have cos $\phi=\frac{1}{2}$. The maximum power for $E_2=E_1$ being then no longer 22,550 watts, the output of the motor can scarcely exceed 15 k.w. Besides, it is sure that every increase of the E.M.F. E_2 above 2200 volts will *diminish the stability.* This will be all the more true if R is higher. We can thus understand that in using machines of 50 k.w. with an external or line resistance of 63 ohms, Mordey could only operate these machines without load, and that the slightest increase of the voltage E_2 above E_1 (20 per cent) would cause the machines to fall out of step.

The preceding analysis therefore explains perfectly the anomalies of Mordey's experiments.

We would be brought to analogous conclusions by the consideration of sudden overloads and of oscillating changes of load, which will be discussed later.

[The amounts of sudden overloads allowable are discussed in the author's preceding work, "The Coupling of Alternators," and also in *La Lumière Electrique*, Vol. XLV, p. 474.]

III. COMPARISON OF POSSIBLE OUTPUTS AT CONSTANT LOAD WITH VARIOUS EXCITATIONS. CONSTANT POTENTIAL SUPPLY

Existence of a Current-Minimum. There exist several values of current and E.M.F. which permit a motor subjected to an external constant E.M.F. E_1 to develop a given electrical power P_2. These are all values for which the point A_2 (Fig. 27) falls on the line of equal power corresponding to that value P_2.

These values differ from each other by the values of the E.M.F. E_2 necessary to produce them, and also by the phase-angles θ and ϕ of the corresponding current I, and, particularly, by the reactive current.

If, to simplify matters, we suppose the motor to be supplied from a constant potential source whose impedance is negligible, the diagram in Fig. 27 will give the solution.

The current being represented by the vector A_1A_2 according to the scale of amperes, its minimum evidently corresponds to the point

F on the power-circle $G\,G'$, situated on the phase-line AN. As soon as the point A_2 is displaced on the circle to either side of the line AN, the current increases. There is therefore a value of the internal E.M.F. E_2 which corresponds to a minimum current, and all values greater or less than this value necessitate a stronger current.

It is evident from Fig. 27, without any explanation, that the minimum value is that which brings the current into phase-coincidence with the E.M.F. E_1, i.e., with the E.M.F. at the terminals, since the motor is supposed to be supplied from a constant potential source having negligible impedance.

[This condition gives, immediately, for the corresponding algebraical value of E_2 the following

$$E_2^2 = E_1^2 + (ZI)^2 - 2E_1RI. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Suppose, for example, a 40 k.w. motor connected to a 2000-volt circuit, its efficiency being $\eta = 0.85$, and the constants being $R = 2$ ohms, and $Z = 40$ ohms. The minimum current necessary, at full load, will be about

$$I = \frac{40{,}000}{0.85 \times 2000} = 23.5 \text{ amperes};$$

whence $$ZI = 940 \text{ volts}.$$

The corresponding E.M.F. will be

$$E_2 = \sqrt{(2000)^2 + (940)^2 - 2000 \times 47 \times 2} = 2166 \text{ volts}.$$

The corresponding value of the exciting current will be obtained by reference to the excitation-curve of the alternator.

V-Curves. The foregoing conclusions, which had already been formulated by the author in a paper in 1892 (*La Lumière Electrique*, Vol. XLV, pp. 423 and 563), were confirmed experimentally by Mordey in 1893. Mordey's results were represented graphically by a curve, in rectangular co-ordinates, to which the name of V-curve has been given, owing to its characteristic appearance (Fig. 30). This curve is obtained by taking as abscissæ the values of the E.M.F. E_2 and as ordinates the corresponding current-values. It is readily seen that the current does pass through a minimum Mm for a certain value Om of the voltage.

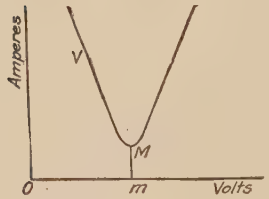


FIG. 30.

Use of Diagram of the First Kind. The diagram of the first kind lends itself more rigorously than the other to the study of voltage and current values when the motor is working at constant load; and it also applies equally well in the case where the impedance of the generator can be neglected.

It is sufficient to cause the segments $OA_2, O'A_2, O''A_2, O'''A_2$, (Fig. 31), representing the E.M.F. E_2, to vary in inverse ratio with respect to the segments A_2D, A_2D', A_2D'', representing the active current I_w, in such manner that the product E_2I_w shall be constant and equal to the given power P_2. If, from each point, O, O', O'', an arc of circle

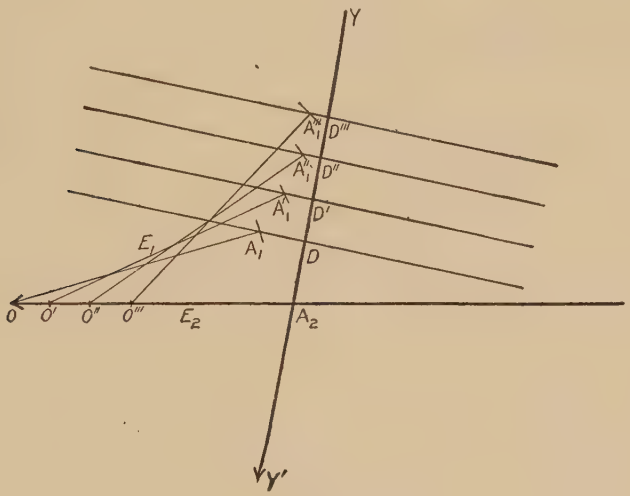


FIG. 31.

is described, having for its radius the generator E.M.F. E_1, their respective intersections with the lines passing through D, D', D'', D''' and perpendicular to A_2Y, determine the load-points A_1, A'_1, A''_1, A'''_1; and, consequently, the current-values I corresponding to the various values of E_2. This diagram also applies in the case where E_1 varies according to some given law.

Predetermination of V-Curves. The preceding diagrams enable these curves to be obtained by a simple graphical transformation, without calculation. It is sufficient to measure, on the diagram itself, the radial vectors for corresponding values of E.M.F. and current, and to plot them as rectangular abscissæ and ordinates. Each of the power-lines of Fig. 28, for example, also gives a *transformed curve*

which is the corresponding V-curve (Fig. 32). These curves show clearly that the V becomes more rounded as the corresponding power P_2 increases. This fact is, moreover, verified by experience, as shown,

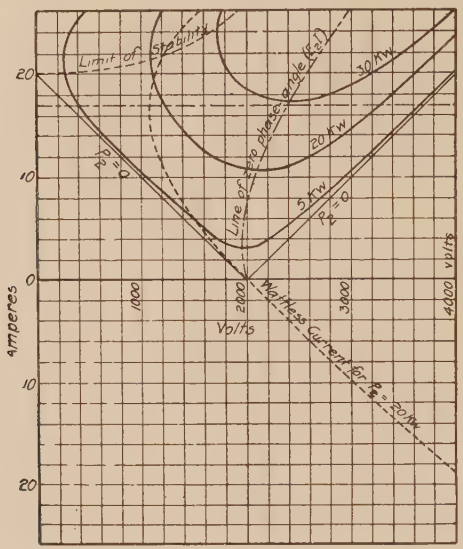


FIG. 32.

for example, by the curve in Fig. 33, obtained experimentally from a synchronous motor of Labour type running at constant load of 5 k.w. (half load).

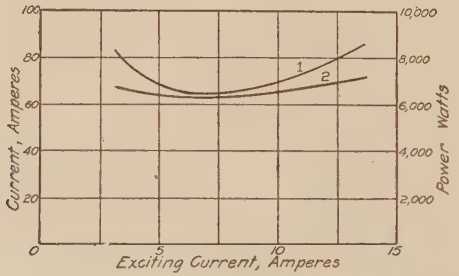


FIG. 33.

The sharpest V is obtained with no load. Theoretically, the curve $P_2=0$ (Fig. 32) ought to present an angular point at the axis of amperes; but, in practice, the power consumed by friction, eddy currents etc., when running without load, is not negligible, and, for these rea-

sons, the curve of no-load has a form analogous to the 5 k.w. curve of this figure.

It will be noticed that, in practice, it is possible to obtain only a rather limited portion of curve, under load, because the values become more and more unstable as they depart from the minimum current value, either way.

Theoretical Form of V-Curves. Although, for various reasons considered later, the V-curves always differ slightly in practice from those deduced from the diagram, it is interesting to determine their theoretical algebraical equation.

If we note that the current I and the power P_2 are given, by eqs. (4) and (8), in terms of E_1, E_2, γ, and ϕ, it will be seen that we only need to eliminate ϕ between these two equations to obtain the desired relation between E_2 and I.

From eq. (4), expanding and replacing $\sin\gamma$ and $\cos\gamma$ by their values $\frac{X}{Z}$ and $\frac{R}{Z}$, we have

$$\left.\begin{aligned} E_1^2 &= E_2^2+(ZI)^2+2E_2[ZI\cos\phi\cos\gamma+ZI\sin\phi\sin\gamma] \\ &= E_2^2+(ZI)^2+2P_2R+2X\sqrt{E_2^2I^2-P_2^2} \end{aligned}\right\}. \qquad (m)$$

The equation obtained on squaring to remove the radical sign is

$$[(ZI)^2+E_2^2-E_1^2+2P_2R]^2-4X^2(E_2^2I^2-P_2^2)=0.$$

This equation is of the 4th degree in I and in E_2. The V-curves obtained, in rectilinear co-ordinates, taking E_2 as abscissæ and I as ordinates, are therefore more or less complicated curves of the 4th degree. They may be calculated by points by solving with respect to I, for example, but the calculation is unnecessarily complicated; and the graphical construction shown in Fig. 31 is preferable.

In the particular case where $P_2=0$ the equation reduces to

$$(ZI)^2+E_2^2\pm 2XE_2I=E_1^2,$$

i.e., an assemblage of two easily constructed ellipses whose axes intersect each other as shown in Fig. 34. The V-curve for no load is therefore formed of two branches of an ellipse which intersect each other on the axis of amperes. The sections of it which are in the useful portion are almost rectilinear.

[Since there is no term of the first degree, these ellipses have the origin of the co-ordinates for their center. If we take as ordinates the product ZI, the inclination of their axes is constant and equal to

$$\frac{ZI}{E_2}=\tan 45^\circ=1.$$

If, instead of the product (ZI), we take, as ordinates, the values of I, the inclination of their axis becomes

$$\frac{I}{E_2}=\frac{Z^2-1}{2X}\pm\sqrt{\left(\frac{Z^2-1}{2X}\right)^2+1}.$$

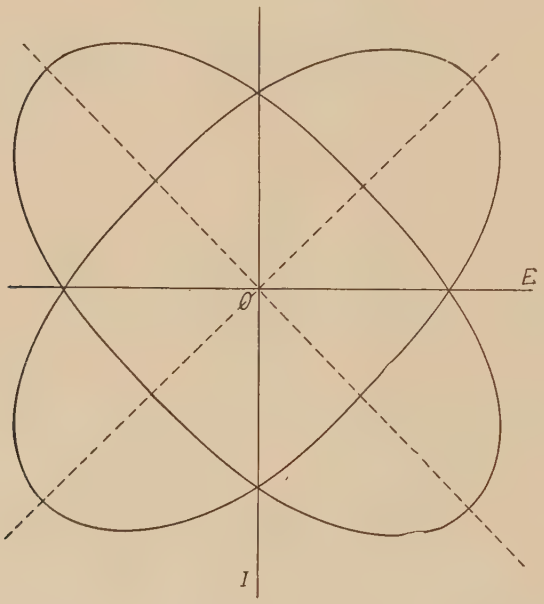


FIG. 34.

This inclination increases inversely as the reactance X, i.e., a given variation of the exciting current produces a current-variation which is all the greater the higher the inductance of the motor. This conclusion is supported by other considerations, as we have already seen.]

The conditions for which the current is a minimum are those for which the phase is zero with respect to E_1. They are obtained by taking, on the diagram of Fig. 27, all the values situated on the right line A_1N. For these points the triangle OA_1A_2 gives

$$E_2^2=E_1^2+(ZI)^2-2E_1ZI\cos\gamma$$
$$=E_1^2+(ZI)^2-2E_1RI.$$

In rectangular co-ordinates the relationship between E_2 and I is represented by the hyperbola [1] passing through the point of intersection of the two arcs of the ellipse of zero-power, and having its principal axis horizontal and situated above the axis of abscissæ at a height equal to

$$I=\frac{RE_1}{Z^2}.$$

The excitation which renders the current a minimum must therefore decrease at first when the power required, P_2, increases; and it must then increase more and more. The greater the impedance the sooner this increase will occur. These results are confirmed by experience, but, in practice, this critical value of E_2 may be considered constant.

Curve of Reactive Current. Fig. 32 would be incomplete if only the total current I were represented therein, because the diagram would not then give information in regard to the phase of the current. It is therefore quite useful to add, to the V-curves, other curves indicating the value of the corresponding positive or negative reactive current. The values of the reactive current are deduced from the diagram by constructing the lines corresponding to each value of the phase-angle θ. The curve of reactive current corresponding to $P_2=20$ k.w. has been drawn as an example in Fig. 32. In practice this curve can be plotted from measurements made by a wattmeter.

Expression for Reactive Current. The expressions for currents which are reactive with regard to E_1 or E_2 can be calculated in like manner. The reactive current as a function of E_2 will be obtained by starting from eq. (m). This equation can be written

$$E_1^2=E_2^2+Z^2\left[I_d^2+\left(\frac{P_2}{E_2}\right)^2\right]+2P_2R+2E_2XI_d.$$

This is also an equation of the 4th degree in E_2, but it can be easily solved with respect to I_d, if written in the following form:

$$Z^2I_d^2+2XE_2I_d-(E_1^2-E_2^2)+\left(\frac{ZP_2}{E_2}\right)^2+2P_2R=0.$$

[1] The conditions of zero phase-angle of the current with respect to the E.M.F. E_2 would be the same on an ellipse such as

$$E_1^2=E_2^2+(ZI)^2+2E_2RI,$$

but they present no special interest.

For the particular case $P_2=0$, this equation reduces to

$$Z^2I_d^2+2XE_2I_d+E_2^2=E_1^2,$$

which represents one of the ellipses of zero-load already determined.

The reactive current j, with respect to E_1, is obtained by writing the relation between E_2 and E_1, as it is depicted in Fig. 26, thus

$$E_2^2=E_1^2+(ZI)^2-2E_1Z(i\cos\gamma+j\sin\gamma),$$

or

$$E_2^2=E_1^2+Z^2\left[\left(\frac{P_2}{E_2}\right)^2+j^2\right]-2E_1\left(R\frac{P_2}{E_2}+Xj\right).$$

With no load, when $P_2=0$, this equation reduces to

$$E_2^2=E_1^2+Z^2j^2-2E_1Xj.$$

The curve of j as a function of E_2, drawn in rectangular co-ordinates, would therefore be a hyperbola; we can write

$$E_2^2-\left(Zj-\frac{E_1X}{Z}\right)^2=E_1^2\left(1-\frac{X^2}{Z^2}\right).$$

The horizontal axis is situated at a distance

$$\frac{E_1X}{Z^2},$$

above the axis of volts.

These formulæ enable the reactive currents to be calculated algebraically with more precision, but also much more laboriously, than by the graphical method.

Comparison of Outputs which the Same Alternator Can Develop with the Same Terminal Voltage, when Used Either as a Generator or as a Motor. Two different cases have to be considered:

(1) Let us suppose the case of an alternator connected to a constant potential *system* of great power, presenting a negligible impedance, and let the excitation be regulated in such a manner as to give the minimum current. Assuming a sufficient margin for regulating the excitation, the output of the machine will be limited in all practical cases by the value of the armature-current.

Let r equal the armature-resistance, x the armature-reactance, z the armature-impedance, U the voltage at the terminals, I the maximum allowable current. The internal E.M.F. of the corresponding generator, necessary to produce this current, will be obtained (Fig. 35) by combining the vector U with a vector $A_1A = zI$ drawn below U at an angle $= \gamma$, such that $\tan \gamma = \frac{x}{r}$. In order that the same machine, when operating as a motor, at the same voltage, U, may furnish the maximum power, UI, with the same current, I, the latter must have the same difference of phase, γ, with respect to U, but in the opposite direction. The internal E.M.F., $E_2 = OA_2$, will therefore be obtained by compounding OA with a vector zI equal, but opposed, to the preceding one.

Fig. 35 thus shows that the alternator corresponds to the same

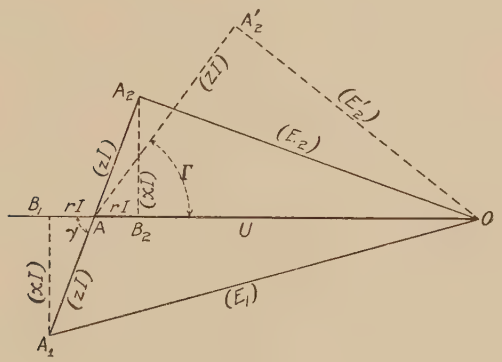


FIG. 35.

electric power at its terminals when operating as a motor and as a generator, and this with substantially the same excitation. Assuming the losses to be the same in both cases, it is seen that by increasing slightly the E.M.F. E_2, the same normal power may be counted upon for a motor as for a generator. (The normal power here designated is the power which the alternator can furnish on a dead resistance. This is the figure given in the catalogues of manufacturers of synchronous motors.)

(2) If now we consider the case where power is to be *transmitted to a distance*, the impedance Z which counts in the operation of the motor is that of the *entire circuit* and it can be notably superior

to the impedance of the armature. The segment ZI then extends as far as $A^1{}_2$, for example, making an angle Γ, which is such that $\tan \Gamma = \frac{X}{R}$; and the E.M.F., OA'_2, necessary to obtain this output may be superior to the preceding value. Eq. (25) gives the value thereof. It is necessary to foresee this condition, in studying an electric transmission project, whenever the impedance of the line is of the same order as that of the alternators.

IV. INFLUENCE OF MOTORS ON THE GENERAL OPERATION OF AN A.C. ELECTRICAL DISTRIBUTION SYSTEM

Effect of Current of Synchronous Motors on Distribution-Systems. There is a tendency, at the present time, and with good reason, to use synchronous motors under conditions which permit them, as if they were condensers, to compensate for the inductance of neighboring motors. This method, which consists in overexciting the synchronous motors, whether they be utilized for the production of mechanical power, or whether they be running without load, appears to have been first advocated in 1891 by Swinburn; and it has been used since by many engineers in Europe and in America. Following the advice of Dolivo-Dobrowolsky it was used for the Bulach-Oerlikon power-transmission in 1893 (see letter of Dolivo-Dobrowolsky to the Elektrotechnische Zeitschrift, 4. Oct., 1894, p. 555); and Lahmeyer also used it at Bockenheim (see Deutscher Verband der Elektrotechnischen Ingenieure, Congress of June, 1894), where he was thus enabled, it would seem, to double the output of the generators by compensating all inductance-effects in the distributing system by the reaction of synchronous motors. Some time after, Professor S. P. Thompson called attention to this method ("Some Advantages of Alternating Currents", by Professor S. P. Thompson, British Association, 1894).

To study this question, we must ascertain how the reactive current varies with respect to the voltage at the terminals of the generator.

A polar diagram referred to the line-voltage as a fixed axis gives the necessary values most accurately.[1] Let us take (Fig. 27, p. 44)

[1] We use here this form of diagram to take the ohmic loss of the motor into account. In practice, the latter can be neglected without great inconvenience, and the diagram referred to the motor E.M.F. E^2, which is more simple, will answer the purpose. We can then see more readily that the conditions corresponding to

on a power-line, P_2, two output-points, A_2 and A'_2, situated respectively at the right and at the left of the phase-line A_1N; and let us project them on that line, at a.

From what has already been seen, the lengths $\overline{A_2a}$ and $\overline{A_2'a}$ will represent the values of the reactive currents corresponding to the two outputs; but while the first, $\overline{A_2a}$, is lagging with respect to the working current, which is equivalent to the effect of a self-induction with all of its objections, the second, $\overline{A_2'a}$, is leading, which is equivalent to the effect of a condenser.

We therefore see that *for all values whose representative point is situated on the left of the phase-line A_1N there will be a reactive leading current.* It is this current, equivalent to that of a condenser, which enables the lagging reactive current to be neutralized, including the magnetization-current necessary for various translating appliances, such as induction-motors, transformers, induction-coils, etc., on the same portion of the circuit.

As we shall see later, it is very easy to calculate these compensation-phenomena with a view to their practical application.

In studying the effects produced by the reaction of synchronous motors, we should, at the outset, distinguish between two very different things, i.e., the load (current) on the line and the load (current) on the generating station supplying that line.

(1) **Compensation with Respect to the Line or Circuit.** Many ordinary translating appliances, notably transformers, arc-lamps, and especially induction-motors, have a power-factor, cos ϕ, which is lower than unity, and which increases uselessly the heating of the conductors of the distributing system or of the line, in the case of a simple transmission-system. [The loss of voltage in the line is likewise increased in greater proportion than the current, in consequence of lag, as the author has shown elsewhere. See articles on "Line-inductance for Alternating Currents", *l'Eclairage Electrique.* Oct., Dec., 1894.]

The aim of the compensation sought should be to increase this power-factor as much as possible, by producing the reactive currents locally instead of transporting them over the line.

The evaluation of these currents is not difficult, owing to the fact that in the translating appliances just mentioned the magnetizing

points situated on the left of the reference-line OY are those which produce a leading current with respect to the voltage at the terminals. It is therefore on the left of OY that the motor plays the role of a condenser.

(or reactive) currents are practically constant at all loads and can even be predetermined, at least roughly, according to the output of the appliances, without knowing exactly what these are. Let J_d be the total value of the reactive currents, and J_w be the total corresponding active current, at any given instant.

To neutralize completely the lag of the line, it would be necessary to produce, by means of the synchronous motors in that part of the line, a reactive current which is leading in phase and equal to

$$j = J_d,$$

(in absolute units). But it is not usually necessary to fulfill strictly this condition, because it may be, as a rule, considered sufficient to have a power-factor 0.95 to 0.97, corresponding to reactance-factors (tan γ) ranging in value from $\frac{1}{3}$ to $\frac{1}{4}$, respectively. Let us take, for example, tan $\gamma = \frac{1}{3}$. It is then sufficient to satisfy the equation

$$\frac{J_d - i}{J_w + i} = \frac{1}{3},$$

in which i equals the active current absorbed by the synchronous motors, and which must be added to that of the apparatus, J_w (useful load).

From this we have

$$j = [J_d - \tfrac{1}{3}(J_w + i)],$$

This means that the reactive current which the synchronous motors must produce to render the reactance-effects of neighboring motors practically negligible need not be equal to the current actually consumed by these appliances, but may be reduced more and more as their useful load J_w is increased. Owing to this circumstance, one or more relatively small synchronous motors will suffice, in practice, to neutralize the harmful effects of reactance on the line.

Let us suppose, for simplicity, that there is only one such compensator-motor, and that it is also serving, at the same time, as a motor doing mechanical work. Its total armature-current will be equal to the resultant

$$I = \sqrt{i^2 + j^2} = i\sqrt{1 + \left(\frac{j}{i}\right)^2}.$$

The current I being limited by the heating of the motor, the useful current i and, consequently, the useful corresponding power P_2, will

have to be all the lower the greater the value of j. If it is satisfactory for example, to take $j=\frac{1}{3}i$, the increase of I will be almost negligible.

The graphical determination of I and E_2, is obtained readily from the fundamental diagram (Fig. 26) by constructing (Fig. 36), according to the scale of volts, the lines Zi in the direction A_1N, and Zj in the perpendicular direction, and by drawing A_1A_2 and OA_2. If I and j are known beforehand, E_2 and i can be deduced inversely. [All that is necessary is to project the broken line A_2A_1O on the line A_1N and on the line perpendicular thereto; the two projections are

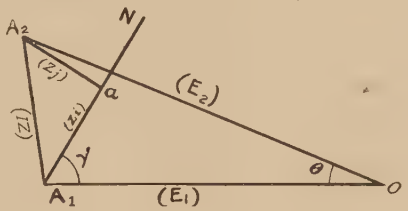


FIG. 36.

$$Zi-E_1\cos\gamma$$

and

$$Zj+E_1\sin\gamma.$$

The sum of their squares gives E_2^2, from which we deduce the formula, (26)].

This diagram also gives the algebraical expression of E_2. thus

$$E_2=E_1\sqrt{1+\left(\frac{ZI}{E_1}\right)^2+\frac{2Z}{E_1}(j\sin\gamma-i\cos\gamma)}. \quad . \quad . \quad (26)$$

From this, given the excitation-curve of the motor, the necessary ampere-turns can be deduced.

In order that the compensation may be really advantageous for the line, it is necessary that the resultant currents should be notably weaker than before, i.e., we must have

$$\sqrt{(J_w+i)^2+(J_d-j)^2}<\sqrt{J_w^2+J_d^2}.$$

In practice, compensation of this kind is especially advisable when synchronous apparatus (motors or converters) is already connected on the distribution-system, and serves for other purposes; because, all that is then necessary is to increase slightly the excitation-current of each of them, which involves no special expenditure of energy other than a slight supplementary loss in the fields and in the armature. The companies which operate alternating-current systems should therefore

seek to encourage as much as possible the use of such apparatus by their patrons.

When no such apparatus exists on the system, it may be desirable to place synchronous motors running without load at the distributing centers for the special purpose of furnishing reactive current. Let us see what the nominal power of such a motor should be.

The reactive current i being then only the current necessary to keep the apparatus in motion, if we designate by p the sum of the losses without load (i.e., friction, hysteresis, eddy currents and power necessary for excitation), we will have, approximately,

$$i=\frac{p+rI^2}{E_1}.$$

If the apparatus is efficient i^2 will be very small with respect to j^2 and consequently we can write, as an approximation,

$$I=j.$$

[In a good motor the active current with no load, i, does not exceed 0.10 or 0.20 of the full-load current. The lag of the current of an over-excited motor running without load can therefore approach very near $\frac{\pi}{2}$, without nevertheless approaching as closely thereto as would the current of a condenser.]

Since θ is very small, under these conditions, E_2 becomes approximately

$$E_2=E_1+Zj.$$

The power for which the motor is rated by the manufacturer is equal to the product of the ordinary E.M.F. (usually with a margin of 15 to 25 per cent increase), by the normal current. For example, under this hypothesis, it would be possible to use a motor having a catalogue voltage of $\frac{E_2}{1.25}\left(\text{more generally, } \frac{E_2}{1+\varepsilon}\right)$ and having the normal current I. Its rated power will be

$$P_1=\frac{1}{1.30}j(E_1+Zj),$$

in which j has the value already given. This value of j shows that, in an installation comprising a certain number of machines in permanent operation, the capacity of synchronous motor necessary for compensation may be taken lower in proportion as the load-factor will be higher,

and that, if sufficient compensating effect has been obtained at light loads, the compensating effect will be still better with heavier loads.

First Numerical Example. Assume a distribution-system supplying 500 k.w. of transformers at 5000 volts. The magnetizing current is a small fraction of the full-load current, varying usually from 2 per cent to 5 per cent, according to whether the transformers are large or small. Let the average be taken at 3.5 per cent. The full-load current being 100 amperes, the total magnetizing current, which is approximately constant at all hours of the day, will be 3.5 amperes. The power-factor with no load being, on the average, 0.70, the active current with no load will also be 3.5, and the resultant current will be $\frac{3.5}{0.7} = 5$ amperes. To neutralize the lag by means of a synchronous motor having a rated efficiency of 0.80 when running without load, it would be necessary to produce a reactive current of 3.5 amperes at the expense of 0.7 ampere of active current corresponding to the resistance-losses. The current taken from the distribution-system would then be $3.5 + 0.7 = 4.2$ amperes, instead of 5 amperes. This saving is not sufficient to justify the purchase and maintenance of a motor intended to serve solely as a compensator; but the use of a condenser would not be much more advantageous, as the current would always remain slightly above 3.5 in consequence of dielectric hysteresis.

Second Numerical Example. Assume a distribution-system comprising, at the end of the line, where the tension is 2000 volts, some translating devices composed of 200 k.w. of induction-motors and 100 k.w. of synchronous motors, the latter having a total reactance of 15 ohms and a resistance of 0.8 ohm. Assume the synchronous motors to have an efficiency of 80 per cent, the induction-motors to have a magnetizing current equal to one-third of the normal current, an efficiency of 85 per cent, and a power-factor of 0.95, at full load.

[The magnetizing current generally varies between one-third and one-quarter of the load-current in polyphase motors and between one-half and one-third in single-phase motors.]

The problem assumed is: to compensate the effects of the lagging current in the induction-motors when running with no load, and also with a quarter-load, it being assumed that all the motors are constantly in operation.

The total current necessary for the induction-motors at full load is

$$\frac{200000}{0.85 \times 0.95 \times 2000} = 124 \text{ amperes.}$$

The reactive current will be, at full load, approximately

$$=\frac{124}{3}=42 \text{ amperes.}$$

It can be assumed that the corresponding active current, at no load, is 10 per cent of the total, or 12 amperes.

Since the synchronous motors, in this case, run without load, it would be necessary, in order to compensate absolutely for the phase-lag, to make them produce also a reactive current

$$j=42 \text{ amperes;}$$

and since their active current attains, at most,

$$i=\frac{0.20\times 100000}{2000}=10 \text{ amperes,}$$

these motors will take a maximum current equal to

$$I=\sqrt{(10)^2+(42)^2}=42.1 \text{ amperes,}$$

while their normal-load current might attain

$$\frac{100000}{0.80\times 2000}=62.5 \text{ amperes.}$$

The E.M.F. is given by the approximate formula

$$E_2=2000+(15\times 42)=2630 \text{ volts.}$$

The line-current, which, without compensation, would be

$$\sqrt{(10+12)^2+(42)^2}=50.5 \text{ amperes}$$

(or at least 42.1 amperes, if the synchronous motors were not operating) would thus be reduced, by the compensation, to

$$10+12=22 \text{ amperes.}$$

It might even be reduced to 19 amperes, if only two-thirds of the synchronous motors were being operated.

Let us suppose, now, that each group of motors shall furnish 50 k.w. The active current for the induction-motors can be estimated at 25 amperes, plus 8 amperes for losses, or in all 33 amperes; and the active current for the synchronous motors can be estimated at 25 amperes, plus the losses, which are at most 10 amperes, or, in all, 35 amperes.

The total active current is therefore $33+35=68$ amperes, while the reactive current of the induction-motors remains, as before, 42 amperes. The resultant current is

$$\sqrt{(42)^2+(68)^2}=92.5 \text{ amperes},$$

and it could not be less than 68 amperes.

Suppose it be required to bring up the power-factor to 0.95, i.e., to bring the current down to 72 amperes. It will be sufficient to reduce the reactive current to $\frac{68}{3}=23$ amperes by producing a current out of phase which is equal to

$$j=42-23=19 \text{ amperes}.$$

The current of the synchronous motors will then become equal to about

$$\sqrt{(19)^2+(35)^2}=40 \text{ amperes}$$

instead of 35 (i.e., it would be slightly increased). We will have $\tan\gamma=\frac{19}{35}=0.543$ and the E.M.F. will be equal to

$$E_2=2000\sqrt{1+\left(\frac{15\times 40}{2000}\right)^2+\frac{15\times 2}{2000}(19\times 0.4771-35\times 0.8788)}$$
$$=1728 \text{ volts}.$$

instead of 1486 volts, which the motor should develop, when the phase-angle γ equals zero.

It is seen that, by overexciting to a maximum of 30 per cent, it is possible to compensate for the full-load reactions, without overloading the synchronous motors. To obtain the same result by means of a motor running without load it would be necessary to use an apparatus having a rating of about 40 k.w.; which would not be very practical.

Economic Study of Compensation for the Line by Means of Motors Running without Load. For simplicity, let us suppose the following to be constant: the voltage at the sending and receiving ends, the power to be transmitted, and the loss of energy in the line. It is required to determine the economy resulting from compensation by overexcited synchronous motors running without load, for the different values of cos ϕ.

Two points have to be considered:

(1) The saving in the capital invested;

(2) The saving in annual cost of operation.

These two points lead, as will be seen, to conclusions which are appreciably different:

(1) **Saving in Cost of Equipment.** Let (for a single-phase current), E_0 = the voltage at the transmitting end of the line; E_1 = the voltage at the receiving end; E_2 = the E.M.F. of the synchronous motors; I_w = the useful active current; i = the active current of the synchronous motors; $I_d = I_w \tan \phi$ = the reactive current when there is no compensation; j = the reactive current of the synchronous motors; I_1 = the resultant current of the synchronous motors; p_0 = the cost of the generators per kilovolt-ampere; p_1 = the cost of the synchronous motors per kilovolt-ampere; p_2 = the price of line-copper per kilovolt-ampere transmitted.

The price of line-copper, when there is no compensation, is given by the formula

$$p_2 \frac{E_1 I_w}{\cos \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

(which includes the return-conductor). In this formula the factor p_2 is equal to

$$p_2 = 65 \times b \frac{1+\varepsilon}{\varepsilon} \times \frac{l^2}{E_0^2} \times 10^4, \quad . \quad . \quad . \quad . \quad . \quad (28)$$

wherein l = length, in kilometers; b = price of copper per kilogram, (assumed = 2.50 francs).

ε = the percentage of loss allowed in the line; E_0 = the initial E.M.F. in volts.

The voltage being supposed constant at the transmitting and receiving ends, it follows that, in order to transmit the same power with the

same losses in the line for the different values of cos ϕ, the weight of the line-copper will have to vary.

The cost, in francs,[1] of the line-copper, *when there is no compensation*, is

$$p_2 \frac{E_1 I_w}{\cos \phi};$$

and, *when there is compensation*, it is

$$\frac{(I_w+i)^2+(I_w \tan \phi - j)^2}{\frac{I^2}{\cos^2 \phi}} \times p_2'' \frac{E_1 I_w}{\cos \phi}.$$

The cost of the generators, *without compensation*, is

$$p_0 \frac{E_0 I_w}{\cos \phi},$$

and, *with compensation*,

$$\sqrt{\frac{(I_w+i)^2+(I_w \tan \phi - j)^2}{\frac{I_w}{\cos \phi}}} \times p_0 \frac{E_0 I_w}{\cos \phi}.$$

Finally, the cost of the synchronous motors is

$$p_1 E_1 I_1,$$

wherein

$$I_1 = \sqrt{i^2+j^2}.$$

The total saving in cost of equipment, as the result of compensation, will be (in francs)[1]

$$p_2 \frac{E_1 I_w}{\cos \phi}\left[1 - \frac{(I_w+i)^2+(I_w \tan \phi - j)^2}{\left(\frac{I_w}{\cos \phi}\right)^2}\right] + \\ + p_0 \frac{E_0 I_w}{\cos \phi}\left[1 - \frac{\sqrt{(I_w+i)^2+(I_w \tan \phi - j)^2}}{\frac{I_w}{\cos \phi}}\right] - p_1 E_2 I_1. \quad . \quad (29)$$

[1] The corresponding cost value in *dollars* will be given by the same formula by introducing the factor 0.20 in the formula. The same remark applies to all the formulæ which follow in which cost-values are given in francs.

(2) **Saving in Annual Operating Cost.** The annual operating expenses include depreciation and interest on the capital invested, and the cost of the extra energy expended in the generators and synchronous motors.

The energy expended in the line is supposed to be the same with or without compensation.

Let us assume the following figures:

Depreciation of copper	3 per cent;
" of machines	10 per cent;
Interest on capital	5 per cent;
Price of one k.w.-hour of energy	10 centimes (2 cents).

Let I′ equal the resultant current at the generators when there is compensation; and let us assume that the resistance-loss in the generators, at full load, amounts to 4 per cent of the normal effective output.

Under these conditions, the saving in resistance-loss in the generators *when there is compensation*, will be (in francs)

$$0.04 E_1 I_w \left[\left(\frac{1}{\cos \phi} \right)^2 - \left(\frac{I'}{I_w} \right)^2 \right] \times \frac{0.10}{1000} \times h, \quad . \quad . \quad . \quad (30)$$

wherein h is the total number of hours of operation of the installation.

The saving in interest and depreciation, for the machines, will be given by the sum

$$p_2 \frac{E_1 I_w}{\cos \phi} \left[1 - \frac{(I_w + i)^2 + (I_w \tan \phi - j)^2}{\left(\frac{I_w}{\cos \phi} \right)^2} \right] \times (0.03 + 0.05) +$$

$$+ \left\{ p_0 \frac{E_0 I_w}{\cos \phi} \left[1 - \frac{\sqrt{(I_w + i)^2 + (I_w \tan \phi - j)^2}}{\frac{I_w}{\cos \phi}} \right] - \right.$$

$$\left. - p_1 E_2 I_1 \right\} (0.10 + 0.05). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Finally, the cost of the energy used by the synchronous motors will be (in francs)

$$E_1 i \times \frac{0.10}{1000} \times h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Numerical Example. Let us apply the preceding formulas to a particular case.

Assume a transmission of 500 k.w. to a distance of 24.75 kilometers with 5000 volts at the receiving end, and a loss of 10 per cent in the line.

Suppose it be desired to effect complete compensation by means of synchronous motors running without load. These motors must produce a reactive leading current

$$j = I_w \tan \phi.$$

Let us suppose, moreover, that the active current of the motors, when running without load, is

$$i = 0.10 I_1.$$

Since we have

$$I_1 = \sqrt{i^2 + j^2} = \sqrt{i^2 + (I_w \tan \phi)^2},$$

we find

$$i = \frac{I_w \tan \phi}{\sqrt{101}},$$

and

$$I_1 = \frac{I_w \tan \phi}{\sqrt{0.99}}.$$

Formula (28) gives

$$p_2 = 436.36 \text{ francs } (\$87.27).$$

The voltage at the receiving end being 5000 volts, and the loss in the line 10 per cent, the initial voltage must be

$$E_0 = \frac{E_1}{0.9} = 5550 \text{ volts.}$$

Let us assume the cost of the generators and motors to be

$$p_0 = p_1 = 100 \text{ francs } (\$20) \text{ per kilovolt-ampere.}$$

TABLE I

SAVING IN COST OF EQUIPMENT

[N.B.—The capital values given in columns *H*, *I*, *J* are in francs. To obtain the values in dollars remove the decimal points one place to the left and multiply by 2, i.e., apply the factor 0.20.]

Power factor, cos ϕ.	Reactive Current to be Neutralized: $i=I_w \tan \phi$.	Resultant Current of Synchronous Motors: $I_1=\frac{i}{\sqrt{0.99}}$.	Active Current of Synchronous Motors: $i=0.1I_1$.	Corresponding E.M.F. of Synchronous Motors: $E_2=E_1+ZI_1$.	Line Current (*with* Compensation), $I_w+i=I'$.	Saving in Cost of Copper. (*a*)	Saving in Capital Invested for		
							Generators. (*b*)	Synchronous Motors. (*c*)	Resultant Total. $a+b+c$
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
0.99	14.25	14.33	1.433	5215	101.433	− 1851.2	− 235.7	− 7473.0	− 9560.0
0.97	25.00	25.15	2.515	5375	102.515	+ 2519.2	+ 302.0	− 13518.0	− 10696.8
0.95	32.88	33.044	3.3044	5495	103.304	+ 8500.0	+ 1093.6	− 18009.0	− 8415.4
0.93	39.5	39.61	3.961	5544	103.961	+ 15210.1	+ 1983.3	− 21959.8	− 4766.5
0.91	45.57	45.8	4.58	5687	104.58	+ 22609.4	+ 2949.0	− 26086.5	− 528.0
0.88	53.98	54.25	5.425	5814	105.425	+ 34537.2	+ 4564.4	− 31541.0	+ 7560.6
0.85	61.96	62.27	6.227	5934	106.227	+ 47409.6	+ 6346.3	− 36951.0	+ 16805.0
0.80	74.99	75.37	7.537	6130	107.537	+ 70881.8	+ 9215.3	− 46202.0	+ 33895.0
0.75	88.19	88.63	8.863	6330	108.863	+ 96960.0	+ 13592.6	− 56103.0	+ 54449.6
0.70	102.0	102.5	10.25	6537	110.25	+126046.7	+ 18114.4	− 67004.0	+ 77157.0
0.65	116.9	117.5	11.75	6762	111.75	+158568.0	+ 23389.0	− 79453.5	+102503.5
0.60	133.3	134.0	13.4	7010	113.4	+195272.0	+ 29592.6	− 93934.0	+130931.3
0.55	151.8	152.6	15.26	7290	115.25	+237263.0	+ 36980.0	−111445.0	+162798.0
0.50	173.2	174.1	17.41	7610	117.41	+285993.0	+ 46550.0	−132490.0	+200053.0
0.40	229.0	230.2	23.02	8453	123.02	+413564.0	+ 70555.5	−194588.0	+289531.5

TABLE II

SAVING IN OPERATING COST

Power-factor, cos ϕ.	Reactive Current to be Neutralized: $i = I_w \tan \phi$.	Resultant Current of Synchronous Motors: $I_1 = \frac{i}{\sqrt{0.99}}$.	Active Current of Synchronous Motors: $i = 0.1 I_1$.	Corresponding E.M.F. of Synchronous Motors: $E_2 = k_1 + Z I_1$.	Line Current (*with* Compensation): $I_w + i = I'$.	Saving in Cost of Operation per Year.							Assuming 2000 Working Hours per Year.		
						Assuming 3000 Working Hours per Year.							Annual Loss of Energy.		Resultant Total Annual of Cost.
						Interest and Depreciation.		By Resistance Heating of Generators: $\frac{0.04 E I_w}{10 \times 1000}\left[\left(\frac{1}{\cos \phi}\right)^2 - \left(\frac{I'}{I_w}\right)^2\right]$.	In Synchronous Motors: $\frac{0.10 \times E I_1 \times 0.10}{1000} \times 3000$.	Resultant Total of Annual Cost.	By Resistance Heating of Generators.	In Synchronous Motors.	
A	B	C	D	E	F	G	H	I	J	K	L	M	N
0.99	14.25	14.33	1.433	5215	101.433	— 148.09	— 1056.3	— 35.4	— 2149.5	— 3389.3	— 23.6	— 1433.0	— 2661.0
0.97	25.00	25.15	2.515	5375	102.515	+ 201.5	— 1082.4	— 70.8	— 3772.5	— 5483.5	— 47.2	— 2515.0	— 4248.7
0.95	32.88	33.044	3.304	5495	103.304	+ 680.0	— 2537.3	+ 246.0	— 4956.0	— 6567.9	— 164.0	— 3304.4	— 4997.7
0.93	39.5	39.61	3.961	5544	103.961	+ 1216.8	— 2996.5	+ 409.2	— 5941.5	— 7312.0	— 272.6	— 3965.0	— 5468.0
0.91	45.57	45.8	4.58	5687	104.58	+ 1868.75	— 3470.6	+ 683.4	— 6870.0	— 7848.4	— 455.6	— 4580.0	— 5781.2
0.88	53.98	54.25	5.425	5814	105.425	+ 2762.98	— 4046.5	+ 1079.4	— 8137.5	— 8341.6	— 719.6	— 5425.0	— 5989.0
0.85	61.96	62.27	6.227	5934	106.227	+ 3792.77	— 4590.7	+ 1536.0	— 9340.5	— 8602.4	— 1024.0	— 6227.0	— 6000.0
0.80	74.99	75.37	7.537	6130	107.537	+ 5670.54	— 5548.0	+ 2436.6	— 11305.5	— 8746.4	— 1624.4	— 7537.0	— 5790.0
0.75	88.19	88.63	8.863	6330	108.863	+ 7756.8	— 6376.5	+ 3536.8	— 13294.5	— 8765.4	— 2371.2	— 8863.0	— 5111.5
0.70	102.0	102.5	10.25	6537	110.25	+ 10083.74	— 7333.4	+ 4953.0	— 13375.0	— 7071.7	— 3302.0	— 10250.0	— 4197.7
0.65	116.9	117.5	11.75	6762	111.75	+ 12685.4	— 8409.7	+ 6799.2	— 17625.0	— 6040.0	— 4473.4	— 11750.0	— 3001.9
0.60	133.3	134.0	13.4	7010	113.4	+ 15621.8	— 9051.2	+ 8946.0	— 20100.0	— 5183.4	— 5964.0	— 13400.0	— 1405.4
0.55	151.8	152.6	15.26	7290	115.25	+ 18981.0	— 11109.7	+ 11866.2	— 22890.0	— 3152.7	— 7910.8	— 15260.0	+ 522.1
0.50	173.2	174.1	17.41	7610	117.41	+ 22879.4	— 12891.0	+ 15729.0	— 26115.0	— 397.6	— 10486.0	— 17410.0	+ 3064.4
0.40	229.0	230.2	23.02	8453	123.02	+ 33085.12	— 18064.8	+ 28419.6	— 34530.0	+ 8370.0	— 18946.4	— 23020.0	+ 10406.7

TABLE III

NUMERICAL EXAMPLE

(Data for curves in Figs. 39 and 40.)

Cos ϕ.	Reactive Current with Synchronous Motors only: $\frac{I_w}{2 \cos \phi}$.	Resultant Reactive Current with Synchronous Motors: $\frac{I_w}{2 \cos \phi}$.	Corresponding E.M.F. of Synchronous Motors: e_1.	Saving in Capital Invested in				Saving in Annual Cost for				
				Line Copper.	Generators.	Synchronous Motors.	Total Saving.	Interest and Depreciation for Copper.	Interest and Depreciation for Machines.	Loss by Resistance of Generators.	Loss Due to Lower Efficiency of Synchronous Motors.	Total Saving.
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*
1.00	0.00	50.0	4250									
0.99	14.25	50.5	4280	− 4393.6	+ 550.0	− 150	+ 4763.6	+ 349.0	+ 60.0	+ 138	−6000	− 5453.0
0.95	32.88	52.63	4407	− 22363.6	+ 2832.5	− 785	+ 24411.1	+ 1789.0	+ 307.1	+ 648	−6000	− 3256.0
0.93	39.5	53.76	4440	− 31636.4	+ 4125.0	− 950	+ 34811.4	+ 2530.9	+ 476.2	+ 936	−6000	− 2057.0
0.88	53.98	56.82	4588	− 55937.8	+ 7502.0	− 1690	+ 61749.8	+ 4475.0	+ 871.8	+ 1748	−6000	+ 1095.0
0.80	74.99	62.5	4822	− 94141.4	+13750.0	− 2860	+105031.4	+ 7531.3	+1633.5	+ 3378	−6000	+ 6543.0
0.70	102.0	71.43	5140	−158967.3	+23573.0	− 4450	+178090.3	+12717.4	+2868.4	+ 6246	−6000	+15832.0
0.60	133.3	83.33	5488	−232734.5	+36668.5	− 6190	+236213.0	+18618.8	+4571.9	+10668	−6000	+27858.7
0.50	173.3	100.0	5895	−327272.7	+55000.0	− 8225	+374047.7	+26181.8	+7016.25	+18000	−6000	+45198.0
0.40	229.0	125.0	6423	−454141.4	+82500.0	−10865	+525776.4	+36331.3	10745.25	+31344	−6000	+72421.0

To estimate the size of the motors required, we will suppose them constructed for an E.M.F. equal to that which they should produce when run as compensating machines.

For the sake of economy, the manufacturers tend more and more to increase the magnetic density, i.e., to 'saturate" their machines, and it is scarcely prudent at present to count upon being able to increase very much their counter E.M.F. by increasing the excitation.

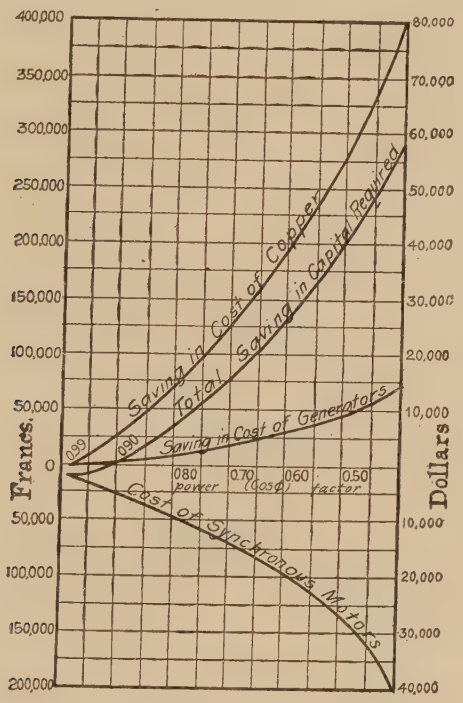


FIG. 37.

From the calculations corresponding to the values of cos ϕ comprised between 1 and 0.40, we obtain the curves given in Figs. 37 and 38. The curves in Fig. 37 give the saving in cost of equipment and the curves in Fig. 38 give the saving in annual expense of operation. It will be seen that if, from the standpoint of cost of equipment, the use of motors running without load becomes rapidly advantageous as cos ϕ decreases, it is different, in any particular case, from the standpoint of the annual cost of operation.

The depreciation of the machinery, which is higher than that of copper and also the consumption of energy required by the motors, (which latter is not compensated by the decrease in the losses of the generators), have the effect, together, of offsetting the saving made in copper, even through it may be somewhat important.

Economy of Compensation for the Distributing System by Means of Synchronous Motors Running with Load. There may be some

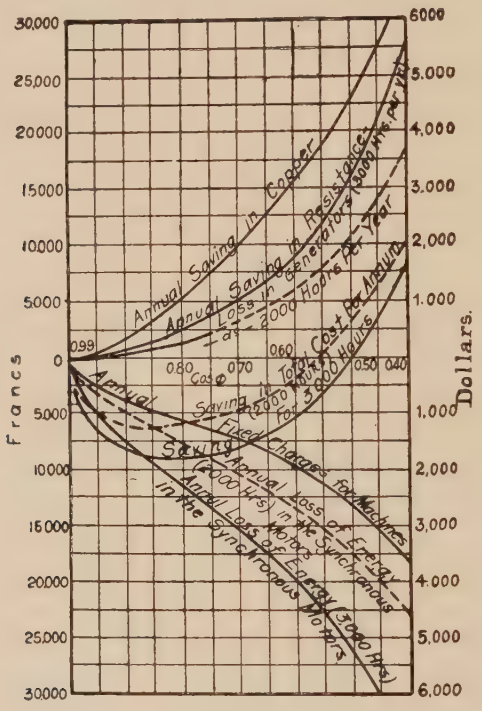


FIG. 38.

hesitation in regard to the choice, in this case, because the efficiency of synchronous motors is generally lower than the efficiency of induction motors, and, besides, in order to produce a leading current, they must have a counter E.M.F. higher than that of induction-motors. The effect of this is to increase their rating and their cost. It is therefore better, in each particular case, to consider whether the use of synchronous motors is advantageous or not, from an economic point of view, due consideration being given to the necessities of the case.

Suppose a long-distance power-transmission project in which it is possible to connect, with the distributing system, either some induction motors working with a certain lag, or else an equipment composed of one-half induction motors and one-half synchronous motors.

The line-current, with the induction motors alone, and with a phase-angle ϕ, would be equal to $\frac{I_w}{\cos \phi}$. With half the capacity in synchronous motors added, and effecting complete compensation, the current would be equal to I_w.

It should be noted immediately that the loss by resistance-heating in the motors will be exactly the same in the two cases, because the magnetizing current of the synchronous motors has the same value as the reactive current of the induction-motors, it being, however, a current leading in phase.

Under these conditions, in order to maintain the loss of energy constant in the line, for different values of cos ϕ, the cost of line-copper, when the induction-motors alone are used, should be

$$p_2 \frac{E_1 I_w}{\cos \phi};$$

and, with complete compensation by means of synchronous motors which carry half the load, the price of line-copper would be

$$p_2 E_1 I_w.$$

For equal losses the cost of line-copper is proportional to the square of the current; the saving effected in the cost of copper, by introducing synchronous motors, would therefore be

$$p_2 \frac{E_1 I_w}{\cos \phi} - \frac{p_2 E_1 I_w}{\cos \phi} \times \frac{I_w^2}{\frac{I_w^2}{\cos^2 \phi}},$$

or

$$p_2 E_1 I_w \left(\frac{1}{\cos \phi} - \cos \phi \right).$$

The cost of generators, without compensation, would be

$$\frac{p_0 E_0 I_w}{\cos \phi};$$

and, with compensation, it would be

$$p_0 E_0 I_w.$$

The saving in cost of generators, in the second case, would, therefore, be

$$p_0 E_0 I_w \left(\frac{1}{\cos \phi} - 1 \right).$$

Let e equal the counter E.M.F. of synchronous motors operating with a magnetizing current $\frac{I_w}{2} \tan \phi$ which is *lagging*. Let e_1 equal their counter E.M.F. with the same magnetizing current *leading* in phase.

The values of e and e_1 are deduced from formula (26) which gives

$$e = E_1 \sqrt{1 + \left(\frac{ZI}{E_1} \right)^2 + \frac{2Z}{E_1} (j \sin \phi - i \cos \phi)},$$

in which E_1 is the potential difference applied at the terminals of the motor and I the resultant current.

The nominal power of the motors and, consequently, their cost, would be increased, in the second case, in the ratio $\frac{e_1}{e}$, and the increase in purchase-price of the synchronous motors, resulting from the introduction of compensation, and on the assumption that the price per k.w. of the synchronous motors and induction-motors is equal, would be

$$p_1 \left(e_1 \frac{I_w}{2 \cos \phi} - e \frac{I_w}{2 \cos \phi} \right).$$

Let us see, now, what would be the saving in annual operating expenses:

The saving in interest and depreciation, for copper, is

$$p_2 E_1 I_w \left(\frac{1}{\cos \phi} - \cos \phi \right) (0.03 + 0.05).$$

The saving in interest and depreciation, for the machines, is

$$\left[p_0 E_0 I_w \left(\frac{1}{\cos \phi} - 1 \right) - p_1 \left(e_1 \frac{I_w}{2 \cos \phi} - e \frac{I_w}{2 \cos \phi} \right) \right] (0.10 + 0.05).$$

The saving in loss by resistance-heating in the generators (which is proportional to the square of the current), is

$$0.04 E_1 I_w \left(\frac{1}{\cos^2 \phi} - 1 \right) \times h.$$

Finally, we will assume the efficiency of the synchronous motors to be 8 per cent lower than that of induction-motors, which is rather

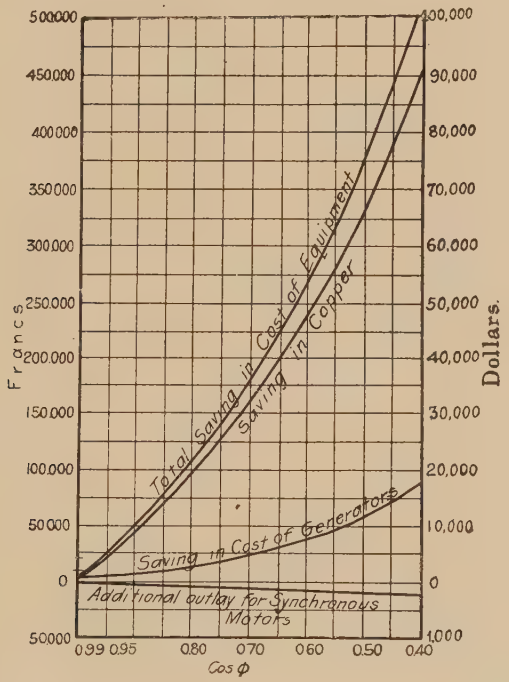


FIG. 39.

exaggerated, because synchronous motors are actually constructed whose efficiency approaches that of induction-motors. We will assume, however, that the efficiency is lower, in order to justify the assumption of equality of cost per k.w. for the two classes of motors.

The increase in annual cost of the energy expended in the synchronous motors, resulting from that difference in the efficiency, would be

$$0.08 \times \frac{I_w}{2} E_1 \times \frac{0.10}{1000} \times h,$$

in which, as before, h is the total number of hours that the plant operates per annum.

Numerical Example. Let us take the same case of power-transmission already mentioned in which the total power to be transmitted k.w. is 500 the distance 10 kilometers, and the potential difference, at the receiving point, 5000 volts.

Let us assume that the total impedance of the synchronous motors

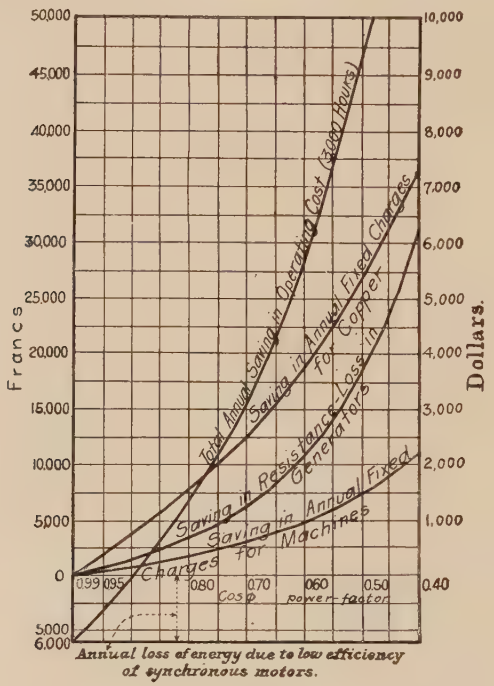


FIG. 40.

TABLE III

is 15 ohms, and that the plant is in operation a total of 3000 hours per annum. On substituting the corresponding numerical values in the preceding formulæ, and making the calculations we obtain data from which Table III (page 78) has been prepared, the results being shown graphically in Figs. 39 and 40.

The line parallel to the axis, in Fig. 40, represents the annual loss of energy due to the decrease of efficiency of the synchronous motors.

It is seen that the curve of the saving which results from the use of synchronous motors working jointly with induction-motors, rises rapidly, and that it becomes positive for values of cos ϕ which are all the greater the better the efficiency of the synchronous motors.

Regulation of Distribution-Voltage. The over-excitation of synchronous motors can, incidentally, enable the voltage to be raised

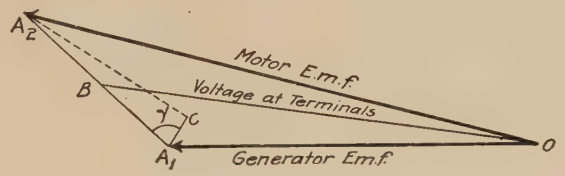


FIG. 41.

at the terminals of motors and transformers connected with the line, and even at the terminals of the generator. The possibility of this is readily seen in the case where the line has a negligible resistance and reactance, and where there is only one generator and one motor which are both alike. The difference of potential at the terminals (Fig. 41), is OB (B being at the middle of A_1A_2), and by increasing

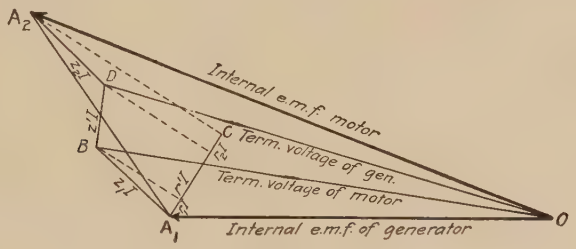


FIG. 42.

OA_2 this difference of potential can always be made as large as desired.

In the general case, by drawing, successively, with their phases, three vectors equal to the products of the current $\overline{A_1C}$ by the impedances of the generator, of the line, and of the motor, we obtain a broken line A_1BDA_2 (Fig. 42). The voltages at the terminals of the generator and of the motor are OB and OD; and A_2 can always be sufficiently increased so as to make OD and OB greater than OA_1.

Lahmeyer made an interesting application of this peculiarity at Bockenheim, where polyphase converters were operated like true synchronous motors. By increasing their excitation, the induced E.M.F. can be raised in such a way as to obtain a secondary voltage of distribution which is constant at all loads. The regulation can be made by hand or automatically. From this point of view rotary converters present a distinct advantage over stationary transformers, which introduce lags and whose secondary voltage necessarily decreases with the load.

(2) **Compensation with Respect to the Generators.** It is a two-fold disadvantage, for the generators at the power station, to have the current out of phase, first, because the total current is increased

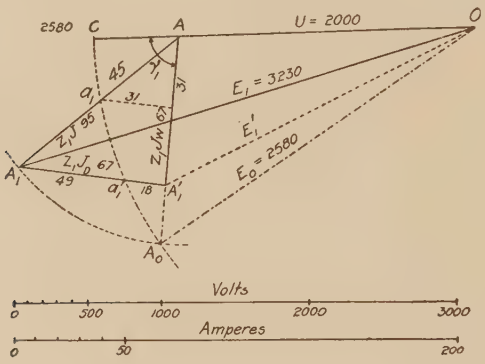


Fig. 43.

and, secondly, because the armature-reaction is greater. To demonstrate this beyond question, let us consider a generator supplying a distribution-system at constant potential U. Let J_w and J_d be the active and reactive currents supplied to that system, J being the resultant current. Let R_1, X_1, Z_1, respectively, be the resistance, reactance and impedance of the generator and of the feeders; and let them be considered constant, and *as if located in a single apparatus.*

Let $\tan \gamma_1 = \frac{X_1}{R_1}$ be the corresponding reactance-factor. Let us construct a diagram (Fig. 43) starting from the distribution-voltage U, which is to be maintained constant at all loads.

The necessary E.M.F. E_2 will be obtained by compounding, with the voltage $\overline{OA} = U$, two vectors representing the volts absorbed by the impedance under the action of the two currents J_w and J_d. The

first $\overline{AA_1'}=Z_1J_w$, makes, it is known, the angle γ_1 with the voltage U. The second, $\overline{A_1A_1'}=Z_1J_d$, will lag $\frac{\pi}{2}$ behind the first, since J_d is lagging $\frac{\pi}{2}$ behind J_w. The resultant A_1A corresponds to the real current

$$J=\sqrt{J_w{}^2+J_d{}^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and the line OA_1 represents the internal E.M.F. which the generator must produce in order that the distributing voltage shall equal U. The diagram gives the following formula, which is analogous to formula (26):

$$E_1=U\sqrt{1+\left(\frac{Z_1J}{U}\right)^2+2\frac{Z_1}{U}(J_w\cos\gamma_1+J^d\sin\gamma_1)}. \quad . \quad . \quad (34)$$

In this case γ_1 being always very large, A_1A_1' is almost parallel to U. In general, we can write, approximately,

$$E_1=\sqrt{(U+Z_1J_d)^2+(Z_1J_w)^2}. \quad . \quad . \quad . \quad . \quad . \quad (35)$$

It is seen that the reactive current requires a much greater increase of E.M.F. than the active current. If we had $J_d=0$, the alternator would develop the same power with a lower current, $\overline{AA_1'}$, and a lower E.M.F., $\overline{OA_1'}$.

Let us suppose the current J to be the maximum which the alternators can produce; and let us describe, around A as a center, a circle of radius $\overline{AA_1}=Z_1J$. The manufacturer must have allowed a sufficient margin in the excitation to attain the E.M.F. E_0, which enables this current to be produced through a dead resistance while keeping the distribution-voltage at the value U; but if he confined himself to doing that, as was his right, it would be necessary, when the lag increases, to reduce the output in such manner that the point A_1 will remain on the circle A_0C, described from O as a center, with E_0 as radius.

In such cases the effect of the reactive current becomes absolutely objectionable, and it is then very necessary to reduce it by employing means such as those just considered.

Fig. 43 can serve as a numerical example in the case of a 2000-volt distribution where the receiving apparatus has a mean power-factor of 0.70 and the generator has an impedance of 15 ohms (the

two scales being therefore in the ratio of 15 to 1). We will now study this example in detail.

Numerical Example. Suppose a case where energy is to be distributed for mechanical purposes by means of a line supplying a great number of induction-motors. During the day, when a portion of these motors are running at light load, their mean power-factor is about 0.70, i.e., the active current J_w is approximately equal to the reactive current. Let us assume that it has been decided that the total power thus distributed will be about 100 kw.

The distribution-voltage is 2000 volts; the line-current has been predetermined on the assumption of a power-factor of 0.70; and, therefore, assuming an average efficiency of 0.75, the current would be

$$J = \frac{100000}{2000 \times 0.75 \times 0.70} = 95 \text{ amperes.}$$

The line, having one ohm of resistance (its efficiency being about 95 per cent), and being supposed to have negligible inductance, gives a voltage-drop of 100 volts. An alternator has therefore been provided, of sufficient capacity to produce $95 \times 2100 = 200$ k.w., and it is believed that ample allowance has been made. This alternator will have, for example, $X_1 = 18$; $R_1 = 1.6$; whence $Z_1 = 13$ ohms, and tan $\gamma_1 = 8.3$. The machine furnished by the manufacturer is regulated for a dead-resistance load and, consequently, it is capable of giving, with its fields almost saturated, the E.M.F. which seems necessary for the full load, by making $OA = U = 2000$ volts, and $E_0 = OA_0 = 2580$ volts.

On putting the machine in operation an unexpected drop of voltage is noted, and it is impossible to obtain the 95 amperes calculated. In fact, if we draw the segment AA_1' at an angle of 45°, starting from OA_1, it is not 2580 volts which would be necessary, but practically,

$$E_1 = \overline{OA_1} = 3230 \text{ volts.}$$

Since the machine can only produce 2580 volts, the load-point will have to be moved to the point of intersection, a_1, of the angle-line γ, with the circle A_0C. The machine cannot, therefore, deliver more than 45 amperes at the voltage of 2100 volts; and, since the corresponding active current is only 32 amperes, it will be seen that the alternator, although bought for 200 k.w. and expected to be good for 100 k.w., can give, in actual service, only $21 + 2100 = 46$ k.w.; and yet the manufacturer is not to blame.

To overcome the difficulty, it will be sufficient to move the load-point to a_1', because the power will be that needed (the active current being $A_1'a_1'$) without requiring that the total current, Aa'_1, should materially exceed the predetermined current. This only requires that 49 amperes of reactive current should be eliminated from the 67 which exist. The most advantageous method of obtaining this result consists in replacing a few induction-motors by synchronous motors. If the change is made, for example, with a certain number of motors representing 0.75×2100 volts $\times 25$ amperes $= 39.5$ k.w. of mechanical power, the reactive current suppressed is already 25 amperes; and it is sufficient, then, to produce an equal current in the opposite direction, by means of synchronous motors. If we assume the latter to have the same efficiency. they will also absorb 25 amperes of active current, so that their total current will be about 35 amperes (same as that of the induction-motors which they replace.

Their E.M.F., calculated, in the manner above indicated, with the object of producing the 25 amperes of reactive current, will be only 2660 volts, if a single motor of 35 amperes be used, and 2525 volts if two motors of 17.5 amperes each are used, etc. These voltages can be attained, as a rule, with motors bought for running at 2000 volts.

These motors will necessarily be working under imperfect conditions, since they will be only partly loaded. The installation, as a whole, will, therefore, be losing an output of $\frac{35-25}{25} \times 40 = 16$ k.w. But, on the other hand, it will be gaining, in the generator-output, $(67-31)2100 = 76$ k.w.; hence there should be no hesitation in adopting this method.

The conditions taken, in this example, are not exaggerated, considering the apparatus which has been used until recently, and in which a drop of voltage of 30 per cent in the alternators under full load on a dead resistance was a frequent occurrence. These conditions occurred at Bockenheim, where, according to Lahmeyer, the output of the generators was doubled by introducing synchronous motors. These conditions would occur on many of the actual distribution systems if induction-motors were extensively introduced.

With the more recent types of alternators the conditions are better, and, consequently, the use of synchronous motors may be less advantageous. Nevertheless the reduction of generator-output by the use of synchronous motors can still, easily, amount to as much as 25 to

30 per cent. For this reason it is always desirable, in any extensive installation, to consider the expediency of utilizing a sufficient number of synchronous motors, to avoid this reduction of output.

Comparison between Synchronous and Induction-Motors. The preceding remarks show the great advantages which synchronous motors present in comparison with induction-motors, in certain cases. Their stability of operation is not inferior to that of induction-motors, because a single-phase induction-motor, and even a polyphase induction-motor, also falls out of step when too much overloaded, unless the secondary resistance has been made quite large, at the expense of efficiency. The two points of inferiority of synchronous motors are, first, the fact that they are not self-starting, and, second, their low efficiency. In reality, their inferiority in these two respects is not as great as supposed, because it is possible to make synchronous motors which are self-starting without load, and as for their efficiency, it can be made as high as that of induction-motors, if they are properly designed.

On comparing a synchronous motor of Kapp type, with non-laminated poles and expensive excitation, consuming, when running without load, a current equal to one-third the full-load current, with an induction-motor of the most perfect Oerlikon type (see article of Kolben, *Elektrotechnische Zeitschrift*, *Nov.* 1, 1894) it is not difficult to establish the superiority of the latter. But, if, in the said induction-motor, the bars of the rotor were to be transformed into a D.C. exciting circuit, it would be observed that the difference in efficiency becomes very small.

Synchronous motors have a great advantage in construction because they admit of much higher magnetic densities than induction-motors.

[**Use of Synchronous Motors to Raise Power-Factor in America.** The compensating action of over-excited synchronous motors is utilized extensively in America for improving the power-factor of distribution-systems.

As an interesting example of the practical application of synchronous condensers some figures obtained from the Detroit Edison Company are given in Appendix C. C. O. M.]

CHAPTER III

ADDITIONS TO THE THEORY. SECOND APPROXIMATION

Imperfections of the Theory. Owing to the hypotheses on which it is based the preceding elementary theory, in common with all theories concerning alternating current machines, has certain imperfections. We shall discuss these briefly.

(1) When the currents are not sinusoidal, the results may be materially modified. At this point, it may be stated that we should precisely endeavor to make machines giving sinusoidal currents for power transmission,[1] because, if their E.M.F. do not comprise exactly the same harmonics, the circuit becomes the seat of parasite currents, which are often very important. For example a certain Labour synchronous motor having a sinusoidal E.M.F. when run without load, being supplied with current from the distribution system of the Champs Elysées district, where the E.M.F. was non-sinusoidal, required a current almost double that which it consumed at the factory when supplied from an appropriate generator. This fact, among many others, shows that the effect of harmonics cannot, by any means, be considered negligible.

(2) The reactance X of the motor is not constant, but varies with the excitation, with the strength of the armature-current, and with its phase-difference with respect to the E.M.F. E_2.

In practice, the angle γ differs sufficiently from 90° to make its variations, under the influence of the preceding effects, very small. Consequently, the line of zero phase-difference, A_1N, in Fig. 27, is a curve which is always sufficiently flattened to warrant our continuing, without material error, to compare it to a straight line. But the power-

[1] This question was discussed by various authors in the *Electrical World*, which had the happy idea of opening its columns to that very interesting controversy. The greater portion of those who expressed their opinion, and among them the author of this work, were in accord in recognizing the important advantages of a sinusoidal E.M.F. from various points of view

circles will be more or less deformed by these variations of reactance. The saturation of the fields, for strong excitation-currents, reduces these deformations and raises the right-hand branch of the V-curves more rapidly toward the top than calculation would indicate. It is generally admitted that there is no other way of taking these deformations into account than by plotting the V-curves experimentally.

(3) No attention has been paid to the variation of losses in the armature other than the resistance-loss. It may be remarked that these losses will vary but little, since the friction is substantially constant, and, in a motor supplied from a constant potential, the resultant E.M.F. in the motor remains almost constant. Likewise the induction and the hysteresis losses and the eddy currents remain approximately constant.

The only variable losses are those due to eddy currents in the fields, and the energy expended for excitation, neither of which have been taken into consideration in what precedes.

Practically, therefore, no great error has been made in supposing the losses in the iron to be constant while increasing slightly the no-load losses (by way of compensation).

Attention is, therefore, called to this last source of imperfection merely to show its influence. In reality, the same thing might also be said in regard to the first two causes of imperfection, because the true object of this theory is only to elucidate *the general course of the phenomena*, and to give a method of calculation which will be sufficient for practical purposes. However, the effects to which attention is called above may assume sufficient importance, at times, to justify a somewhat more detailed investigation. This is more especially true in regard to variations of reactance.

Variations of Reactance with Lag of Current and Saturation of Fields. Armature-Reaction. The author has shown, elsewhere (*L'Industrie Electrique*, 1899, p. 481) that the self-induction of any alternator can be represented by two armature-reactions:[1]

(1) An armature-reaction directly opposed to the exciting ampere-turns and which is equivalent to

$$\tfrac{1}{2}KN'I_d\sqrt{2} = \frac{KN'I_d}{\sqrt{2}},$$

counter-ampere-turns. In this equation I_d = the effective amperes of reactive current, with respect to E_2; N' equals the number of armature *conductors* in the induced field; and K equals a coefficient which

[1] See Part II, Chapter I.

depends on the shape of the pole-pieces. The induction-flux is produced by the algebraical resultant of the exciting ampere-turns and of these counter-ampere-turns.

(2) A *transverse* reaction producing a flux which closes itself transversely in the pole-pieces, without penetrating the field-windings and having a numerical value equal to

$$\frac{L'I_w}{\frac{1}{\sqrt{2}}N'},$$

wherein L' = a coefficient.

This resolution, of which the validity has been shown elsewhere, takes into account the lag, since the current intervenes by its two components (active and reactive) with respect to the E.M.F. of the motor. [The words "active" and "reactive" are here taken relatively to an E.M.F. induced with open circuit. The phase-angle thus obtained is therefore slightly different from that which would be found at the terminals.] It also takes into account the saturation of the fields, since the resultant flux of the fields is calculated from the ampere-turns themselves, by reference to an excitation-curve obtained experimentally.

As to the transverse flux, it is substantially independent of the saturation, because the reluctance of the magnetic circuit through which it finds its path is substantially constant. That is why L' can be considered constant.

With this assumption, it is easy to establish a new diagram for synchronous motors which will take these reactions into account. It will be sufficient to segregate the motor-reactances. Let z and x equal the impedance and the reactance, respectively, of the generator and of the line, and of the magnetic leakage of the motor; and let R equal the resistance of the circuit, including the motor-resistance. (The generator-impedance may be neglected when the current is taken from a distribution-system.) Let ε_2 equal the effective motor E.M.F., i.e., the E.M.F. produced by the resultant fields of the field and armature ampere-turns.

The generator E.M.F. E_1 will be equal to the resultant of the following three components: First, the effective *counter E.M.F.*, $A_2O = \varepsilon_2$, with its sign reversed; second, the transverse reaction − E.M.F. $A_2B = \omega L'I_w$ with its sign reversed and having a phase-lead of $\frac{\pi}{2}$ ahead of ε_2; and,

third, all the load-losses produced by the impedance z; this latter being represented by a vector $BA_1 = zI$ (Fig. 44) which makes the angle ϕ with a line BD that is itself drawn at an angle δ with OA_2 such that

$$\tan \delta = \frac{\omega l}{R}.$$

The line BA_1 can serve to define the current I, in magnitude and in phase, if BY be taken as the axis of reference, with a scale Z times greater than the scale of volts.

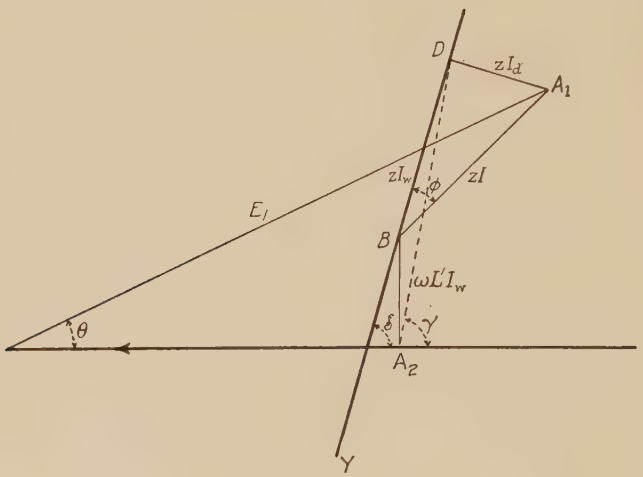


FIG. 44.

This vector zI can be resolved into two E.M.F.'s, viz.: $BD = zI_w$, produced by the active current, and $DA = zI_d$, produced by the reactive current.

The line A_2D, closing the triangle A_2BD, then represents the combination of the reaction – E.M.F.'s due to the active current, and it makes, with OX, the angle γ defined by the equation

$$\tan \gamma = \frac{\omega(L' + l)}{R}.$$

[It will be noted that the diagram differs from the preceding by the definitions of z and δ, which do not include the motor-reactance.]

Moreover, we will have

$$\varepsilon_2 = j\left(Ni - \frac{1}{\sqrt{2}} KN' I_d\right),$$

wherein Ni equals the exciting ampere-turns, and f equals the function which defines the law of variation of the induced E.M.F. with the total inducing ampere-turns (this law being defined, in practice, by means of a curve). The ampere-turns of the armature are, in fact, demagnetizing, whenever I_d lags behind ε_2, i.e., when the point A_1 is at the right of the axis BY; and they are, on the contrary, magnetizing, when A_1 is at the left.

The segments zI_w and zI_d again measure, in magnitude and in phase, the active and reactive amperes by reference to some suitable scale; and the phase-angles (lags) are measured from the line BY, which serves as axis of reference.

The output is the sum of the outputs corresponding to the currents I_w and I_d. The induced E.M.F. is composed of the E.M.F. ε_2, produced by the direct field, and of the E.M.F. $(\omega L' I_w)$, produced by the field of transverse reaction. The first is in phase with I_w and the second with I_d. We therefore have

$$P_2 = \varepsilon_2 I_w - \omega L' I_w I_d.$$

The diagram therefore gives us again all the conditions of operation of the motor without material increase in complication.[1]

First Application of Corrected Diagram. Determination of Reactive Current as a Function of the Excitation, with Constant Active Current. The preceding diagram enables us easily to find the values of the reactive current as a function of the excitation and of the active current, by the additional assistance of the excitation-curve for open circuit, either obtained experimentally or else predetermined by calculation. This curve (Fig. 45) gives, for each value, F, of the ampere-turns, the corresponding induced E.M.F.

[1] The problem is really much more complex, especially when the load changes suddenly, since the effect on the field-flux due to the armature-reaction does not then appear as fast as the change of armature-current. The flux may indeed remain constant for several seconds, even if the armature-current changes considerably. Hence, for sudden changes of load the phenomenon is largely governed by the true self-induction of the armature only. Thus, the armature-reactance may be only from 1 per cent to 3 per cent, whereas, for slowly changing loads, it would be equal to the synchronous reactance, say, 40 per cent. Therefore, machines which have good mutual induction between field and armature, such as high-speed motors and turbogenerators, may be unstable if the line-loss is great and if the load changes suddenly. The remedy in such cases is to install external reactances.—E. J. B.

Let, for example, I_w equal the given active current. Knowing the angle δ (Fig. 46), the segment $A_2B=\omega L'I_w$ perpendicular to the

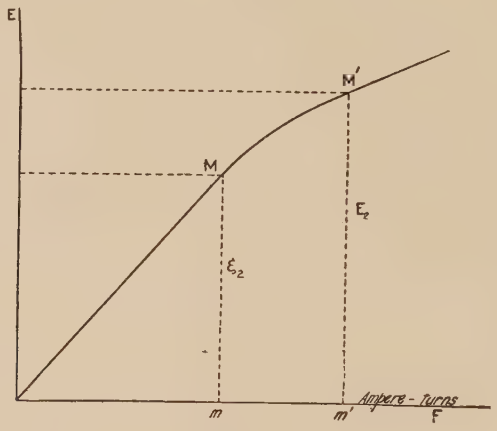


Fig. 45.

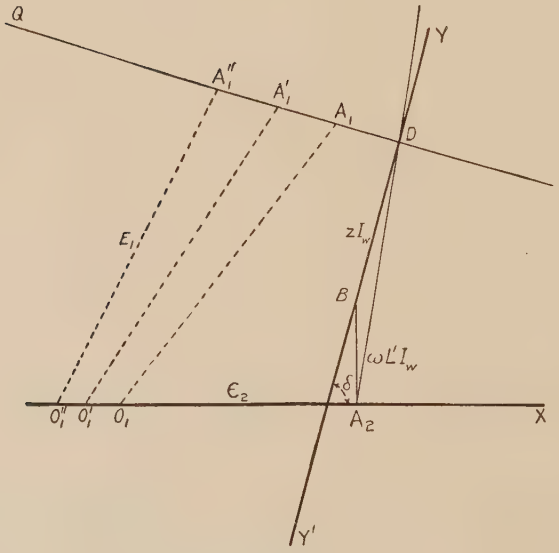


Fig. 46.

axis A_2X can be drawn, and then the segment $BD=zI_w$ making the angle δ with A_2X can be also drawn.

We thus obtain the line of reference BY, which represents the condition when the current is in phase with respect to ε_2. The locus of the characteristic point A_1 is a straight line DQ perpendicular to BD, while the locus of the point O_1 is the straight line OX. For each value I_w, we therefore know the two straight lines at which the vector A_1O_1, equal to the supply E.M.F. E_1, should always terminate.

All loads corresponding to the values I_w will therefore be determined by taking points such as A_1, A_1', A''_1, etc., on DQ, and by drawing circular arcs of radius E_1. The respective intersections O_1, O_1', O_1'', of these lines with O_1A_2, give some E.M.F.'s values, ε_2, which are proportional to A_2O_1, A_2O_1', A_2O_1'', while the corresponding reactive currents, I_d, are proportional, respectively, to the lengths DA_1, DA_1', DA_1''.

For each pair of values of I_d and ε_2, the corresponding exciting ampere-turns F will then be obtained by referring to the excitation-curve. Let us take, on this curve, the point M, which has for its ordinate the resultant E.M.F. ε_2, thus determined. The corresponding abscissa, Om, measures the resultant ampere-turns.

$$F \pm \frac{1}{\sqrt{2}} KN'I_d.$$

It is therefore sufficient to subtract, from this abscissa, the counter-ampere-turns of the armature, $= KN'I_d$, in order to obtain the abscissa, $Om' = F$, which represents the excitation, and, consequently, represents E_2, the E.M.F. induced on open circuit which would be obtained with this excitation.

It must not be forgotten that the reactive current I_d is demagnetizing and positive if counted to the right of the axis YY', and magnetizing and negative if counted to the left (see page 94). The upper signs in the above formulæ apply in the first case, and the lower signs in the second case. We will therefore have $E_2 < \varepsilon_2$ if the point A_1 is to the right of the point D and, on the contrary, $E_2 > \varepsilon_2$ if it is to the left.

In the particular case where a motor is connected directly, without additional rheostat or self-induction, on a constant potential circuit of great output, and, when the impedance z of its leakage is negligible, D coincides with B, and the line QD then passes through the point B itself; which simplifies the diagram still further.

The diagram which has just been explained is most precise in the case of an alternator working at the knee, or above the knee of the

characteristic curve, or presenting unequal coefficients of direct and transverse reactions. In the elementary theory it was assumed, on the contrary, that the permeability of the field-circuit is practically constant and that the two reaction-coefficients are equal.

Particular Case where the Permeability of the Field-Circuit is Constant and the Two Reaction-Coefficients are Equal. It is interesting to note how, in this case, the new and more general diagram comprises the previous forms as a particular solution. It is only necessary, indeed, to observe that the coefficient of transverse reaction, L', then

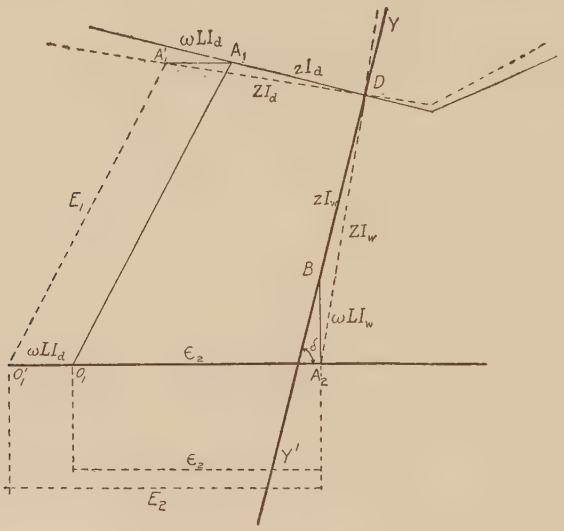


FIG. 47.

becomes equal to what we have termed the coefficient of mean self-induction L. Likewise, the direct armature-reaction, which produces $\frac{1}{\sqrt{2}} KN'I_d$ counter-amperes-turns due to the reactive current, gives rise to a counter E.M.F. which can be represented by ωLI_d. Therefore, between the effective E.M.F., ε_2, and the open circuit E.M.F. obtained with the same excitation, we have the relation

$$E_2 = \varepsilon_2 - \omega LI_d,$$

I_d being always taken as positive on the right-hand side of BY and as negative on the left-hand side. Suppose, for example, I_d to be negative.

It would be necessary, in order to obtain E_2, to add to the segment O_1A_2 (Fig. 47), a section

$$O_1O_1' = \omega L I_d.$$

It amounts to the same thing if this segment is drawn parallel to and to the left of A_1, along $A_1A'_1$, and if a circular arc, of radius E_1, is drawn from A'_2, to cut OA_2 precisely at the point O'_1.

Now if we join DA'_1, it is seen that the triangles $DA_1A'_1$ and DA_2B are "similar" triangles, since the side DA_1 and DB, also $A_1A'_1$ and BA_2 are respectively perpendicular and proportional each to each. It may be deduced from this that the line DA'_1 is per-

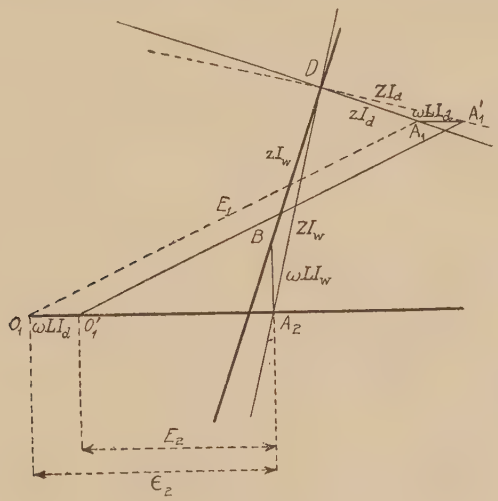


FIG. 48.

pendicular to DA_2. On the other hand, the vectors A_2D and DA'_1 being the resultants of the loss of potential in the motor and in the rest of the circuit, evidently represent ZI_w and ZI_d, respectively, when Z equals the total impedance of the circuit. The diagram composed of the right lines A_2D, DA'_1, $A_1A'_1$, A_2O, is, therefore, nothing more than the diagram which would have been obtained directly by the first method. Q.E.D.

Fig. 48 shows the same analysis for the case where I_d is *lagging* (I_d being positive). The precision of the power-formula already given can also be verified; for, in this formula,

$$P_2 = \varepsilon_2 I_w - \omega L I_w I_d,$$

if we substitute for ε_2, its value,

$$\varepsilon_2 = E_2 + \omega L I_d,$$

we have

$$P_2 = I_2 I_w,$$

as before.

Second Application. Operation with Constant Excitation, on Constant-Potential-Supply System. When once the V-curves have been drawn it is very easy to obtain from them the different load-

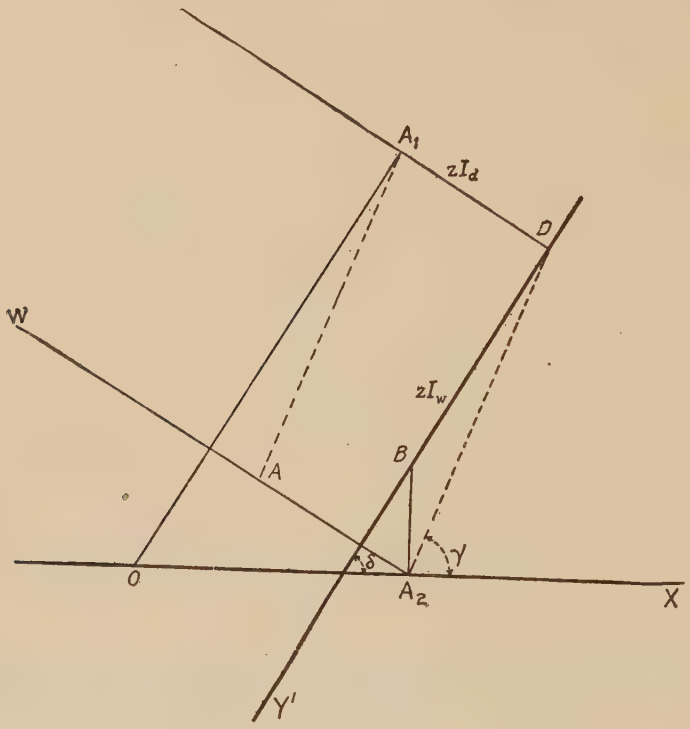


FIG. 49.

values corresponding to any given constant excitation. These values can also be determined directly from the diagram in the following manner:

Let us take the reactive current I_d and let us find the active current I_w corresponding to each value thereof. The direction A_2D (Fig. 49) being, therefore, constant, can be drawn once for all. Likewise, the direction A_2W, to which the reactive segments DA_1 are parallel, makes

with OA_2 a known angle, $\frac{\pi}{2}-\delta$. Since we assume the values of I_d, we can calculate the values zI_d and $\frac{1}{\sqrt{2}}KN'I_d$. (As before z represents the impedance of the armature only).

Let us draw, on A_2W, a segment $A_2A=zI_d$. The point A must fall on a right line, AA_1, which passes through the point A and which is parallel to A_2D. Again, knowing E_2, the excitation-curve gives the excitation ampere-turns F. Subtracting $\frac{1}{2}KN'I_d$, we have the resultant ampere-turns, and consequently, the effective E.M.F. ε_2. Let this be drawn on A_2X, to A_2O. The load-point A_1 must fall on a circle described around O as a center with E_1 as radius. It is therefore determined by the intersection of that circle with the right line AA_1 previously drawn.

We thus obtain the value of $zI_w=BD$, and, consequently, the current I_w. From this we deduce the electric power applied to the motor,

$$P_2=\varepsilon_2 I_w-\omega L'I_w\times I_d.$$

On the other hand, $I=\sqrt{I_d^2+I_w^2}$. We therefore obtain, by points, the relations between I and P_2, both expressed as a function of ε_2 which serves as an intermediary variable.

V-Curves. The diagram does not lend itself to the direct determination of the V-curves for constant power, owing to the complicated expression, which is not explicit as a function of E_2 and I_w, as it was in the elementary theory.

But these curves can be deduced from the curves of constant excitation, when these are drawn by the process indicated in the preceding paragraph.

Influence of Field-Saturation on Stability. The stability is evidently greater the higher the E.M.F. of the motor at the limit of lag. Now, for the same reactive current, the loss of potential produced by the direct reactance is all the less the higher the degree of excitation above the knee of the characteristic. It would therefore be advantageous, from the standpoint of stability alone, to have the fields saturated. But there is a limit, here, owing to the amount of energy required for excitation, and the desirability of a certain margin to enable the voltage to be raised.

The cut-and-try process involved in finding the co-ordinates of the V-curves may be obviated, *without constructing any curves*, by

utilizing Fig. 46. Knowing the value of P and assigning a value to I_w, the equation,

$$P_2 = \varepsilon_2 I_w - \omega L' I_w I_d$$

can be written as follows:

$$\varepsilon_2 - \omega L' I_d = \frac{P_2}{I_w} = \text{constant} = M. \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Taking, in Fig. 46, a point A_1 on the right line DQ, as a center, let a circle be described with a radius equal to E_1. Let O_1 be the intersection of this circle with the right line A_2X. The values of ε_2 and I_d can then be immediately deduced. We find

$$\varepsilon_2 = \overline{A_1 O_1}$$

and

$$I_d = \frac{i}{z} \overline{DA_1}.$$

These values introduced in equation (a) will give, for example,

$$\varepsilon_2 - \omega L' I_d = N.$$

Should this value be greater than M, it is because the point A_1 was located too near D. If, on the contrary, the value N is less than M, it is because the point A_1 is too far from D. The point A_1 may be moved until the values found for ε_2 and I_d give a value for N equal to M. We can also proceed otherwise. Having found, quite easily, by the above method, two positions such as A_1 and A_1' which give, for the polynomial N, two values, one a little too large (N) and the other a little too small (N''), a point, A_1', can be obtained by interpolation such that

$$\frac{A_1' A_1}{A_1'' A_1'} = \frac{N - M}{M - N''}.$$

This gives the desired value of the power, P, with a sufficient degree of approximation for practical purposes.

Influence of the Wave-Form of E.M.F. The shape of the E.M.F. curves of the generator and motor should approach the sinusoidal form as much as possible in order to avoid instability and low efficiency. Let us first suppose that only one of the two machines departs from the sinusoidal form. By virtue of Fourier's theorem, its E.M.F. can be considered as the resultant of the superposition, upon a principal sinusoidal, of higher harmonics, i.e., of sinusoidals of higher frequency (odd multiples).

The fundamental can alone produce useful power, because it is known that energy cannot be produced by the action of an E.M.F. until the current has the same frequency. Since the harmonic terms exist only in one of the machines they will simply produce parasite currents therein which will increase the heating by resistance-loss without any useful result.

It is known that the square of the effective current is equal to the sum of the squares of the effective currents of the component sinusoidals. The heating produced separately by each of the harmonics will therefore be added to the principal heating. This effect makes itself readily apparent in the V-curves by an increase in the minimum current, which will be much greater than would be necessary to obtain the power actually developed. This can easily be observed by means of a wattmeter, which will indicate a real power no longer equal to, but really lower than, the apparent power (product of the effective current by the effective voltage). It cannot be possible that cos ϕ is lower than unity, since the current is in phase, as we know, under these conditions.

It can even happen, as noted by Bedell and Ryan (*Bedell and Ryan, Journal of Franklin Institute, Mar.*, 1895, *Fig.* 9), on a motor having a strong third harmonic, that the V-curve breaks up into two branches, one of which is normal, at high excitations, the other being parasite, and very high at low excitations. Between these there is no stable load possible, without adding a strong self-induction in series. The author, who has himself observed similar phenemona (*A. Blondel. Electrotechnische Zeitschrift, April* 25, 1895), attributes this favorable effect of a large self-induction (an effect which is contrary to the theory given on page 53) to the obliteration (throttling) of the harmonics, which meet, in passing through the self-induction, a reactance that is proportional to their own frequency.

In the case where it is the generator which has a distorted E.M.F., it can be eliminated, as M. Perot has shown (*Perot, Comptes-Rendus*, 6 *Aug.*, 1900, p. 337), by connecting in parallel, on the line supplied by this generator, a synchronous motor (or a converter) having a sinusoidal E.M.F. This apparatus, opposing no harmonic to those of the generator, short-circuits them without reducing the principal sinusoidal. If, therefore, we take Z equal to the generator-impedance (or that of the group of generators) and z equal to the motor-impedance, the amplitude of the harmonics, at the terminals, will be substantially reduced to $\frac{z}{Z+z}$ of their value on open circuit. Perot has

thus been able, in a practical case, to reduce these harmonic terms to one-eighth of their value.

The case is different when the E.M.F.'s of the two machines are equally distorted, because the harmonics which produce the parasite current are then resultants of harmonics of the same order in the two machines. Since these harmonics are all of odd order, the two sets will oppose each other, if the principal sinusoidals are themselves opposed, as is approximately the case under normal operation when there is no lag.

Consequently, if like machines are used as generators and as motors, a considerably distorted E.M.F. curve would produce parasite effects which are much less important than in the first case, and almost negligible if care is taken to have the operation correspond to the lower part of the bend of the V-curve.

Simplified Diagrams. The diagrams shown in Figs. 44, 46, 49 correspond to the most general case where it is desired to take into account both the direct and the transverse armature-reactions.

Inasmuch as the direct armature-reaction alone affects the total excitation ampere-turns, it may seem, to some, an unnecessary complication to introduce the transverse reaction in the diagrams. For the benefit of those who prefer Kapp's simplified theory, other diagrams have been prepared (Figs. 44*a*, 46*a*, 49*a*) in which the transverse reaction is neglected, i.e., where it is assumed that $I_w = 0$.

In constructing these simplified diagrams it is important, as before, to determine the values of the resultant E.M.F. ε_2 by reference to the law of magnetic saturation as represented by the E.M.F. characteristic curve shown in Fig. 45*a*. In this diagram the abscissæ represent excitation ampere-turns, and the ordinates represent E.M.F.'s. If the point M' represents the E.M.F. E_2 with open circuit, and if Om' represents the corresponding excitation ampere-turns, then, on subtracting from Om the ampere-turns corresponding to the reactive current I_d or making $mm' = \frac{KNI_d}{\sqrt{2}}$ the ordinate Mm corresponding to the abcissa Om will be the E.M.F. ε_2.

The impedance involved (z) is that which corresponds to the magnetic leakage of the armature. Fig. 44*a* shows how the corresponding potential difference zI can be resolved into two components obtained by projecting A_1A_2 on YY' and on the line DA_1 perpendicular thereto.

The component DA_1 gives the value of I_d when z is known.

Fig. 46*a* shows how the values necessary for constructing the *V*-curves are obtained; and Fig. 49*a* represents the case where the synchronous motor is operating with constant excitation and at constant potential.

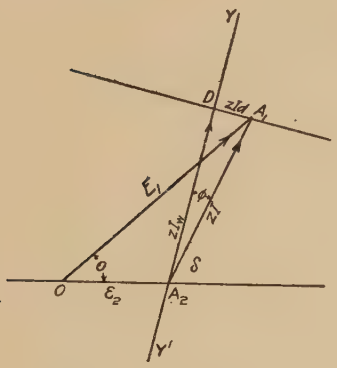


Fig. 44*a*.

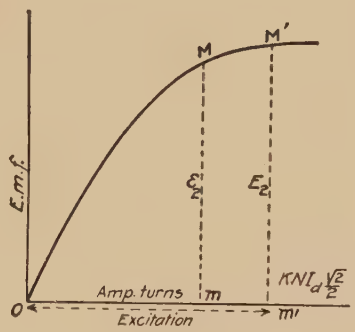


Fig. 45*a*.

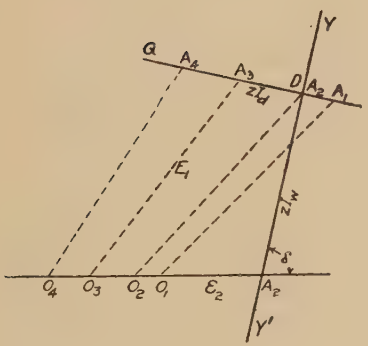


Fig. 46*a*.

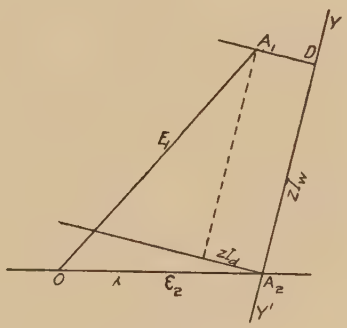


Fig. 49*a*.

These diagrams give a degree of precision which is generally sufficient for practical purposes, being intermediate between that obtainable with the simplified methods used in Chapter II and the more rigorous method used in Chapter III. Moreover, the diagrams 44*a*, 46*a*, 49*a* express armature-reactions in terms of magnetomotive force more precisely than if the total *zI* resultant were to be expressed by a magnetomotive force.

CHAPTER IV

OPERATION OF SYNCHRONOUS MOTORS. HUNTING

Synchronous motors are not self-starting, the same as other electric motors, but they have to be started by some artificial means such as described hereinafter.

Starting by Direct Current. The simplest way, when there is a source of direct current and the synchronous motors are provided with a commutator for producing direct current for self-excitation, is to start the machine as if it were a D.C. motor, with the armature and fields connected in parallel. This plan can be especially adopted in transformer stations using motor-transformers, so as to start a set of motors operating dynamos as soon as one has been put in operation.

Starting with Alternating Current by Polyphase Motors. When there is no D.C. source, the alternating currents themselves must be used for starting the motor, either by mounting on the shaft of the machine a small induction-motor which serves to run it without load, or, more frequently, by using the machine itself as an induction-motor. (When a small motor is used its capacity should be about 10 per cent of that of the machine. It can be even less, if it can withstand overloading for a few seconds.)

When using the machine itself as an induction-motor, the first step is to suppress the regular excitation, which would prevent the armature from moving. The second step is to connect the armature with the source of alternating supply, taking care to lower the voltage sufficiently to prevent excessive current through the armature. The armature then behaves as if it were the primary winding of a revolving field machine in which the fields, and more especially the pole-pieces, play the role of secondary circuit. To obtain a torque equal to a quarter the normal torque, it usually requires, by this method, a current at least double the normal current. To complete this action, it is well to short-circuit the field-winding. Certain manufacturers have, for this purpose, arranged in the pole-pieces, as shown in Fig. 50, a series of slots containing windings or bars of copper, connected together

like the armature of an induction-motor. In that case, the motor starts normally, and with a current which depends on the resistance. In any case this current is higher than in an induction-motor because the air-gap is greater and requires a stronger magnetizing current. For this reason, it is well to start the motors without load, in order to avoid excessive starting currents.

The production of the revolving field results from the eddy currents, but in the absence of these, the pole-pieces, even when laminated, give rise to hysteresis effects which are often sufficient for starting the motor without load. In certain motors having laminated poles the starting current is about double the normal full-load current.

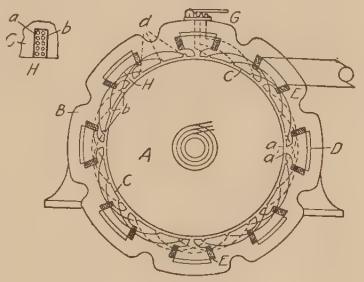


FIG. 50.

This effect is explained by the fact that, during the rotation of the field, there is a stronger attraction in the direction of motion than in the opposite direction owing to the lag of the magnetization behind the changes of field producing it.

Synchronism. In proportion as the armature-speed increases, the magnetic pulsations produced by it in the pole-pieces become less numerous, as can be seen on connecting a lamp to the field-circuit and noting its variations of brightness. It is well to provide a centrifugal regulator which connects this lamp in circuit only when the speed approaches synchronism, because, at lower speeds, it would be subjected to excessive voltage. When synchronism is almost attained—(the speed generally remains slightly lower,)—the fields are excited when the phases come into opposition, the motor falls into step, and the current immediately diminishes, owing to the disappearance of the reactive current which was absorbed up to that time. The more carefully the time has been selected for closing the excitation-circuit, the more easily the motor will fall into step. It is well, as a rule, to include, in the circuit, a variable self-inductance, which has the effect of preventing excess of current and of damping objectionable harmonics.

When once the motor is in synchronism, it can be loaded progressively, by shifting the belt from the idler to the driving pulley. This should be done very carefully when the mechanical resistance (load)

In the particular case where it is possible to start the generator and the motor at the same time, the operation of starting becomes easier and there is no necessity of suppressing the excitation of the motor. The motor starts of itself by giving it a slight impulse, it falls into step at the very first period, owing to the low reactance at low speed, and its motion then accelerates synchronously with that of the generator, increasing gradually to full speed.

Observations on the E.M.F. Induced in the Poles. During the starting period, the fields are the seat of an important alternating flux whose frequency attains, at first, the same frequency as the alternating currents themselves. The result is the production of a very high E.M.F. in the field-winding, which is generally made of fine wire. If, therefore, this winding remains on open circuit, its insulation should be particulary good, and contact should not be made with its terminals. It is preferable to short-circuit this winding. This alternating E.M.F. naturally vanishes when synchronism is attained. It is obvious that the armature-brushes on the D.C. side must also be disconnected, since, otherwise, the pulsating current would produce a braking effect which would prevent the motor from starting.

Accessory Starting Apparatus. Installation of Synchronous Motors. The starting process involves the use of two kinds of accessory apparatus, i.e., phase-indicators and current-controllers.

It is also well to provide fuses and *automatic circuit-breakers* which break the main circuit whenever there is no excitation.

Phase-Indicators. Phase-indicators play the same role in this case as in coupling alternators in parallel. They are of the same kind and are connected in the same way. Their use is recommended for large-size motors.

When a synchronous motor is started by a direct current or by an auxiliary motor, a lamp of higher voltage than that of the supply-source may be connected in series with the motor. Owing to its high resistance, this lamp burns feebly at first under the action of the E.M.F. of the source of supply. In proportion as the motor increases in speed, if the field-winding is connected with a D.C. source, the E.M.F. which acts on the lamp, being the resultant of the generator and motor E.M.F.'s, will produce beats or pulsations which will gradually become less frequent. The proper time to connect the armature directly to the circuit is when the beats have become extremely slow, and when the lamp goes out, thereby indicating that the E.M.F.'s are in opposition.

When the motor is started without field-excitation, as an induction-

motor, this method is no longer applicable and the phase-lamp (or else several lamps in series) must be connected to the terminals of the field-circuit, which is then subjected to the inductive action of the variable flux of the armature. The lamp burns brightly at first, then its brightness diminishes gradually, and the extinction of the lamp shows that synchronism is approaching; the direction which the E.M.F. will have when the circuit is closed is not determined, but this is generally of very slight importance.

With a well-constructed synchronous motor there is no need of a phase-indicator, because, when once it has attained its highest speed as an induction-motor (which may sometimes require several minutes), the powerful reactions which occur when the excitation-circuit is closed are sufficient to insure the machine falling into step.

Current-controllers. It is necessary, in order to reduce the amount of current taken from the line during the starting period, to employ a current-controller, owing to the absence of counter E.M.F. in the armature. Use may be made for this purpose of a simple rheostat or a choke-coil. Rheostats are more frequently used, on account of their simplicity. Switching appliances are used for the purpose of connecting them into circuit and then short-circuiting them, an armature circuit-breaker being interposed to break the circuit in case the field-excitation fails.

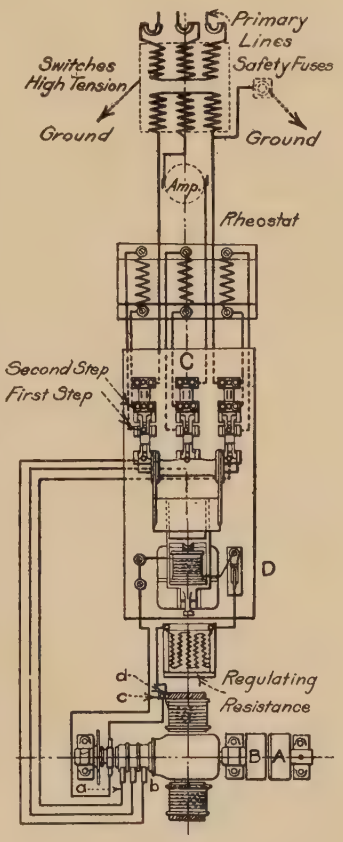
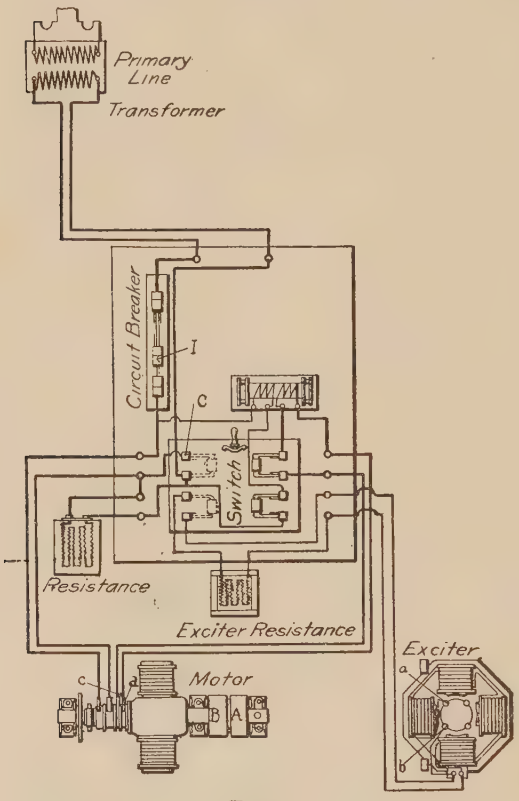


FIG. 51.

Fig. 51 shows, for example, the diagram of the starting apparatus of a certain type of three-phase motors ranging from 1 to 22 H.P. When the motor is at rest, the pulley is on the idler *A*, the triple-switch *C* and the excitation circuit-breaker *D* are open.

To start the motor the triple-pole switch *C* is closed on the first steps, and the motor then receives a current from the supply-trans-

former through a rheostat which limits the amount of current. The motor starts and attains synchronous speed in from 20 to 30 seconds. The field-excitation circuit is then closed, and the counterweight of the circuit-breaker is lifted, which enables the triple-pole switch to be brought to the second position. The motor then receives the current



At Rest:

The belt is on the loose pulley *A*. The circuit-breaker *I* is open. The switch *C* is to the left.

Starting:

1. Throw switch *C* to the right.
2. Close circuit-breaker *I*.
3. When synchronism is reached throw switch *C* to the left.
4. Shift belt from pulley *A* to pulley *B*.

Stopping:

1. Shift belt to pulley *A*.
2. Open circuit-breaker *I*.

N. B.

To change direction of rotation transpose wires *a* and *b*, and *c* and *d*. (Fig. 51.)

FIG. 52.

directly, and it attains full speed. The belt can then be transferred gradually from the idler pulley to the driving pulley *B*.

To stop the motor we proceed in inverse order. The belt is shifted to the idler pulley *A*, the counterweight is dropped (which causes the three-pole switch to open), and finally, the circuit-breaker *D* of the field-excitation circuit is opened.

For motors of 25 to 50 H.P. the single resistance is replaced by a variable rheostat and the switches have but one step. Once the speed is

attained the rheostats are short-circuited. For these large motors, separate exciters are generally used, as already seen (Fig. 52).

The operation of starting without load does not require more than 20 to 30 seconds, and it is effected with a current not exceeding the normal full load current.

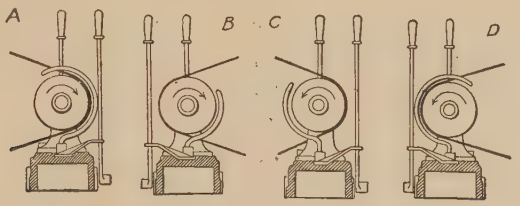


Fig. 53.

It will be observed that the process of starting involves the use of high-tension switches on the primary circuit, to isolate the transformer; also fuses on both the primary and secondary circuit.

The belt is shifted from the idler to the driving pulley in the ordinary way by a belt-shifter. Fig. 53 shows the manner in which this shifter should be placed to correspond properly with the direction of rotation and the direction of driving, for an observer placed at the commutator end. Figures *A* and *B* refer to the case where the direction of rotation (indicated by the arrows) is the same as the hands of a watch; and Figs. *C* and *D* refer to the case where the rotation is in the opposite direction.

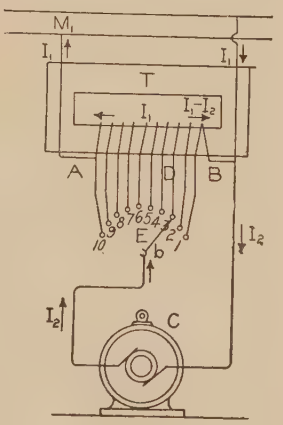


Fig. 54.

The use of a reactance-coil instead of a rheostat has the objection of increasing the lag of the current taken from the supply-source, which current already has too much lag. It is therefore preferable to replace it by an auto-transformer which enables the current of the motor to be increased at low voltage while reducing the amount of current taken from the line. Fig. 54 shows an example of this kind of arrangement which is in extensive use. The diagram represents the simplest case, i.e., that of a single-phase alternating current machine. It is a transformer with a single wind-

ing having taps connected to contacts, over which moves the switch *b*. The two ends, *A*, *B*, of the winding are connected to the source of current-supply, while the motor is connected to the end *B* and the switch *b*.

The winding operates like a potentiometer by which any fraction whatever of voltage can be obtained at will. At the beginning, very low voltage is obtained by placing the switch handle to the right-hand side. The handle is then moved gradually toward the left in proportion as the speed increases, and gradually raising the voltage applied to the motor, while reducing the current consumed by it. The current I_1 taken from the supply-source is a fraction of the current I_2, consumed by the motor, which is represented by the inverse ratio of voltages

$$I_1 = I_2 \frac{E_2}{E_1}.$$

The winding of the transformer is traversed by the current I_1, between *A* and *D*, and by the current $I_2 - I_1$, between *D* and *B*.

In the case of polyphase apparatus it is sufficient to install as many transformers with commutating switches as there are phases. All the levers of these switches are operated by one handle. An arrangement of this kind was utilized with polyphase current in the power-transmission installed for Messrs. Menier, at Noisiel in 1893 (*Power-Transmission to Noisiel, Lumière Electrique, May*, 20, 1894).

Starting of Single-Phase Machines. These machines can also be started by the first two methods, i.e., by means of an auxiliary D.C. motor, or by a small induction-motor which starts by means of an auxiliary phase such as, for example, "shaded pole" single-phase motors. It is desirable to be able to increase the speed of the machine gradually while keeping constant the speed of the small auxiliary motor, because the latter would be stalled if it were made to slow-up. To overcome this difficulty the motor is mounted on guides, and it transmits power to the larger machine by means of a friction-cone bearing against a large disk mounted on the shaft of the machine itself. The motor is gradually shifted on the guides in such manner that the friction-cone approaches the center of the disk until the moment when synchronism is attained.

The starting of the machine as an induction-motor is more difficult, but it can be accomplished by transforming temporarily the armature into a two-phase armature, by means of a second winding suitably

provided for that purpose, and which can be of small wire, inasmuch as the time required for starting is so short. By sending into this auxiliary winding a current which is more or less out of phase with respect to that in the principal winding, the armature produces a revolving field which is more or less imperfect and pulsating, but which is, nevertheless, sufficient to start the motor.

To produce the phase-difference between the two currents, recourse may be had to some one of the methods known for producing induction-motors by means of a single alternating force. For example, use may be made of the Tesla method of introducing a self-induction into one branch of the circuit, and a resistance in the other, or use may be made of the Leblanc method, which is more perfect, and which consists in introducing a self-induction in one side and a capacity or a polarizer in the other.

But in order to reduce to a minimum the current taken from the line, there is again a great advantage in inserting a reducing transformer, built according to the principle already explained, and supplying current either for one phase or for both phases of the motor. One arrangement consists in supplying only the principal winding by means of the reducing transformer and in connecting the secondary winding directly to the switchboard bus-bars. In such a case, the secondary winding is made with a very high number of turns so as to have a very high self-induction and to absorb only a weak current having much lag with respect to the current of the other winding.

The arrangement employed by Labour (Fig. 52) is the reverse of this. In this case, it is the supplementary winding of the armature, made of fine wire, which is supplied by a reducing transformer, while the principal winding receives current direct from the switchboard through a rheostat. The double-throw switch *C* being moved to the right (starting position) as shown in the diagram, the supplementary winding will then be in circuit with the secondary of the reducing transformer, and at the same time the rheostat is inserted in the principal circuit, which is closed by the circuit-breaker *I*. The motor starts as an induction-motor. When the speed approaches synchronism, the switch *C* is suddenly thrown to the left; by this operation the current in the supplementary winding is broken, and the rheostat of the principal winding is short-circuited, at the same time that the fields are excited. The motor is then in step.

In large motors the circuit-breaker *I* is replaced by a starting rheostat, the object being to avoid fluctuations in the lights supplied from the same

distributing system. Sometimes an electromagnetic circuit-breaker is interposed in the excitation-circuit, its function being to open the main-circuit whenever the excitation-current fails.

The operation of starting and synchronizing requires about 30 seconds. The maximum current taken from the mains during this period does not exceed the normal current by more than 20 per cent; and it falls down to $\frac{1}{4}$ or $\frac{1}{5}$ the normal current when synchronism is attained.

Besides these general methods, there is still another, which can be used only in the case of motors having laminated field-poles. The method consists in sending alternating currents through the field-coils and through the armature-coils, which are then provided with a commutator. The latter can be adapted either to the principal winding, or to a supplementary winding, which serves only for starting and for excitation. This arrangement has been used by the Fort Wayne Company in the United States [*Churchward*, *Eclairage Electrique*, Vol. XVII, p. 41]. At the time of starting, the current is sent through the armature and field-circuits connected in series; the machine then operates like a series-motor having a commutator; the direction of the current changes, both in the armature and in the field, and consequently the torque is always in the same direction and can thus cause the motor to start.

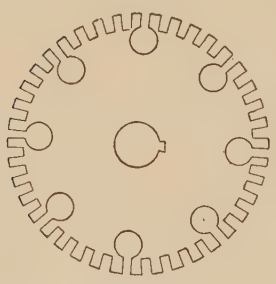


Fig. 55.

Both the field and armature of the Fort Wayne motor are laminated; the field has two windings, one being a fine wire winding for synchronous operation, the other being a coarse wire winding which serves for starting. The armature itself also has two circuits wound on a core composed of sheet-iron disks of the form indicated in Fig. 55. The principal winding, which serves for synchronous operation, is placed in the round holes, there being as many coils as there are field-poles, and the winding is connected with two collector-rings. The other winding, distributed in the slots, is similar to that of an ordinary D.C. drum winding with its commutator.

The operation of starting is accomplished by means of a single lever controlling the switches. If it is first placed in the position indicated in Fig. 56 the motor will be connected to the source of current

like a series-motor, and it will start as such. When its speed attains synchronism, a centrifugal device closes the circuit with a third collector-ring on the right, causing an incandescent lamp to be lighted; the switch-lever is then moved, causing the normal connections to be made; the armature is then connected directly with the circuit; and the commutator circuit is connected with the fine wire excitation-circuit. The inductance in circuit is adjusted in such a manner as to permit the maximum allowable current to pass through the circuit when starting. This maximum current is about two to two and a half times the full load-current.

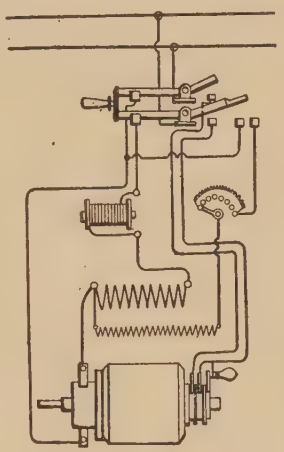


FIG. 56.

The efficiency of this type of motor is 70 per cent for a $2\frac{1}{2}$ H.P. motor, and 80 to 85 per cent for motors of 10 to 15 H.P. They start readily under load, without excessive sparking at the brushes, the latter being of carbon and having a fixed position. The power-factor, which is very high when starting, does not fall below 0.95 to 1 under load.

For small motors, of 1 to 5 H.P., the Gramme commutator just mentioned can be replaced by a less complicated commutator having only one segment per pole of the motor. The small Ganz motors are of this kind. Their fields are excited by commutated currents obtained from a transformer connected in shunt to the line. The even segments are connected to one terminal of the field-circuit, and the odd segments to the other. To start the motor, which can only be done without load, only one set of brushes need be used on the commutator, the others being put off. The current thus changes direction in the field-coils and in the armature coils, as already mentioned; but there is much sparking at the brushes during the whole time that the motor is starting. When the synchronous speed has been attained, which requires at least a minute, the brushes previously removed are again put on, by hand or else by a centrifugal regulator, or they may be put in circuit by means of a switch; sparking is then reduced to an allowable amount. Fig. 57 gives the diagram of the arrangement actually used (Blathy system). The field and armature windings are in parallel. The armature-current, coming from the transformer at 100 volts,

through the wires *I*, *II*, enters into the armature by the terminal *A*, leaves by the terminal *B* and passes through the secondary-winding S_1, S_2 of the transformer *E*, called compensator, which plays the role of self-inductance for throwing the armature-current out of phase.

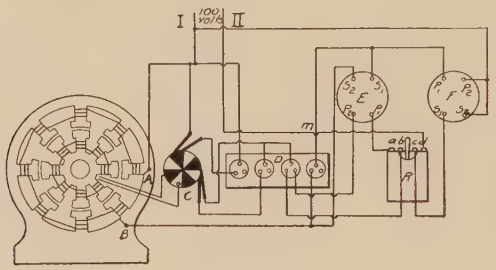


Fig. 57.

The field-circuits are wound for a low voltage (25 volts) supplied by the secondary circuit S_1, S_2 of another transformer *F*.

At the time of starting, to overcome the induction of the field-circuit, the potential-difference at its terminals is raised by the primary winding of the compensating transformer *E*, whose secondary has the armature-current passing through it, as already stated. The field-current also passes through a fixed resistance, *ab*, in the rheostat, *R*, the object of which is to reduce the lag in the field-circuit, and to put it in phase with the armature-current.

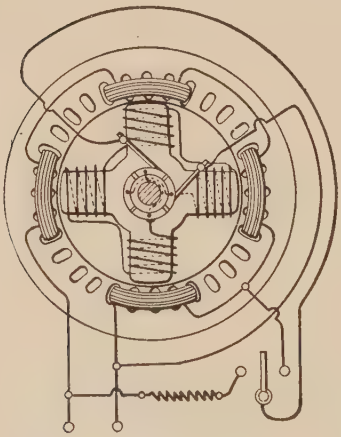


Fig. 58.

The motor starts. When it attains synchronism, the current in the field is commutated; the connections are then changed by means of the double-throw switch *D*, which short-circuits the secondary of *E*, connects the two neighboring brushes together and sends the secondary current of *F* into the variable resistance *cd* before it reaches the field-coils.

A motor of the same kind was put on the market some years ago by the firm of Brown-Boveri (Fig. 58). The motor starts by simply coupling in parallel the armature and the field-circuit and a resistance

which is in series with it. To bring the phases of the two currents into proper relation, the brushes are moved an angle equal to $\frac{\pi}{2}$ during the operation of starting. When synchronism is attained, the brushes are brought back to their place, and the excitation-circuit is connected in shunt to one coil alone of the armature, so as to reduce the voltage.

Theory of Initial Synchronizing. Two cases must be considered, depending on whether the field, when separately excited, maintains its normal value independently of the speed, or whether the field is produced by means of commutated currents.

Separate Excitation. Let us again suppose the motor to be connected to a constant potential source E_1 and let us retain the same symbols as before. Let us designate by T_2 the duration of the period of the motor, which will differ from T_1 the period of the supply-current, so long as synchronism is not attained. For simplicity, let us write

$$\frac{2\pi}{T_1}=\omega_1, \quad \frac{2\pi}{T_2}=\omega_2.$$

The equation of the current proceeding from the source of supply toward the motor is

$$r_2 i+l_2\frac{di}{dt}=E_1 \sin \omega t-\frac{\omega_2}{\omega_1}E_2 \sin (\omega_2 t-\theta),$$

where E_2 equals the E.M.F. induced at the speed of synchronism, and θ represents, as before, the lag in phase of the motor. From this, neglecting the exponentials which occur in the integral, but which always neutralize each other quickly, and taking

$$\tan \phi=\frac{\omega_1 l}{r}, \quad \tan \psi=\frac{\omega_2 l}{r},$$

we have

$$i=\frac{E_1 \sin (\omega t_1-\phi)}{\sqrt{r^2+\omega_1{}^2 l^2}}-\frac{\omega_2}{\omega_1}\frac{E_2 \sin (\omega_2 t-\theta-\psi)}{\sqrt{r^2+\omega_2{}^2 l^2}}.$$

The power supplied to the motor at each instant is

$$e_2 i=\frac{\omega_2}{\omega_1}E_2 \sin (\omega_2 t-\theta)\left[\frac{E_1 \sin (\omega_1 t-\phi)}{\sqrt{r^2+\omega_1{}^2 l^2}}-\frac{\omega_2}{\omega_1}\frac{E_2 \sin (\omega_2 t-\theta-\psi)}{\sqrt{r^2+\omega_2{}^2 l^2}}\right].$$

Integrating between t_0 and t, the work done is

$$T=\frac{E_1E_2}{\sqrt{r^2+\omega_1{}^2l^2}}\frac{\omega_2}{\omega_1}\int_{t_0}^{t}\sin(\omega_2t-\theta)\sin(\omega_1t-\phi)dt$$
$$-\frac{E_2{}^2}{\sqrt{r^2+\omega_2{}^2l^2}}\frac{\omega_2{}^2}{\omega_1{}^2}\int_{t_0}^{t}\sin(\omega_2t-\theta)\sin(\omega_2t-\theta-\phi)dt.$$

While the second term increases indefinitely, the first is doubly periodical and has the form

$$\int\sin\omega_1x\sin\omega_2xdx.$$

It is known that an integral of this form, taken within limits which comprise a whole number of periods of ω_1x and of ω_2x, is equal to zero, so long as ω_1 is different from ω_2. If ω_1 and ω_2 are very different from each other, their zeros are very near each other and the amplitude is very small. On the contrary when ω_2 is very near ω_1 the periodicity of the integral increases in length to the point of becoming infinite.

Two cases are therefore to be considered:

1) So long as the speed of rotation is low, the first term is negligible in comparison with the second, provided that the inertia in the motor is sufficient and that only negative work is done. We then have

$$T=-\frac{1}{2}\left(\frac{\omega_2}{\omega_1}E_2\right)^2\frac{(t-t_0)}{\sqrt{r^2{}_2+\omega_2{}^2l_2{}^2}}\cos\phi$$
$$=-\frac{1}{2}\left(\frac{\omega_2}{\omega_1}E_2\right)^2\frac{r(t-t_0)}{r^2+\omega_2{}^2l^2}.$$

It is therefore necessary to apply mechanical power to turn the alternator and with Ω_2 = the speed corresponding to the period I_2, and Ω_1 = the speed corresponding to the condition $\left(\Omega_2=\Omega_1\frac{\omega_2}{\omega_1}\right)$, the motor torque C_ω will have the following expression:

$$C_\omega=-\frac{1}{2}\left(\frac{\omega_2}{\omega_1}E_2\right)^2\frac{r}{r^2+\omega_2{}^2l^2}\frac{1}{\Omega_2}$$
$$=\frac{1}{2}\frac{\omega_2}{\omega_1}\frac{E_2{}^2}{\Omega_1}\frac{r}{r^2+\omega_2{}^2l^2}$$
$$=-\frac{E_2{}^2}{2\Omega_1r}\frac{\frac{\omega_2}{\omega_1}}{1+\left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{\omega_1l}{r}\right)^2}.$$

Taking $x=\frac{\omega_2}{\omega_1}$ we will have

$$C_\omega = -\frac{E_2^2}{2\Omega_1 r}\frac{x}{1+x^2\tan^2\phi}.$$

The curves in Fig. 59 represent C_ω as a function of x, when the constant $\frac{E_2^2}{2\Omega_1 r}$ is taken as unity, and when $\tan\phi$ is given values equal to 5 and 10.

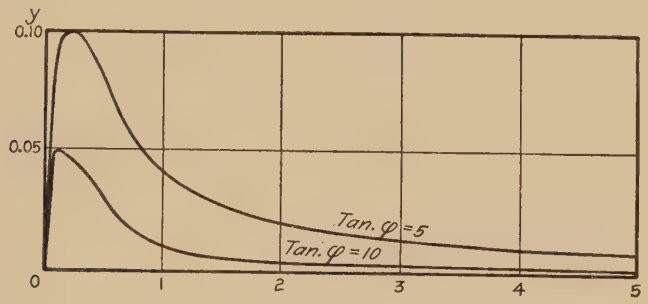


FIG. 59.

It is seen that the torque C_ω increases rapidly, at first, with the speed, and then decreases more slowly with a maximum

$$x=\frac{1}{\tan\phi};\quad C_\omega = -\frac{E_2^2}{2\Omega_1 r}\frac{1}{2\tan\phi} = -\frac{E_2^2}{4\Omega_1\omega_1 l}.$$

To bring back the motor to a speed near that of synchronism, it would be necessary to exert a mechanical force (torque) near that which corresponds to $x=1$, or

$$C = -\frac{E_2}{2\Omega_1 r}\frac{1}{1+\tan^2\phi},$$

i.e., the same as would be required to make the alternator rotate when short-circuited upon itself.

2) When once ω_2 is near ω_1, the first term is no longer negligible. The integral can no longer be evaluated except by supposing ω_2 to be constant within the limits of integration, i.e., during a great number of periods.

Then, taking t_0 as the origin, $(t_0=0)$, we will have

$$\int_0^t \omega_2 \sin(\omega_2 t-\theta)\sin(\omega_1 t-\phi)dt$$
$$=\frac{\omega_2}{2}\int_0^t \cos[(\omega_1-\omega_2)t+\theta-\phi]\,dt$$
$$-\frac{\omega_2}{2}\int_0^t \cos[(\omega_1+\omega_2)t-(\theta+\phi)dt$$
$$=\frac{1}{4}\frac{\omega_2}{\omega_1-\omega_2}\sin[(\omega_1-\omega_2)t-(\phi-\theta)]$$
$$-\frac{1}{4}\frac{\omega_2}{\omega_1+\omega_2}\sin[(\omega_1+\omega_2)t-(\phi+\theta)].$$

Noting that

$$\frac{\omega_2}{\omega_1\Omega_2}=\frac{1}{\Omega_1},$$

the total value for the torque, at each instant, would therefore be

$$C=\frac{E_1E_2}{\Omega_1\sqrt{r+\omega_1^2l_2^2}}\left\{\frac{1}{4(\omega_1-\omega_2)}\sin[(\omega_1-\omega_2)t-(\phi-\theta)]\right.$$
$$\left.-\frac{1}{4(\omega_1+\omega_2)}\sin[(\omega+\omega_2)t-(\phi+\theta)]\right\}-\frac{E_2^2}{2\Omega_1}\frac{\frac{\omega_2}{\omega_1}r}{r^2+\omega_2^2l^2}-C_{t_0}.$$

When ω_1 is very near ω_2 the amplitude of the first term becomes very high, and this is the case so long as ω_1 is not exactly equal to ω_2. At that moment, there is a discontinuity; and the term in question vanishes.

Under these conditions, the inertia is low compared with the enormous torque which is developed, and the motion of the alternator can undergo pulsating variations of speed which are considerable. The hypothesis which forms the basis of our reasoning concerning the nature of the motion is no longer fulfilled and the equations no longer have any value.

Only one conclusion, therefore, can be drawn, i.e., it is necessary to bring the motor, by some process, to a speed *at least equal to that of synchronism*, before connecting it with the source of current-supply.

A second method of starting a motor without load, and which can be used with success every time there is only one power-transmission, consists in starting simultaneously the generator and the motor. A very weak impulse then suffices to make $\omega_1=\omega_2$, at the moment of

starting. The two machines therefore fall readily into step, and if the resisting couple due to passive mechanical resistances (air-friction, bearing-friction, hysteresis, etc.) is not too high, the generator drags the motor, in its progressive acceleration, the two always remaining in synchronism.

Field Due to a Commutated Current. In this case, the effect produced is altogether different. Two periods must again be distinguished, i.e., whether ω_2 is very different from ω_1, or whether, on the contrary, it is of the same order of magnitude.

In the first case, i.e. at the beginning of the operation of starting, the current is reversed a great number of times, both in the armature and in the fields, while the armature is moving an angular distance corresponding to a period T_2. The motor behaves like a commutator-motor, and produces a torque which is always in the same direction and which gradually increases the speed.

In the neighborhood of the normal speed, the effect of the commutator on the excitation becomes apparent. The alternations of flux are commutated in a dissymmetrical manner, with the consequence that a very powerful field with *slow oscillations* is produced, on which pulsatory undulations that are more feeble are superposed. This effect may be expressed by the formula,

$$\Phi = \Phi_0 \cos \omega t_0,$$

in which t_0 has a value which is variable as a function of the time.

As soon as the two periods T_1 and T_2 are near each other, the small undulations disappear before the large undulation, namely, the undulation of t_0. The frequency, $\frac{1}{T}$, of this great oscillation is evidently equal to the least common multiple of $\frac{1}{T_1}$ and $\frac{1}{T_2}$. The period, T', therefore, lengthens indefinitely, when T_2 approaches T_1, at the same time that the value of the field thus produced increases.

Under these conditions, the power furnished by the motor varies according to a complicated function. The oscillations of the field can be taken into account on replacing ωt_0 by $\omega' t$ and substituting the value

$$E_2 = -\frac{d\Phi}{dt} = -\omega' \Phi_0 \sin (\omega' t - \zeta),$$

in the expression $e_2 i$. But the equations are then no longer rigorous and can only be used to predict the occurrence, in the neighborhood of synchronous speed, of heavy fluctuations resulting in alternate accelerations and retardations which are very marked and for which the inertia of the system cannot compensate. As soon as one of these accelerations becomes strong enough, it will, at one bound, bring the speed to its normal value, leaving the motor synchronized.

The starting of motors of the Ganz type may thus be explained approximately, without our being able to take exactly into account the effects of inertia. It is known, moreover, that synchronous motors do not always run at synchronous speed, but at a speed near it, which is periodically variable, owing to the production of the alternating field already mentioned. It is an extreme case of oscillations of load, which we now proceed to discuss.

Oscillations of Synchronous Motors. It is easily observable, by means of a dead-beat ampere-meter connected in the armature-circuit, that the operation of a synchronous motor is always accompanied by fluctuations above and below the normal current, which are due to oscillations above and below the synchronous speed. With a good motor, operating in a satisfactory manner, these oscillations are rapid and slight, except when the load undergoes sudden variations. But, with certain motors, these oscillations attain a considerable amplitude and they are complicated by slow variations of very high amplitude, giving rise to exaggerated current-values like those due to a short-circuit, which usually end by the motor falling out of step; it may even be that synchronous operation is altogether impossible.

There are here two distinct phenomena, which will be examined separately, i.e., short-period oscillations of the motor itself and long-period parasite oscillations due generally to foreign causes.

Short-Period Oscillations. Any variation of the speed of the generator or any sudden variation of the resisting effort at the motor-shaft gives rise to an oscillating condition of motor-speed. If the generator, for example, has taken the lead, the driving torque becomes stronger than the resisting torque, and it causes the armature to accelerate gradually; but, owing to inertia, the acceleration lasts longer than would be necessary to attain the corresponding phase at the condition of equilibrium, and the motor, in its turn, takes the lead in phase. From that moment, as its driving torque diminishes, the excess of the resisting torque tends to produce a retardation; the motor slackens in speed, and its phase is soon behind what it should be; and so on.

Consequently, oscillations of speed occur around the normal speed value, the effects being altogether analogous to the oscillations occurring in the speed of steam engines which are provided with imperfect governors.

The characteristic feature of the short-period oscillations of the motor-speed is that this oscillatory movement occurs without loss of synchronism, in other words, each acceleration in one direction or the other is stopped by an opposing torque which limits the amplitude, as is the case with the oscillations of a pendulum.

This torque is easily determined by the variation of power of the two machines (*A. Blondel*, "*Coupling of Alternators*," *La Lumière Eletrique*, Vol. XLV, p. 352). Let us first suppose these two machines to be similar; let P_1 and P_2 represent, respectively, the generator-power and the motor-power. The variation in the electric power received by the motor and transmitted by the generator, per unit of angle α of angular variation, taking $2P$ to equal the number of poles, is, according to the formulæ on page 38, as follows :

$$\left(\frac{dP_1}{d\alpha}-\frac{dP_2}{d\alpha}\right)=p\left(\frac{dP_1}{d\theta}-\frac{dP_2}{d\theta}\right)$$

$$=p\frac{E_1E_2}{Z}[\sin(\gamma-\theta)+\sin(\gamma+\theta)]$$

$$=\frac{2pE_1E_2}{Z}(\sin\gamma\cos\theta).$$

In this expression Z equals the total impedance, and γ is, as before, the angle of lag between the current and the resultant E.M.F., the relations being such that

$$\tan\gamma=\frac{\omega L}{R}.$$

The correcting torque is obtained by dividing this power by the mean angular velocity

$$\frac{\omega}{p}.$$

From this we have

$$C=\frac{p^2}{\omega}\left[\frac{2E_1E_2\sin\gamma\cos\theta}{Z}\right].$$

If we let K equal the inertia of the motor, the period of oscillation produced by this torque may be deduced by the known formula,

$$\theta = 2\pi\sqrt{\frac{K}{C}},$$

whence, on substituting, and noting that $\frac{2E_2}{Z}$ is nothing more than the short-circuit current, I_{sc}, of one of the machines—the motor—closed on itself, there remains

$$\theta = \frac{2\pi}{p}\sqrt{\frac{K\omega}{E_1 I_{sc}\sin\gamma\cos\theta}};$$

or, if we suppose θ to be small enough so that $\cos\theta$ may be taken as equal to unity, we will have

$$\theta = \frac{2\pi}{p}\sqrt{\frac{K\omega}{EI_{sc}\sin\gamma}}.$$

In the case where the machines are not similar and are running at different speeds α_1 and α_2, it can be shown, in the same way (*A. Blondel*, loc. cit., p. 357), that the equation of oscillatory movement is

$$\omega\frac{d^2\theta}{dt^2} + \frac{(P_1)_\theta - (P_1)_{\theta_0}}{\left(\frac{K_1}{p_1^2}\right)} - \frac{(P_2)_\theta - (P_2)_{\theta_0}}{\left(\frac{K_2}{p_2^2}\right)} = 0.$$

In the case of small oscillations, if we replace the numerators by their values, we have

$$\theta\frac{dP_1}{d\theta} = \frac{E_1E_2}{Z}\sin(\gamma+\theta),$$

$$\theta\frac{dP_2}{d\theta} = -\frac{E_1E_2}{Z}\sin(\gamma-\theta),$$

and assuming $\cos\theta = 1$, and $\sin\theta$ negligible in comparison with $\cos\theta$, we have

$$\frac{d^2\theta}{dt^2} = \frac{E_1E_2}{\omega Z}\left(\frac{p_1^2}{K_1} + \frac{p_2^2}{K_2}\right)\sin\gamma\times\theta,$$

whence

$$\theta = 2\pi\sqrt{\frac{\omega}{EI_{sc}\frac{\sin\gamma}{2}\left(\frac{p_1^2}{K_1} + \frac{p_2^2}{K_2}\right)}}.$$

In the particular case where the motor is supplied from a circuit of great output-capacity the term $\frac{p_1^2}{K_1}$ can be considered as very small in comparison with $\frac{p_2^2}{K_2}$, and, consequently, the expression for the period of the oscillations, as a function of the constants of the machine and of the inertia of the armature and of the parts mounted on the shaft, is materially increased.

This formula shows that the oscillations are all the more rapid and of lower amplitude the lower the self-induction and the mechanical inertia.

Consequently, from this point of view, it would be more profitable to select motors with a low armature-reactance, a conclusion which we had already reached by another process. The oscillations not only have the effect of causing the motor to fall out of step, and of thus limiting its maximum output, but also of producing oscillations of E.M.F. and current, including change of phase (lag). These effects can manifest themselves in a very objectionable manner, on the circuit and on the generator, by producing fluctuations of voltage which are often important, especially when the reactance of the circuit is large and the circuit is thereby made very sensitive to variations in wattless current.

A synchronous motor which is oscillating can therefore cause trouble to its neighbors on the circuit, and it may even render lighting from that circuit impossible. For this reason, great care should be taken to regulate perfectly the speed and phase of the motor by means of a phase-indicator, before connecting it to the supply-circuit.

Every sudden change in load gives rise to a strong oscillation in the phase-angle, causing E_2 to go beyond the position of equilibrium corresponding to the new value of the power. If this oscillation comes from a reduction in load, it can cause no inconvenience; if, on the contrary, it is caused by an excess of load, the phase-variation may, in the first impulse, exceed the angle of stability, and the machine will fall out of synchronism.

Calculation shows (*A. Blondel*, loc. cit., *Lumière Electrique*, Vol. XLV, p. 24) that the relative increase of load Δ which can be made suddenly, starting from an initial power $(-P_2)_{\theta_0}$ does not depend on inertia, but only on the two ratios

$$\frac{\gamma(-P_2)_{\theta_0}}{E_1^2} = y \quad \text{and} \quad \frac{E_2}{E_1}\cos\phi = x;$$

and the curves in Fig. 60 enable its value in each particular case to be calculated as a function of the data. These curves show that Δ varies in the inverse ratio of y and in direct ratio of x, so long as z is lower than a value near $\frac{1}{2}$. The conditions that insure the best stability in the case of sudden overloads are, therefore, substantially the same as those which insure the maximum theoretical output. In particular, in the case where $E_2 = E_1$, we should have $\cos \phi = \frac{1}{2}$, whence $ml = r\sqrt{3}$.

The experiments made on a Ganz motor (*Experiments made by the Frankfurt technical commission, Upperborn, Lumière Electrique*, Vol. XXXVI, p. 315), showed that, starting from no-load, a sudden load equal to 150 per cent of the nominal load may be put on the motor. This load could be much exceeded for alternators of lower inductance, especially if they are provided with dampers on the pole-pieces.

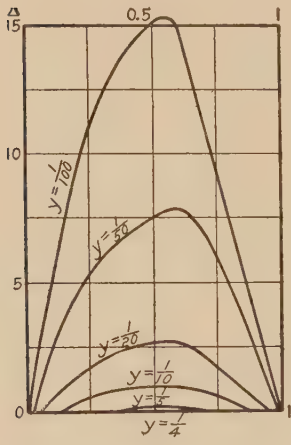


FIG. 60.

Damping of Oscillations. The surest way to prevent oscillations of both kinds from attaining an excessive amplitude consists in damping them rapidly. It is an application of the general principle laid down in a masterly manner by Cornu (*"On the synchronization of oscillating systems," Journal de Physique et Comptes Rendu de l'Académie des Sciences*, 31 *Mai*, 1887; *Bulletin de la Société des Electriciens*), who showed that perfect synchronization can be obtained only when there is damping, i.e., when the speed-variations cause a supplementary expenditure of energy. This supplementary expenditure of energy can only occur to a slight extent in the armature-circuit, but it is possible to bring it about in the field-circuits by allowing eddy currents to be produced in the massive pole-pieces or, better still, in special circuits wound on the fields, and short-circuited. These circuits constitute the damper of Hutin and Leblanc. (*C. F. Guilbert, Lumière Electrique*, Vol. XLVI, 1892, p. 801; *and Leblanc, Bull. de la Soc. Int. des Electriciens*, 1898). This damper consists either of bars of copper surrounding the pole-pieces, as shown in Fig. 61, or, better still, of a series of bars passing through the fields and united on each side by copper rings, as shown in Fig. 62.

The explanation of the role of these dampers is made easy by reference to the elementary considerations set forth on page 11. When there are no speed-oscillations, the armature-reaction flux undergoes no displacement with respect to the field-poles, and, consequently, it does not generate any induced E.M.F. in the damping circuits. On

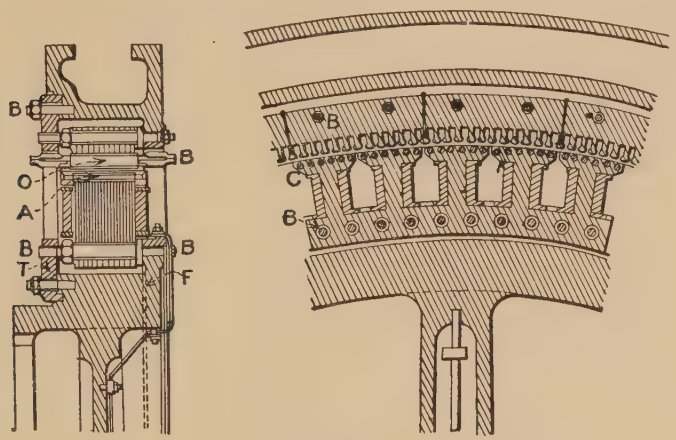


FIG. 61.

the contrary, any variation of speed producing a flux-oscillation gives rise to eddy currents, which are all the stronger the greater the conductivity of the damping circuits and the greater the extent to which they increase the flux developed by the armature. The energy consumed by the induced currents is what produces the damping. It

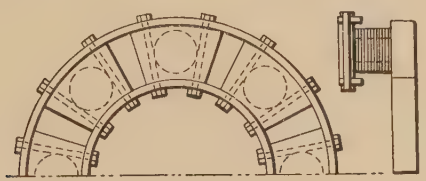


FIG. 62.

manifests itself by the production of a corrective torque exerted on the armature and which tends to bring the speed to synchronism.

The same explanation applies in the case of a single-phase motor, with this difference, that the inverse revolving flux due to the armature should, in consequence of its high velocity relatively to the damping circuit, produce strong induced currents which almost completely

neutralize this armature-flux. The result is that the armature-reaction, or the apparent inductance in the armature, is reduced about one-half and, at the same time, the noise of single-phase alternators disappears. The loss of energy is much lower than might be supposed, because it eliminates, incidentally, the hysteresis-effect in the fields. In any case, the damping effect can be regulated easily to an amount that is deemed sufficient, as can be seen in the following manner:

Suppose the motor departs from synchronism; the period of its induced E.M.F. is no longer the value T, but a slightly different value, T'. The coefficient of mutual induction between the armature-circuit and the damping-circuit, being a periodic function of the angle described by the moving part, can be written in the following form:

$$M = M_0 \sin\left(2\pi\frac{t}{T'} - \gamma\right),$$

in which γ is the phase-angle measured from the origin of time.

The current can be represented by the following expression:

$$I = I_0 \sin 2\pi\frac{t}{T} + I_0' \sin\left(2\pi\frac{t}{T'} - \beta\right).$$

The flux sent into the damping-circuit by the induced current, at each instant will be

$$\Phi = MI = M_0 I_0 \sin 2\pi\frac{t}{T} \sin\left(2\pi\frac{t}{T'} - \gamma\right) + M_0 I_0 \sin\left(2\pi\frac{t}{T'} - \beta\right) \sin\left(2\pi\frac{t}{T'} - \gamma\right),$$

or, substantially, if we confine ourselves to the first term,

$$= \frac{M_0 I_0}{2} \left\{ \begin{array}{l} -\cos\left[2\pi\left(\frac{1}{T} + \frac{1}{T'}\right)t - \gamma\right] + \\ +\cos\left[2\pi\left(\frac{1}{T} - \frac{1}{T'}\right) + \gamma\right] \end{array} \right\}, \quad \ldots\ldots \quad (1)$$

The corresponding E.M.F. will be

$$e = -\frac{d\Phi}{dt} = \left\{ \begin{array}{l} \pi\left(\frac{1}{T} + \frac{1}{T'}\right) M_0 I_0 \sin\left[2\pi\left(\frac{1}{T} + \frac{1}{T'}\right)t - \gamma\right] \\ -\pi\left(\frac{1}{T} - \frac{1}{T'}\right) M_0 I_0 \sin\left[2\pi\left(\frac{1}{T} - \frac{1}{T'}\right)t - \gamma\right] \end{array} \right\}. \quad (2)$$

Consequently, if ρ equals the resistance, and λ the inductance of the damping circuit, the induced current i is due to the superposition of two currents of two different frequencies:

$$i=\left(\frac{1}{T}+\frac{1}{T'}\right)\frac{M_0 I_0 \sin\left[2\pi\left(\frac{1}{T}+\frac{1}{T'}\right)t-\psi\right]}{\sqrt{\rho^2+\left[2\pi\left(\frac{1}{T}+\frac{1}{T'}\right)\lambda\right]^2}}$$

$$-\pi\left(\frac{1}{T}-\frac{1}{T'}\right)\frac{M_0 I_0 \sin\left[2\pi\left(\frac{1}{T}-\frac{1}{T'}\right)t-\psi'\right]}{\sqrt{\rho^2+\left[2\pi\left(\frac{1}{T}-\frac{1}{T'}\right)\lambda\right]^2}}.$$

The energy consumed is represented by the integral

$$W=\int_0^t eidt,$$

which contains products of terms having different frequencies. If the integration is supposed to extend over a sufficiently long period, these terms can be neglected in comparison with the others, which alone increase indefinitely. In that case, the value of the mean power expended per second can be written as follows:

$$P=\left[\pi\left(\frac{1}{T}+\frac{1}{T'}\right)M_0 I_0\right]^2\frac{\rho}{\rho^2+\left[2\pi\left(\frac{1}{T}+\frac{1}{T'}\right)\lambda\right]^2}$$

$$+\left[\pi\left(\frac{1}{T}-\frac{1}{T'}\right)M_0 I_0\right]^2\frac{\rho}{\rho^2+\left[2\pi\left(\frac{1}{T}-\frac{1}{T'}\right)\lambda\right]^2}.$$

The first term is substantially constant, since T' remains near T. On the contrary, the second term, which vanishes when $T'=T$, increases rapidly with the difference of frequencies. It represents the correlative increase of energy consumed, which produces the damping.

Supposing the difference $\frac{1}{T}-\frac{1}{T'}$ to be very small, the inductance may be neglected, in comparison with the resistance, in the second term. In the first term, on the contrary, the inductance is always very large, in comparison with the resistance, and, consequently, the latter

may be neglected. The expression for P is therefore practically equal to the following:

$$P=\rho\left(\frac{M_0I_0}{2}\right)^2+\frac{1}{\rho}\left[\pi\left(\frac{1}{T}-\frac{1}{T'}\right)M_0I_0\right]^2. \quad . \quad . \quad . \quad (3)$$

Consequently, by making ρ very low, as was done by Hutin and Leblanc, the permanent expenditure of energy due to the current of frequency $\frac{1}{T}+\frac{1}{T'}$ can be reduced, at the same time that the damping effect due to the current of frequency $\frac{1}{T}-\frac{1}{T'}$ is amplified.

When the pole-pieces are solid, they already, by themselves, play the role of a damping circuit, and, their resistance ρ being very small, it can be readily understood that the effect thus obtained would be already quite appreciable.

At synchronism, we have $T'=T_1$, and the preceding expressions (1) and (2) (p. 128) reduce to the form

$$\Phi=\frac{M_0I_0}{2}\left\{-\cos\left[2\pi\left(\frac{1}{T}+\frac{1}{T'}\right)t-\phi\right]+\cos\phi\right\}.$$

and

$$e=\frac{2\pi}{T}M_0I_0\sin\left(2\pi\frac{t}{\frac{T}{2}}-\phi\right).$$

The armature-reaction then produces two fluxes in the field-coils, one being constant, which represents the mean reaction, and the other having a frequency which is double that of the alternators.

Long-Period Oscillations. Long-period oscillations, characterized by periodical variations of current which can be observed on an ampere-meter, may occur in consequence of causes which are apart from the peculiarities of the motors themselves. For example, a motor which drives a machine-tool subjected to a variable load, or a motor connected to a generator whose speed varies periodically (for example, a generator driven by a steam engine having a single cylinder), will itself undergo the same corresponding oscillations of currents, which are much slower than the preceding. These slow oscillations have, moreover, the incidental effect of keeping up the real oscillations, and thus they are often the cause of the machines falling out of synchronism.

Other causes more complex, and which are still less well understood, can bring about the systematic and periodic production of this phenom-

enon of successive falling out of and falling into synchronism. For instance, Hutin and Leblanc (*M. Leblanc, Lumière Electrique, Vol.* XXXIII, p. 227, *and F. Geraldy*, ibid., Vol. XLVI, p. 652) showed by an experiment that some Ganz motors of the kind excited by commutated currents did not always attain synchronism, but, in certain cases (especially when running without load), assume an oscillatory movement having long periods. These periods can, in such a case, be attributed to slow alternations of the excitation produced by the current-commutator. This is an additional reason for abandoning definitely this kind of excitation. Certain motors also appear subject to slow oscillations, in consequence of the discontinuity of the V-curve produced by the presence of important upper harmonics in their E.M.F. curve, as already noted above (page 103).

CHAPTER V

TESTS OF SYNCHRONOUS MOTORS

Characteristic Curves. It may be stated, in general, that the characteristic curves of synchronous motors are the same as those of alternators, i.e., the variation of the inducing flux is represented as a func-

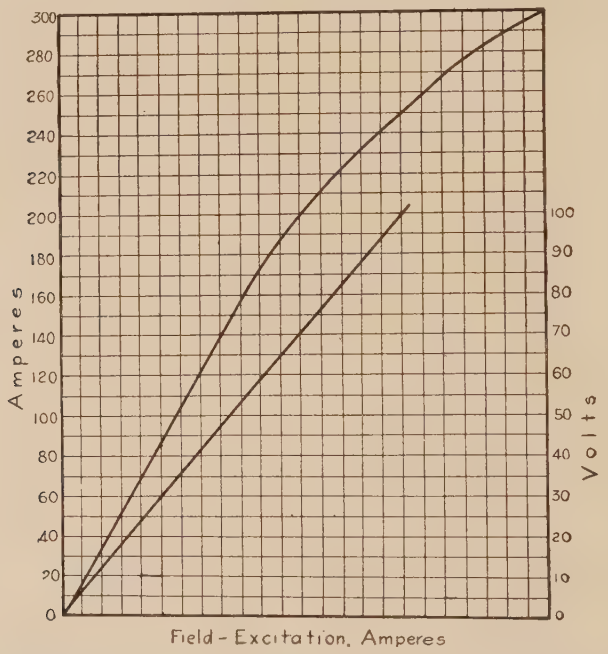


FIG. 63.—Current consumed by a 15 H.P. synchronous motor, run at constant potential with variable field-excitation. The ordinates represent armature current (amperes) per phase.

tion of the exciting ampere-turns by an " excitation-curve," and the armature-reaction is represented by a short-circuit-curve obtained according to the method of Behn-Eschenburg, by measuring the amper-turns necessary to produce increasing currents, when the machine is

short-circuited on itself and is run at a speed near its normal speed. The short-circuit current will be all the smaller, for a given excitation, the greater the armature-reaction. Since a good stability of operation is only obtained, as already seen, with low armature-reaction, it follows that a good synchronous motor will be characterized by a short-circuit curve which is rapidly ascending such as that shown in Fig. 63, which refers to a synchronous motor with revolving iron of Oerlikon type, described by Kolben (*Elektrotechnische Zeitschrift*, 19 *Dec.*

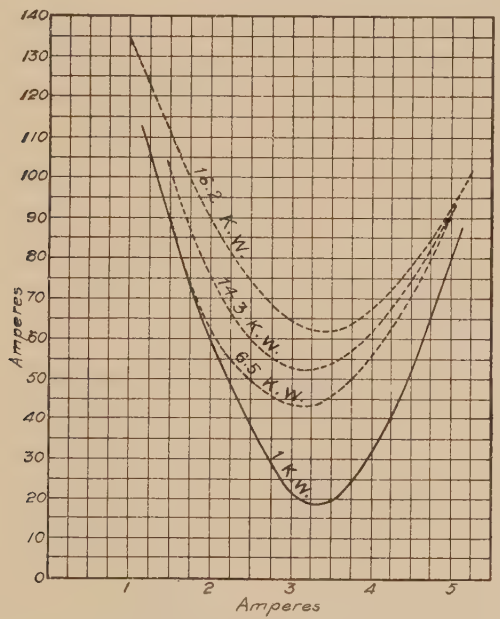


FIG. 64.—Power input for 15 H.P. synchronous motor, run with variable field-excitation.

1895). A motor is good, according to this author, when a short-circuit-current equal to the normal operating current is obtained by an excitation giving not more than one-third the normal voltage, for the induced E.M.F. Finally, from what precedes, there exists, for synchronous motors, a third kind of important characteristic curve, the V-curves, of which the theory has already been given. These curves are drawn for variable loads applied by a Prony-brake, by measuring, for each value of the excitation, the current-values given by an ampere-meter, and the electric-power values given by a watt-meter.

Figs. 64 and 65 give, for example, these curves obtained by Kolben for an Oerlikon three-phase synchronous motor of 15 H.P. These curves show that the minimum current in the armature is obtained with about the same excitation at all loads, and that the efficiency is almost constant within wide limits. For example, the power consumed at full load, to produce 14 k.w., scarcely varies between 1 and 5 amperes of excitation. These favorable conditions characterize a motor having low armature-reaction, Motors having high armature-reaction give rise to eddy currents in the pole-pieces, which involve a considerable increase in the power absorbed, when the current increases in the armature. This peculiarity may be made evident by means of a fourth characteristic curve known as the "characteristic curve of loss by parasite-effect." This curve is obtained by taking as ordinates the power absorbed when running short-circuited, the excitation-currents being taken as abscissæ. Figs. 66 and 67 show, quite clearly, the difference, in this respect, between a motor having a high armature-reaction and a motor having a low armature-reaction. In the latter the eddy currents, which assume great importance, impart a parabolic form to this characteristic curve.

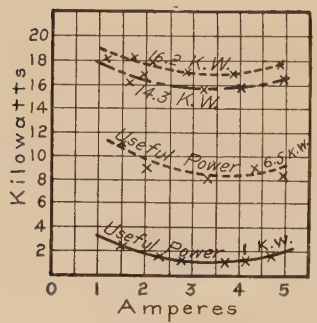


FIG. 65.—Power-curves for three-phase 30 H.P. synchronous motor, run with variable field-excitation.

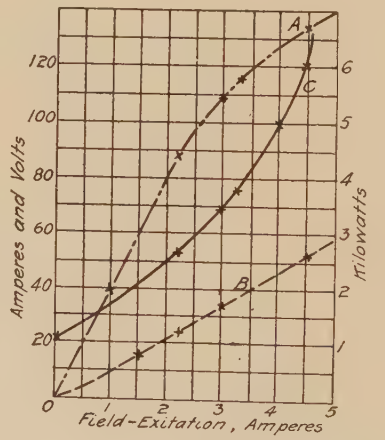


FIG. 66.—Curves for 30 H.P. synchronous motor having high armature-reaction.
A = characteristic curve without load;
B = short-circuit current;
C = power absorbed when short-circuited.

Measurement of Efficiency. The most natural method for the measurement of efficiency is the Prony-brake-dynamometer method, used in the same way as for a D.C. motor. The resisting torque produced by the brake is measured, and, at the same time, the

electric power applied at the terminals of the motor is measured by a watt-meter. This method has the objection of requiring the expenditure of too much energy in the case of large motors.

The author's method, described further on, for the study of conditions of operation, then finds its application. In such cases, the generator and motor are mounted and coupled together in such manner that the generator is driven by its pulley, and also tends to be driven by the motor which is connected to the other end of the shaft and which takes the lead and drags the dynamometer. The latter

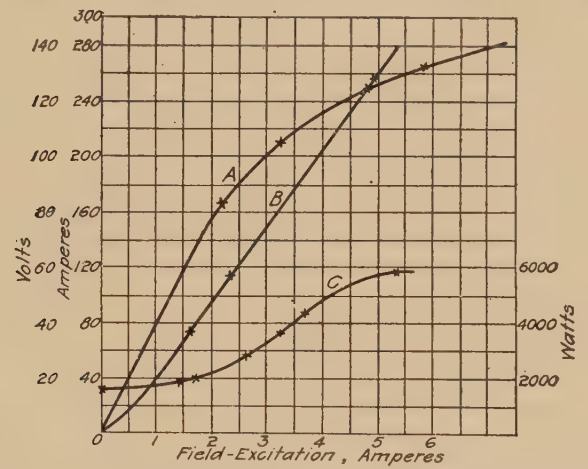


FIG. 67.—Curves of 15 H.P. synchronous motor. Normal voltage 240 volts.
A = characteristic curve without load.
B = short-circuit current.

then measures directly the work done by the motor, and all that is necessary is to measure, by means of a watt-meter, the electric power supplied to the motor by the generator. If we let C equal the resisting torque, n the number of turns per second, and w the electric power measured by the watt-meter, the efficiency, η, will be expressed by the following equation:

$$\eta = \frac{2\pi nC}{w}.$$

The segregation of the different losses, in a synchronous motor, is a much more complex problem than in the case of direct current machines because the parasite effects (hysteresis and eddy currents) in the field-pole pieces depend not only on the induced current and on the

excitation-current, if the armature-core is slotted, but they also depend on the phase-angle of the induced current, especially in the case of single-phase motors. It can be readily understood that, in the case of single-phase motors, the variation in the reaction-flux in front of the pole is all the greater the more rapidly the armature-current varies in the position of maximum mutual induction. For the same current in the armature, the losses will, therefore, be greater the smaller the phase-angle (lag) of the current.

By the aid of these assumptions, the losses can always be approximately separated, when the conditions of operation are precise, such as when the current-supply has a constant voltage and the excitation-current is constant. In such cases, the losses by friction are measured

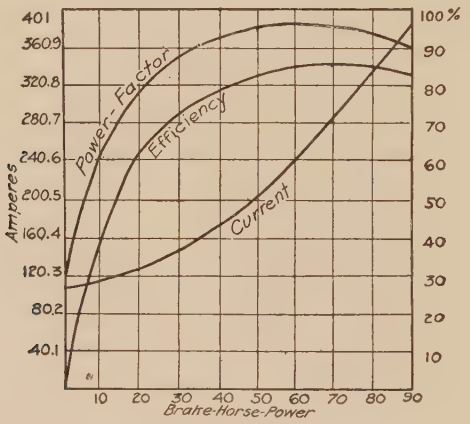


FIG. 68.

by running the machine without excitation, by means of a small D.C. motor whose efficiency curve has been obtained. On exciting the machine, the loss becomes increased by an amount equal to the hysteresis and eddy currents, in the armature and the fields, other than those due to armature-reaction. They may be distinguished, as usual, by running at various speeds. The motor is then run with a load applied at the brake. The difference between the losses noted and those previously measured consists of two portions: the loss due to resistance-heating in the armature, which can be easily calculated from the resistance and the current, which are known; and the increase of loss due to armature-reaction, which is obtained, more or less approximately, by taking the difference.

Fig. 68 gives an idea of the manner in which the power-factor and the efficiency vary as a function of the load in a synchronous motor.

Experimental Tests. It is interesting to compare the results of experiments with those of calculation. The methods which can be used for that purpose will now be indicated.

It should be understood that a satisfactory agreement between theory and practice can only be hoped for when working with machines having E.M.F.'s of the same wave-form, and as nearly as possible sinusoidal. Owing to the want of this precaution, certain authors (*Bedell and Ryan*, loc. cit.) have found discontinuities in operation which are inexplicable by the ordinary theory

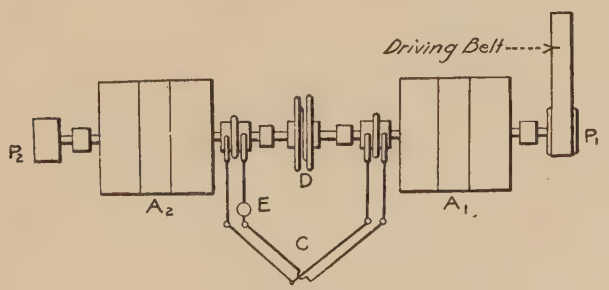


Fig. 69.

The factors to be measured experimentally are the E.M.F.'s induced in the generator and the motor, the current in amperes, and the phase-angles between these various quantities, and also the electrical and mechanical powers applied or developed. The investigation can be more or less complete according to the apparatus available.

The author published, in 1893 ("*Theory of the Coupling of Alternators*," *Bulletin de la Société des Electriciens*, 1893, p. 9), a method, based on that of Hopkinson for D.C. dynamos, which has the advantage of requiring only a small amount of mechanical power (Fig. 69).

The two machines, generator A_2 and motor A_1, are placed end to end, and the two shafts are joined together by a flange-coupling or a torsion dynamometer C (of type such as used by Mascart, Rateau, Bedell, etc.) so arranged that they can be set in various positions for changing the angle of phase of the two shafts with respect to each other. This phase-angle is known, at rest, and all that is necessary is to add to it the angle of torsion of the dynamometer to have the phase-

angle when running. One of the alternators is driven by a belt run over its driving pulley, and it drives the other, mechanically through the dynamometer, as well as electrically through the circuit. The alternator which is belt-driven acts as a motor, the other acting as a generator; the torsion-dynamometer connected between the two machines measures the excess of power consumed by the generator over and above the energy recovered. The power applied at the pulley represents the total losses of the two machines operating as a power-transmission system. From these data the efficiency of transmission may be deduced.

Suppose that, for each phase-angle, the excitation-current and the line-current are read, by means of an electrodynamometer E, also that the voltage at the terminals is read, by means of a volt-meter, and that the electric power is read by means of a watt-meter inserted in the circuit, with its shunt-coil connected to the terminals.

By comparing this "actual" power with the "apparent" power, the phase-angle between the internal E.M.F.'s E_1 and E_2, is obtained substantially. The magnitudes of these E.M.F.'s are known from the excitation-curves, which are drawn once for all. This method is particularly simple when no transmission-dynamometer is used, when the loads are perfectly steady; but the loads are not then obtained in terms of the mechanical power of the motor.

When it is impossible to couple the two machines rigidly, or when they do not have the same number of poles, it is not so easy to determine the phase-difference or lag between the two motors, and it is then necessary to proceed in one of the two following ways:

(1) It is possible to mount on the shaft of the machines some small alternators serving as "tell-tales." They are arranged in such manner that their E.M.F.'s synchronize in phase with the E.M.F.'s induced when running without load. A triple oscillograph record will then give, directly, for each load, the phase-angles between these E.M.F.'s and the current.

(2) When the machines have not the same number of poles the phase-angles between the induced E.M.F.'s can be measured mechanically by means of the ingenious phase-indicator of Bedell and Moler. The two machines are placed end to end, with their shafts almost touching. The phase-indicator consists of two metallic disks 23 centimeters in diameter and 8 millimeters thick, each fastened to one end of the shaft and provided with curved slits symmetrically disposed in the form of an Archimedean spiral. (Fig. 70.) The disks are

arranged in such a way that their spirals are pointed in opposite directions. Owing to the form of these curves, the distance from their points of intersection to the center is proportional to the phase-angle between the two disks, or the lag of one with respect to the other. The point of intersection of two slits forms a small opening traversed by rays of light which, in turning, make a circle that expands or contracts in proportion to the phase-angle. The radius of the circle can be measured on a fixed scale which is graduated directly in angles.

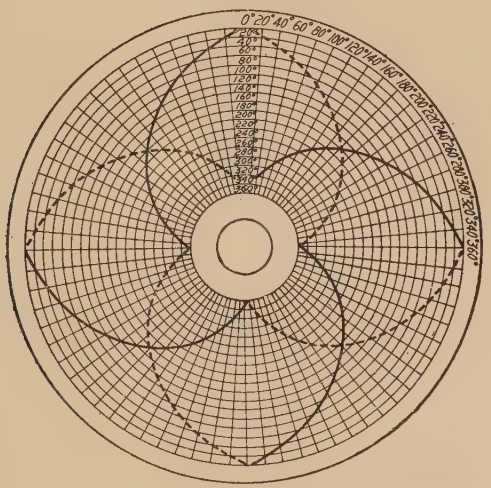


FIG. 70.

Bedell and Ryan have used this method to determine the phase-angle between E.M.F.'s, by bringing this phase-angle to zero when the poles are in line. The currents are measured by means of an electrodynamometer. The E.M.F.'s are deduced from the excitation-curves. Their vectors are then drawn with an angle between them equal to the measured phase-angle, and, from these, the resultant vector is obtained, in magnitude and in phase, by completing the parallelogram. The current is known, in magnitude. Its phase, with respect to the resultant E.M.F., is deduced from the reactance, which is known by means of the expression

$$\tan \gamma = \frac{\text{reactance}}{\text{resistance}},$$

or it can be deduced from an oscillograph diagram. We then have all that is necessary to draw the diagram. The results are sufficiently concordant with the theory, but they indicate no new phenomenon, except the discontinuity already mentioned. For more complete details see the author's original paper.

Advantages and Disadvantages of Synchronous Motors. The characteristic features of synchronous motors, constituting their advantages and disadvantages, may be summarized as follows:

(1) Their construction is as simple as that of alternators, and it enables them to be connected directly to high voltage circuits.

(2) Their specific output and their efficiency are also the same, and are quite satisfactory.

(3) When once put in operation they have perfectly uniform speed.

(4) Their power-factor is as good as may be desired when their E.M.F. is properly selected. By using them, all reactive currents can be eliminated, and currents can even be made to lead in phase, by suitable regulation of the excitation.

(5) It is difficult to start these motors, especially single-phase motors; special methods and apparatus must be employed and also an idler-pulley or clutch. Even when starting without load, which is the only practical way, a very high current is required.

(6) Speed-regulation is impossible.

(7) A very sudden overload causes the motor to fall out of phase and brings about a short-circuit.

(8) Any irregularity of operation of the generator causes great oscillations of speed ("pumping") which make the voltage of the system fluctuate. If the excitation is not well regulated, the motor produces a more or less important reactive current, which may bring about a drop of voltage in the system.

(9) If the E.M.F. curve of the motor is different from that of the supply-system, harmonics will be produced, which will uselessly increase the line-current and compromise the stability of operation. Stability of operation can only be attained by the addition of inductance to the circuit; and it remains weak, at best.

(10) A synchronous motor reacts more or less on the rest of the system, to which it can communicate its oscillations, or of which it can distort the E.M.F. curve. The construction of synchronous motors should therefore be carefully studied, not only with reference to their operation by themselves, alone, but with reference to their effect on the distributing system from which they are to be supplied.

CHAPTER VI

OTHER MOTORS OPERATING SYNCHRONOUSLY WITHOUT DIRECT CURRENT EXCITATION

It is possible to devise other motors which, once brought to a suitable speed, can continue to operate synchronously, without excitation by a direct current or by a commutated pulsating current. These motors, not being used industrially, constitute, properly speaking, theoretical curiosities, but it is interesting, in order to give a general idea of the whole subject, to indicate their principles.

Mention can be made of two types of such motors; synchronous motors without any excitation, known as "reaction-motors," and synchronous motors with alternating-current excitation, or alternating-field motors.

Reaction Synchronous Motors. It is natural to suppose that the constant armature-reaction-field of polyphase motors can be used as an inducing field. It is sufficient, for this purpose, that the armature should have a large number of windings to produce a large number of ampere-turns, and that the current passing through it should be much out of phase, in order that the magnetizing turns may coincide with those of the field-poles. Moreover, the air-gaps should, obviously, be very small, to reduce the reluctance of the inducing circuit. The self-excitation of the latter is then only an exaggeration of the effect of the lagging reactive current, which has already been mentioned several times. The operation of such a motor is therefore quite possible, even at light loads; but the inductance of the armature has been increased at the same time as its reaction on the field; and the consequence is that the stability and, moreover, the power-factor of the current used, will be small, owing to the great lag of this current. Such a motor would, therefore, be bad from a commercial point of view.

The conditions of operation can be easily ascertained by making use of the diagram on page 94, in which the E.M.F. due to external excitation is supposed to be zero.

The operation of motors on the reaction-principle is not limited

to polyphase motors, but it can also be accomplished, although with more difficulty, in the case of single-phase motors. The phenomenon was first observed as early as 1874, by Siemens of London, who found that he could suppress the excitation of a single-phase motor without stopping it. The explanation is simple if we refer to the use already made of the theorem of Leblanc, i.e., that the alternating field of the armature-reaction can be replaced by two fields, each of half the strength, rotating in opposite directions, one of which remains in front of the field-poles and magnetizes them, the other being a useless parasite current, rotating with twice the speed of the poles and producing no effect (except eddy currents).

Another explanation can also be given, based on the variation of the coefficient of self-induction of the armature at each period during its rotation. In consequence of this variation the reaction-flux li is a maximum when the armature coils are exactly in front of the poles.

This position is therefore that which the armature would tend to take if a direct current were passing through it. Hence, if the machine is driven by hand, it will be sufficient that the motion may continue, as in certain toys, to break or to weaken the current each time that the coils move away from a pole, or that it may close the circuit, or strengthen the current, each time that they approach the next pole. Now this effect is precisely produced with an alternating current (because the direction of the current is of no importance here) as soon as the maximum of the periodic current is made to lead in phase with respect to the maximum of l, as can be easily seen. The sum of the effects which attract the armature in the direction of motion during a period is then greater than the sum of the contrary effects.

The motion therefore continues, and it is easy to see that it may be steady, because every acceleration of the motion tends to diminish the phase-angle between i and l, and, consequently, tends to diminish the dissymmetrical action, while retardation increases the phase-angle, and, consequently, increases the useful action of the current.

These considerations can be easily verified by calculation by referring to the equation

$$u = e - ri - \omega l \frac{di}{dt},$$

in which we will let $e = 0$, to take into account the fact that there is no excitation. The alternator having a variable self-induction

$$l = l' + \lambda \cos 2\omega t,$$

and supposing the current passing through it from the inside to the outside to be sinusoidal, and to be out of phase by the angle ψ with respect to the position of coincidence between the poles, we will have

$$I = I_0 \sin(\omega t - \gamma).$$

We will therefore have, for the voltage at the terminals,

$$u = -rI_0 \sin(\omega t - \psi) - \omega\left[\left(l' - \frac{\lambda}{2}\right) I_0 \cos\psi \cos\omega t + \left(l' + \frac{\lambda}{2}\right) I_0 \sin\psi \sin\omega t\right],$$

the harmonics of the order $3\omega t$ introduced by the pulsation of l being neglected.

This equation is represented by the diagram in Fig. 71. It is seen

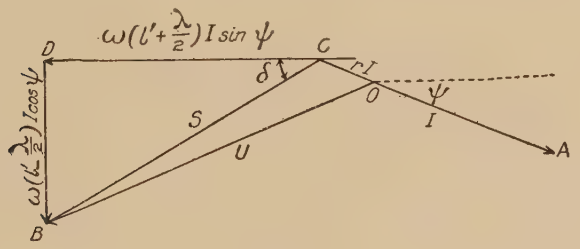


FIG. 71.

that the voltage U at the terminals is the geometrical resultant of the vectors

$$\left[-rI, -\omega\left(l' + \frac{\lambda}{2}\right) I, -\omega\left(l' - \frac{\lambda}{2}\right) I \text{ (perpendicular)}\right].$$

In this diagram the lag ψ of the current is defined as the interval between the zero-value of the current and the position of coincidence of the poles.

It is seen that the E.M.F. of self-inductance S makes an obtuse angle with the current I so long as ψ is comprised between 0 and $\frac{\pi}{2}$ (the two vectors being in opposition when $\psi = 0$), and that it is in quadrature when $\psi = \frac{\pi}{2}$.

The expression for the mean electric power can be easily obtained, according to the usual rule, by multiplying the vector S by the projection of I on S (I and S being effective values). We then have

$$P = SI \cos ACB$$
$$= I^2\omega \cos(\pi - \psi - \delta)\sqrt{\left(l' + \frac{\lambda}{2}\right)^2 \sin^2\psi + \left(l' - \frac{\lambda}{2}\right)^2 \cos^2\psi},$$

on the assumption that

$$\tan\delta = \frac{\left(l' - \frac{\lambda}{2}\right)\cos\psi}{\left(l' + \frac{\lambda}{2}\right)\sin\psi}.$$

The value of the power-factor, obtained from the preceding, is

$$\cos(\pi - \psi - \delta) = -\cos\psi\cos\delta + \sin\psi\sin\delta$$
$$= \frac{-\lambda\cos\psi\sin\psi}{\sqrt{\left(l' + \frac{\lambda}{2}\right)^2 \sin^2\psi + \left(l' - \frac{\lambda}{2}\right)^2 \cos^2\psi}}$$

or

$$= \frac{-\lambda \sin 2\psi}{\sqrt{l'^2 + \frac{\lambda^2}{4} - \lambda l' \cos 2\psi}},$$

whence

$$P = -\omega\lambda I^2 \sin\psi\cos\psi = -\frac{\omega\lambda I^2 \sin 2\psi}{2}.$$

The power is negative and therefore the alternator operates as a motor (consuming electrical energy and producing mechanical power) when the value of ψ is comprised between o and $\frac{\pi}{2}$; and it operates as generator (producing electrical energy and consuming mechanical power) for values of ψ comprised between o and $-\frac{\pi}{2}$, i.e., when the current is leading.

The power is a maximum for

$$\psi = \pm\frac{\pi}{4},$$

and its absolute value is

$$P_{\max} = \frac{\omega\lambda I^2}{2} = \frac{\omega\lambda I}{2} \times I.$$

This power is necessarily very low, since λ is never more than a fraction of the mean self-induction l', and since the E.M.F. which is utilized, $\frac{\omega\lambda l}{2}$, is itself only a small fraction of the E.M.F. of self-induction of the alternator, which latter is always lower than the E.M.F. that would be generated if the alternator were excited by its fields. It can be easily seen that the power-factor always remains very low. Moreover, this motor has the same disadvantage as polyphase reaction-motors. For this reason, this method of using alternators does not appear susceptible of practical application.

Synchronous Motor with Alternating Fields. The invention and the theory of a synchronous motor with alternating fields are due to Galileo Ferraris (*see Memoir presented to the Academy of Sciences of Turin, Dec.* 1893, *and also Industrie Electrique,* 10 *Juin,* 1894). In this apparatus the fields and armature are excited in series or in shunt by alternating currents taken from the same source, and the armature is previously brought to a speed double that which would be suitable for a synchronous motor with constant fields. The motor then runs synchronously, and it can be lightly loaded.

The operation of such a motor is easily explained by the aid of the theorem of Leblanc. Let $2p$ equal the number of poles and ω the speed of pulsation of the currents. The alternating field (the field winding being supposed stationary) can be expressed fictitiously by two fields, H_1, H_2, one rotating to the right, the other to the left, with the contrary speeds $+\frac{\omega}{p}$ and $-\frac{\omega}{p}$. A speed equal to $\frac{2\omega}{p}$ is given to the armature in one direction; the alternating field which the armature produces is itself also equivalent to two revolving fields having the velocities $+\frac{\omega}{p}$ and $-\frac{\omega}{p}$ *relatively* to the armature. The corresponding absolute speeds will be

$$\text{First field, } M_1, \quad \frac{2\omega}{p}+\frac{\omega}{p}=\frac{3\omega}{p} \text{ to the right.}$$

$$\text{Second field, } M_2, \quad \frac{2\omega}{p}-\frac{\omega}{p}=\frac{\omega}{p} \text{ to the right.}$$

The field M_1 will have no action on the fields H_1 and H_2, whose speeds are either different or contrary. Likewise, M_2 will have no

action on H_1. On the contrary, H_2 and M_2, both rotating to the right with the same speed $\frac{\omega}{p}$, will produce a regular torque.

A motor of this type can be started more or less well by the methods of starting already described for ordinary synchronous motors. If the motor is started by means of an induction-motor for two-phase currents, it is well to group the stator-coils so as to reduce the number of poles by one-half, and thus obtain the double speed which is necessary.

A disadvantage of this system is that it gives rise to upper harmonics in the induced E.M.F. of the motor. Moreover, the field M_1 of the armature will induce a harmonic of frequency $3\omega t$ in the field-circuit, while the fields will give, in the armature, an E.M.F. of the form

$$A \sin \omega t \sin 2\omega t = \frac{A}{2}(\cos \omega t - \cos 3\omega t),$$

i.e., also a harmonic of the same frequency, $3\omega t$.

Besides, in consequence of the high self-induction, the power-factor is necessarily very low.

Another type of single-phase synchronous motor with alternating field can also be obtained by employing, in the armature, currents commutated by means of a shell-commutator analogous to that used for the Ganz motors previously described. When once synchronism has been attained, the effect of this commutator is to invert the current in the armature so that the torque resulting from the action of the field on the armature always retains the same sign. This motor is about equivalent to the motor with pulsating field, since the two elements are simply inverted and the armature can be considered as a revolving field. The only difference results from the relative importance of the ampere-turns of the armature in the fields.

CHAPTER VII

BIPOLAR DIAGRAM OF THE SECOND KIND IN AMPERE-TURNS

By Prof. C. A. Adams, S.B., E.E.

INTRODUCTION

In order to make plain the transition from Fig. 27, which is fundamentally an E.M.F. diagram, to the corresponding M.M.F. Diagram, it will be necessary to develop what may be called the General Vector Diagram of the Synchronous Motor.

Referring to Fig. 72, R represents in magnitude and space phase [1] or direction, the resultant of the field M.M.F. F and the armature M.M.F. A. The space phase of F is its direction reduced to a two-pole equivalent; i.e., the direction of the axis of the field poles in two-pole diagram. The space phase of A is, for a single-phase machine, the average direction (with respect to field structure) of the armature M.M.F., and coincides with the direction of this M.M.F. when the current is a maximum, right-handed or clockwise rotation of the armature being assumed. In a polyphase machine the resultant of the M.M.F.'s of the several armature phases is approximately constant in magnitude and direction for all parts of a revolution. The time phase of the armature current I may also be represented by the direction of A; and by choosing a proper scale of amperes, the vectors I and A may be made identical.

Assuming that the reluctance of the magnetic circuit through the armature and air-gap is the same in all radial directions, there will result a flux, Φ, in the same direction as R. The rotation of the armature through this flux will cause to be induced an E.M.F. E_2' in magnitude proportional to, and in time phase 90° behind, Φ. The

[1] A similar analysis for the alternator is given in greater detail in the *Harvard Engineering Journal* for January, 1903.

vector E_2' also represents (on the space phase diagram with right-handed [1] armature rotation) the direction of the axis of an armature coil when the induced E.M.F. is a maximum, just as the vector I represents both the time phase of the armature current and the direction of the axis of an armature coil when its current is a maximum, which is thus the direction or space phase of the armature M.M.F. A. The vector E_1' drawn equal to E_2' and in opposite phase, represents that part of the impressed E.M.F. E_1 necessary to balance the induced E.M.F. (E_2'). The impressed E.M.F. (E_1) must then be equal to E_1' plus the E.M.F. Ir, in phase with I, consumed by the armature

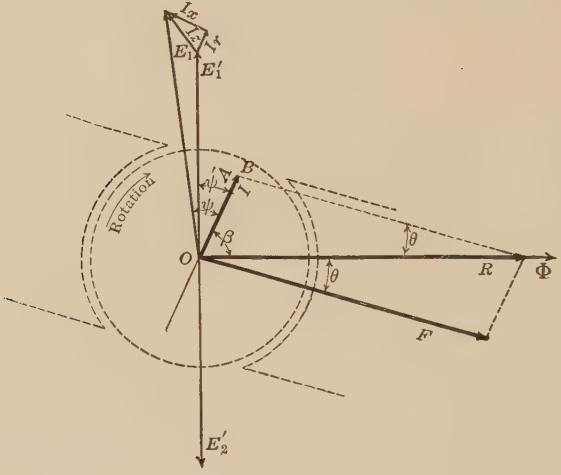


FIG. 72.

resistance, and the E.M.F. Ix, 90° ahead of I, consumed by the leakage reactance of the armature.

The diagram of Fig. 72 gives a fairly complete representation of the relations involved in the operation of the synchronous motor. Two assumptions are involved, however, which should be kept in mind in case of a quantitative analysis: first, it is assumed that the flux Φ is in the same direction as the resultant M.M.F. R; but in the

[1] It will be observed that the right-handed or clockwise armature rotation here assumed for the space phase interpretation of the diagram corresponds to the customary left-handed vector rotation in the time phase diagram. In the case of a revolving field type of machine, the field must revolve left-handedly, in order to give the same relative motion between armature and field.

salient pole [1] machine the reluctance of the magnetic circuit through the armature is least in the direction of F and very much larger at right angles thereto; the flux will therefore tend to lie inside of R, i.e., on the side towards F: second, F is taken as the total M.M.F. of the field coils per complete magnetic circuit; but a part of F is consumed by the reluctance of the field cores and yoke, and is thus not available to compound with A in the armature space. The errors due to these two assumptions partly neutralize each other as far as the direction of Φ is concerned, since the second throws it too far out and the first tends to bring it back.

But even this diagram with its approximations is too complex to be of service in getting a clear bird's-eye view of synchronous motor operation, although it is an excellent starting point for further approximations, and will prove very useful for reference purposes.

The two familiar approximations are obtained as follows:

Diagram Transformations

In any dynamo-electric machine there are in general two M.M.F.'s, those of the field and armature. In computing the E.M.F. of such a machine, it is possible either to consider the resultant of these two M.M.F.'s and the corresponding flux, or to consider the fluxes which would be produced by the two M.M.F.'s if acting separately, and to find the resultant of these two fluxes. The results of the two methods of procedure are identical only when the reluctances of the magnetic circuits in which the several M.M.F.'s act are equal. This condition, though never present except in a very roughly approximate degree, is nevertheless a very convenient one to assume for purposes of approximate analysis. With this assumption, the various fluxes in any case will be proportional to their several M.M.F.'s and in the same direction or space phase; moreover, at constant speed of rotation the E.M.F.'s generated in the armature by cutting through these fluxes will be proportional thereto and in space and time quadrature therewith.

It will be legitimate, therefore, under the above-mentioned assumption, to substitute for any M.M.F. its corresponding flux or E.M.F., or to substitute for any E.M.F. its corresponding flux or

[1] The most accurate diagram for the salient pole machine is Professor Blondel's Two-reaction Diagram employed in Chapter III; but this does not lend itself readily to a simple visualization of the problem in hand, or to simple calculations.

M.M.F., since these are proportional to each other in magnitude and bear to each other a fixed phase relation.

E.g., in Fig. 72 there are shown vectors which represent three M.M.F.'s, A, F, and their resultant R, each one of which would, under the above assumption, produce a flux of the same phase and proportional in magnitude. These fluxes $\Phi_A \Phi_F$ and their resultant Φ are shown in Fig. 73, as well as the corresponding E.M.F.'s E_A', E_F', and E_2', which would be induced by the cutting of the armature conductors through these fluxes, and which are 90° behind their respective fluxes. There are thus three M.M.F.'s, three fluxes, and three E.M.F.'s, each set being proportional to the other two, and

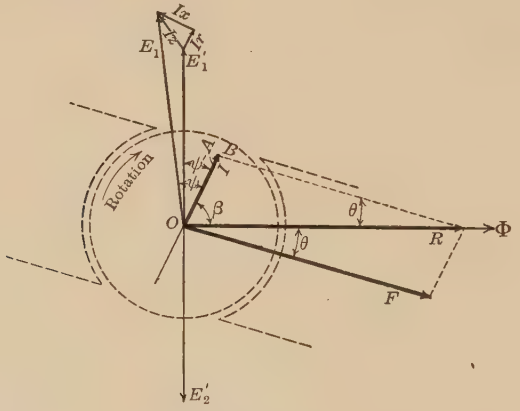


FIG. 72.

the three triangles representing the relations between the elements of the three sets being similar.

E.M.F. Diagram

In Fig. 73 E_F, E_A, and E_1' are the parts of the impressed E.M.F. consumed by the induced E.M.F.'s E_F', E_A', and E_2' respectively, just as Ir and Ix are the parts of the induced E.M.F. consumed by resistance and leakage reactance. Moreover, since E_A is proportional to and in quadrature with A (and therefore I), it is in phase with Ix and may be written,

$$E_A = Ix_A,$$

where x_A (the constant of proportionality) is an equivalent reactance representing the armature M.M.F. The sum $x + x_A = X$ is called

the synchronous reactance, and the corresponding impedance $Z=\sqrt{r^2+X^2}$ is called the synchronous impedance. The use of these quantities reduces the synchronous motor or alternator to a very simple basis, although at the expense of accuracy.

It is then possible to subdivide the impressed E.M.F. E_1 into three components; E_F, necessary to balance the counter E.M.F. E_F' which is due to the excitation F considered as acting alone; Ir consumed by resistance; and IX consumed by the synchronous reactance X: or, the synchronous impedance E.M.F. IZ may be looked

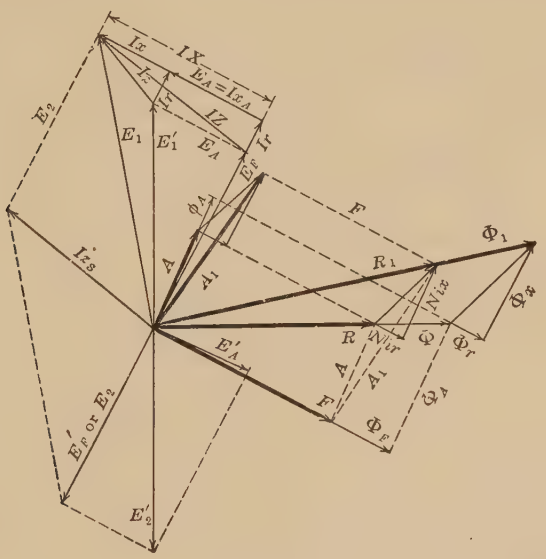


FIG. 73.

upon as the resultant of the impressed E.M.F. E_1 and the counter E.M.F. E_F'. This is shown more clearly in the simplified E.M.F diagram of Fig. 74, where E_2 corresponds to the E_F' of Fig. 73, and $E=IZ$. The current I will then lag behind the resultant E.M.F. E by an angle whose tangent is $\frac{X}{r}$.

The transformation from Fig. 72 to Fig. 73 consists in substituting for the armature M.M.F. A, an E.M.F. E_A', which would be induced in the armature by cutting the flux Φ_A, which in turn would be produced by the M.M.F. A if acting in a magnetic circuit of the same reluctance as that in which R produces Φ.

The triangle E_1E_2E of Fig. 74 is the E.M.F. equivalent of the M.M.F. triangle R_1FA_1 of Fig. 73; or, if r and x be neglected, it would reduce to the E.M.F. equivalent of the triangle RFA of Fig. 73.

This is the basis of the diagrams of Chapter II, which are essentially E.M.F. diagrams.

The M.M.F. or ampere-turn diagram is similarly obtained from the general diagram by substituting for the E.M.F.'s Ir and Ix their equivalent M.M.F.'s Ni_r and Ni_x; i.e., Ni_r and Ni_x are the M.M.F.'s which would produce the fluxes Φ_r and Φ_x, the cutting of which by the armature conductors would induce E.M.F.'s equal and opposite to Ir and Ix (see Fig. 73), where R_1 and Φ_1 are the M.M.F. and flux corresponding to the impressed E.M.F. E_1, and A_1 the M.M.F.

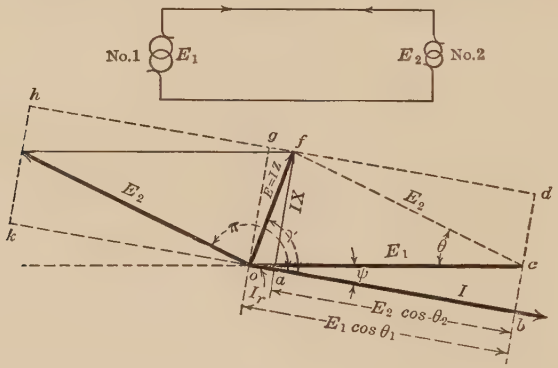


FIG. 74.

corresponding to the E.M.F. IZ. Thus the M.M.F. rectangle A_1R_1F is similar and equivalent to the E.M.F. rectangle IZE_1E_F, and the M.M.F. triangle A_1FR_1 of Fig. 75 is similar and equivalent to the E.M.F. triangle EE_2E_1 of Fig. 74.

It is thus evident that with constant impressed E.M.F., constant load and variable excitation, the point B, Figs. 75 and 76, will lie on the arc of the circle as does A_2 in Fig. 27; but it will be more interesting to go back to fundamental principles and to make this proof independent of that for Fig. 27.

In any direct-current dynamo-electric machine the electromagnetic torque is proportional to the product of the ampere-turns on the armature and the flux upon which the armature current reacts. In an alternating current machine the space phase of the armature

current also enters. E.g., in Fig. 75 the armature current in every coil reaches its maximum value when the axis of the coil has the direction of the vector A or I; i.e., when the plane of the coil is in the line $\overline{Ob}$, which differs from the direction of the flux Φ by the angle ψ'. Thus the torque-producing effect of the current is reduced from its maximum possible value, in the ratio $\cos \psi'$, and the torque is

$$T = k_T A \Phi \cos \psi',$$

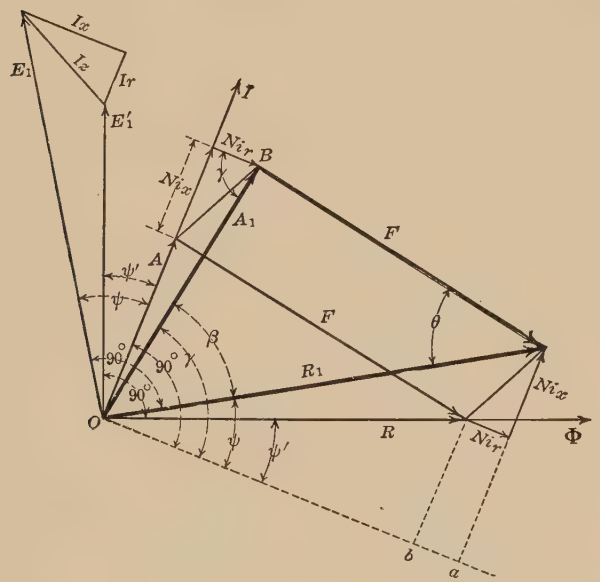


FIG. 75.

where k_T is the constant of proportionality. But

$$\Phi = \frac{4\pi}{10} \frac{R}{\mathcal{R}},$$

where $\mathcal{R}$ is the reluctance of the magnetic circuit and may for the present be assumed constant.

Then the output (which is proportional to the torque at constant speed) may be written

$$P_2 = k_2 A R \cos \psi', \quad \ldots \ldots \quad (1)$$

where k_2 is a constant. But,

$$R \cos \psi' = R_1 \cos \psi - A_1 \cos \gamma,$$

where ψ is the phase difference between E_1 and I, and

$$\cos \gamma = Ni_r \div A_1 = r \div Z$$

also A_1 is proportional to A (on the assumption of constant reluctance in all directions through the armature, see " Diagram Transformations," page 149, etc., therefore

$$P_2 = K_2(A_1 R_1 \cos \psi - A_1^2 \cos \gamma), \quad . \quad . \quad . \quad . \quad (2)$$

where K_2 is the new constant of proportionality.

Referring now to Fig. 76, the isosceles triangle OAC is constructed on R_1 as base, with $\gamma \left(= \tan^{-1} \frac{A + Ni_x}{Ni_r} = \tan^{-1} \frac{X}{r} \right)$ as the base angle. The notation is the same as in the preceding figures.

Locate the point B by the rectangular coordinates u and v measured along the axes $\overline{OC}$ and $\overline{OD}$ (perpendicular to $\overline{OC}$). Then $u = A_1 \cos \psi$, $A_1^2 = u^2 + v^2$ and

$$\frac{P_2}{K_2} = R_1 u - (u^2 + v^2) \cos \gamma, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$v^2 + \left(u - \frac{R_1}{2 \cos \gamma} \right)^2 = \left(\frac{1}{\cos \gamma} \right)^2 \left[\left(\frac{R_1}{2} \right)^2 - \frac{P_2 \cos \gamma}{K_2} \right]. \quad . \quad (4)$$

This equation has exactly the same form as equation (See Ch. II, near Eq. (19)) and its interpretation is likewise the same. The radius of the constant-power circles is

$$\text{Radius} = \frac{1}{\cos \gamma} \sqrt{\left(\frac{R_1}{2} \right)^2 - \frac{P_2 \cos \gamma}{K_2}}. \quad . \quad . \quad . \quad . \quad (5)$$

The general interpretation of Fig. 76 is identical to that of Fig. 27, but the method of quantitative application is not quite so obvious. The simplest method is as follows:

Since A_1, F, and R_1 are measured in ampere-turns, they may in any given case be expressed in terms of the equivalent field amperes, R_1 being the field current taken from the saturation curve at the point corresponding to the impressed E.M.F. E_1, F being the actual operating field current, and A_1 the field current taken from the

short-circuit curve at the point corresponding to the actual armature current I. The reason for this last step is obvious when it is recalled that when the machine in question is operated as generator on short circuit, E_1 and R_1 are zero and $A_1 = F$. This is in fact the definition of A_1.

Equation (5) gives the radii of the constant power circles in ampere-turns; but it is somewhat simpler to compute these radii from the formula given for the E.M.F. diagram, Eq. (19), Chapter II. This formula may be written

$$\text{Radius (volts)} = Z\sqrt{\left(\frac{E_1}{2r}\right)^2 - \frac{P_2}{r}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

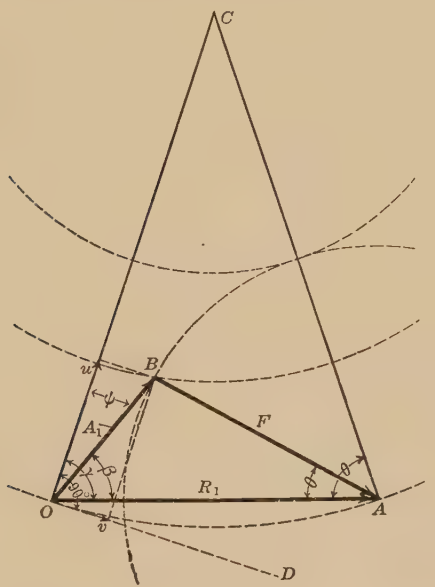


FIG. 76.

or in armature amperes,

$$\text{Radius (armature amperes)} = \sqrt{\left(\frac{E_1}{2r}\right)^2 - \frac{P_2}{r}}. \quad . \quad . \quad . \quad (7)$$

Designate by k_{af} the ratio of short-circuit armature amperes to the corresponding field amperes as taken from the short-circuit curve. Then the radius in field amperes is

$$\text{Radius (field amperes)} = \frac{1}{k_{af}}\sqrt{\left(\frac{E_1}{2r}\right)^2 - \frac{P_2}{r}}. \quad . \quad . \quad . \quad (8)$$

When $P_2=O$ the circle passes through O and A, and the corresponding radius is,

$$\text{Radius (field amperes, } P_2=O)=\frac{E_1}{2k_{af}r}=\overline{CO}.$$

This together with R_1 determines the isosceles triangle of Fig. 76, although the base angles θ are the same as for the E.M.F. diagram (Fig. 27) and can be computed from r and Z, the latter being determined in the usual manner from the saturation and short-circuit curves. The two methods of constructing this diagram give the same results when the value of Z employed is that corresponding to the point E_1 on the saturation curve.

Thus the ampere-turn diagram is completely and easily determined by the same information required by the E.M.F. diagram; namely, the armature resistance, the saturation and short-circuit curves, and the impressed E.M.F.

In the case of the polyphase machine, P_2, E_1, and r should designate the per-phase values.

The ampere-turn or M.M.F. diagram has several advantages over the E.M.F. diagram.

First, the substitution of an equivalent M.M.F. for the leakage impedance E.M.F., involves less error than the substitution of an equivalent E.M.F. for the armature M.M.F., because in the common type of synchronous motor the armature M.M.F. is large as compared with the leakage impedance E.M.F.

Second, in determining the *excitation* or *phase characteristics* (Fig. 79) by means of the M.M.F. diagram, F is given directly in field amperes, the quantity actually measured, and A_1 is given in equivalent field amperes, for which the corresponding armature current can be taken directly from the short-circuit curve, which is usually a straight line; whereas when taken from the E.M.F. diagram the results are in terms of armature current and of E_2, the latter being a hypothetical E.M.F., which would be induced if there were no armature reaction, and assumed constant for constant excitation.

Third, the M.M.F. method gives a clearer picture of the actual phenomena. E.g., if all the constants refer to two similar machines (generator and motor) together with the connecting line, and if these machines are of the revolving field type, R_1 and F are respectively the field M.M.F.'s of generator and motor, and their relative

directions show clearly the relative angular positions of the two revolving fields, θ being the coupling angle as before.

Also the torque-producing mechanism (the flux represented by R_1, the armature ampere-turns represented by A_1, and their phase difference) is a little more obvious.

In both E.M.F. and M.M.F. diagrams the line $\overline{OB}$ is in length proportional to the armature current, and the angle ψ designates the phase difference between armature current and impressed E.M.F.

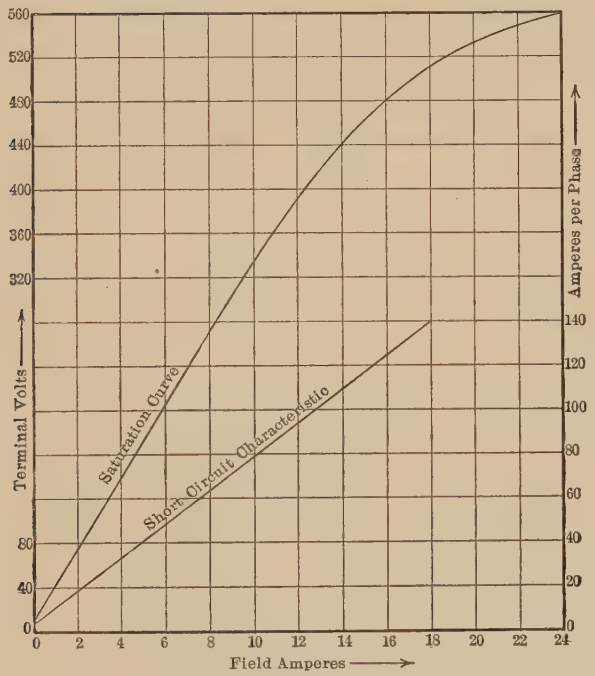


FIG. 77.

In the case of a normal machine, the armature resistance is small as compared with the synchronous reactance X, and the constant-power circles flatten out almost to straight lines (see Fig. 78).

Example. In Fig. 77 are shown the saturation and short-circuit curves of a three-phase 60 H.P. 440-volt synchronous motor.

The M.M.F. diagram is shown in Fig. 78 and the phase characteristics in Fig. 79. If the abscissæ of the phase characteristics, determined from the E.M.F. diagram, were to be transformed from E.M.F. to field current through the medium of the saturation curve,

the results would be found to differ widely from the curves of Fig. 79, the latter being much more accurate.

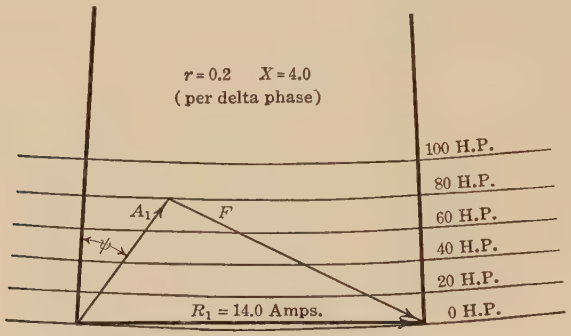


FIG. 78.

Approximate Diagram

If the resistance r be neglected in Fig. 76, $\overline{OC}$ and $\overline{AC}$ will become vertical and parallel, and the constant-power circles will degenerate into straight lines (see Fig. 81).

Following the interpretation of Figs. 27 and (76) we have, as the general interpretation of Fig. 81:—

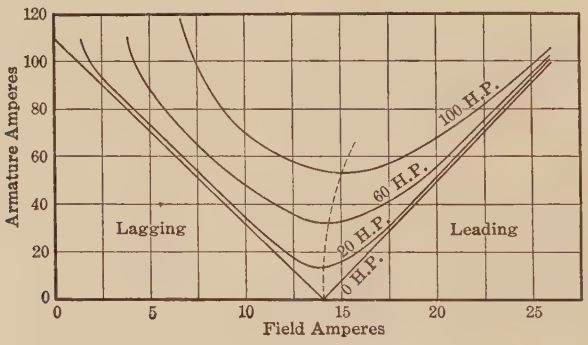


FIG. 79.

The region to the right of the vertical line $\overline{SS}$ corresponds to unstable operation, while the region to the left corresponds to stable operation.

The region above $\overline{OD}$ and between $\overline{LL}$ and $\overline{SS}$ corresponds to

lagging motor current, while that above the line $\overline{OD}$ and to the left of $\overline{LL}$ corresponds to leading motor current.

The region below $\overline{OD}$ and between $\overline{LL}$ and $\overline{SS}$ corresponds to leading generator current, while that below $\overline{OD}$ and to the left of $\overline{LL}$ corresponds to lagging generator current.

Extreme Cases

The assumptions (involved in the above-described diagrams) that the flux Φ has the same direction as R, and that constant Φ means constant R, irrespective of the direction or space phase of Φ, become

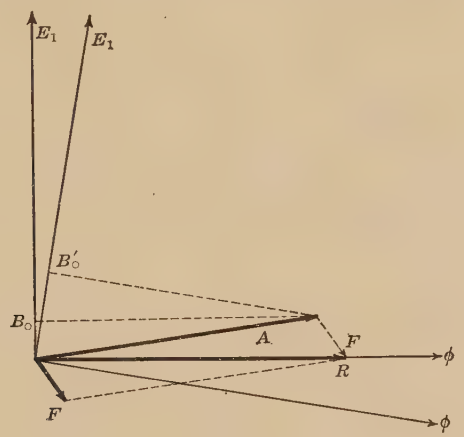


FIG. 80.

less and less warrantable as the conditions of operation depart farther and farther from the normal.

E.g., consider the case of light load and very low excitation (Fig. 80). R makes such a large angle with F (the axis of the field magnets) that Φ will not only be less in proportion to R, owing to the greater reluctance of the magnetic circuit in this direction, but it will also be pulled down more nearly into the direction of F, because that is the direction of minimum reluctance. Then E_1 $(=E_1')$ will also be shifted by an equal angle, since it must be in quadrature with Φ. This is shown by the light lines of Fig. 80. Thus R will lag behind E_1 by less than 90°, and if F be reduced to zero, A $(=R)$ will still have a torque component and the motor will operate under very light load without any excitation.

The physical explanation of this is that the armature current reacts upon a flux produced wholly by said armature current. Ordinarily this self-produced flux would be perpendicular to the plane of the coil, and would therefore have no component in the plane of the coil upon which the current could react to produce a torque; but in this case the unsymmetrical reluctance of the magnetic circuit results in a flux not in the same direction as the M.M.F., and the motor will actually run without excitation at very light load. The armature current is, of course, excessive.

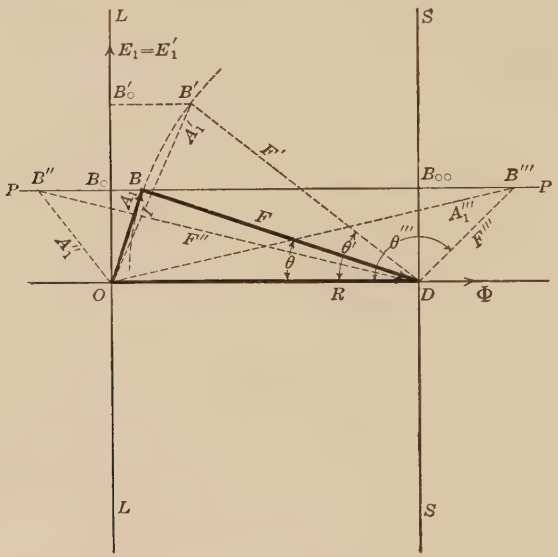


Fig. 81.

Mechanical Analogue

The approximate diagram of Fig. 81 lends itself readily to the explanation of a very interesting mechanical analogue to the synchronous motor.

Consider two shafts, a driver and a driven, placed end to end in the same line and connected together by an elastic coupling consisting of an elastic string or band, the two ends of which are connected to two crank pins on the two adjacent ends of the shafts in question. Assume further that the tension of the elastic band is directly proportional to the distance between the two crank pins which lie in the same plane perpendicular to the shafts. In Fig. 82 is shown a

diagrammatic end view of this coupling: the point D is the center of the shaft; the line $\overline{DO}$ is the radius of the driving crank pin and $\overline{DB}$ that of the driven crank pin; $\overline{OB}$ is thus the distance between the two crank pins, to which the tension of the elastic band is proportional; the lever arm of the tension about the center of the shafts is $\overline{DC}=\overline{DB}\cos\phi$, and the corresponding turning moment transmitted from driver to driven is thus proportional to $\overline{DC}\times\overline{OB}$. But $\overline{DC}=\overline{DO}\cos\psi$, and the torque or turning moment is $T=\overline{OB}\times\overline{DO}\cos\psi$; or since $\overline{OB}\cos\psi=\overline{OB_0}$, $T=\overline{OB_0}\times\overline{OD}$; i.e., the torque is proportional to the area of the rectangle $\overline{DOB_0}$. OB_0 is the tangential

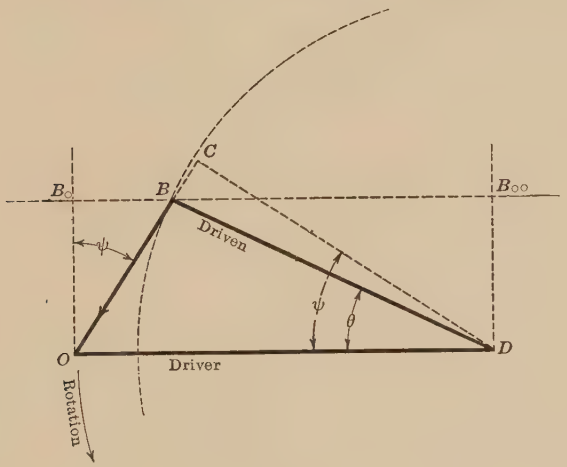


FIG. 82.

component of the tension $\overline{OB}$, and $\overline{OD}$ is the corresponding lever arm.

Consider now the operation of this coupling for given crank pin radii and a given stiffness of the elastic band $\overline{OB}$. At no load or zero torque the crank $\overline{DB}$ will be pulled ahead into line with $\overline{DO}$, the tangential component of the tension $\overline{OB}$ will be zero, the angle θ will be zero, B_0 will coincide with O, and the rectangle DOB_0 will collapse into a line according to the above-indicated relation between the torque and the area of this rectangle. If now the load torque be increased, the point B will fall back to the right, and the angle θ will increase until $\overline{OB_0}$, the tangential crank effort, has attained the the value imposed by the load. A further increase in the load torque will be accompanied by a further increase in θ and in $\overline{OB_0}$; but it

will be observed that there is a limit to the possible increase of $\overline{OB_0}$, namely, when θ is 90° and $\overline{OB_0}=\overline{DB}$, beyond which the tangential crank effort $\overline{OB_0}$ decreases, to zero when $\theta=180°$, to a negative maximum when $\theta=270°$, and back to zero again when $\theta=360°$ or 0°. Thus although the tension $\overline{OB}$ goes on increasing after θ has passed the 90° point, the lever arm decreases more rapidly than the tension increases, i.e., the tangential component ($\overline{OB_0}$) of the tension decreases.

Assume now that with constant load-torque the *driven* crank pin radius is altered. The tangential crank effort, $\overline{OB_0}$, must remain the same, and the driven crank pin B must lie somewhere on the line $\overline{B_0B_{00}}$. Thus for a given driver crank radius, the angle θ and the tension $\overline{OB}$, depend upon the torque demanded by the load (which is proportional to $\overline{OB_0}$), and upon the *driven* crank pin radius $\overline{DB}$. For each value of the load torque and of $\overline{OB_0}$, the tension will be a minimum for that value of the *driven* crank pin radius $\overline{DB}$, which causes B to fall at B_0. It is obvious that this particular value of $\overline{DB}$ varies with the load.

If now the *driver* crank pin radius $\overline{DO}$ be increased, the value of $\overline{OB_0}$ for a given load will be decreased and *vice versa*, since their product must be constant.

It has been assumed thus far that a fixed relation exists between the crank pin displacement $\overline{OB}$ and the coupling tension; but while this is true for a given case, it is evident that a different elastic band or spring could be substituted which would have a different elastic coefficient, and with which the angle θ would be quite different for the same load.

The angle θ will be hereafter referred to as the *coupling angle*. It obviously depends upon the load torque, the two crank pin radii, and the elastic coefficient of the coupling.

Let p = tension in pounds $=k\times\overline{OB}$, where k is the coefficient of the elastic band. Then if $\overline{DO}$ and $\overline{DB}$ are measured in feet, the torque in pound-feet will be

$$T=k\times\overline{OB_0}\times\overline{DO}=k\times\overline{OB}\cos\psi\times\overline{DO}=k\times\overline{DB}\sin\theta\times\overline{DO}.$$

Thus for any given value of the crank pin radii, the torque varies as the sin θ.

A comparison of the diagrams of Figs. 81 and 82 will show their exact mechanical equivalence. In both cases the torque is proportional to $\overline{OB_0}$ for a given value of $\overline{OD}$; therefore the relation of the torque to θ and to $\overline{DB}$ is exactly the same in the two cases. In

Fig. 81 $\overline{OB}$ is a measure of the armature M.M.F., i.e., the strength of the armature considered as an electromagnet lying across the magnetic field whose flux is Φ and whose M.M.F. is R (see Fig. 83). The resulting torque is then proportional to $A_1\Phi \cos \psi$ or approximately proportional to $A_1 R \cos \psi$; but in this approximate analysis, Φ and therefore R is determined by the impressed E.M.F., and is in any given case a rough measure of the exciting M.M.F of the generator which supplies the power, while F is the exciting M.M.F. of the synchronous motor.

Thus the two crank pin radii correspond roughly to the exciting M.M.F.'s of the generator (driver) and the motor (driven), the tension $\overline{OB}$ corresponds to the armature M.M.F., the angle ψ to the phase difference between current and E.M.F. or to the angle by which the armature M.M.F. differs from its position of maximum

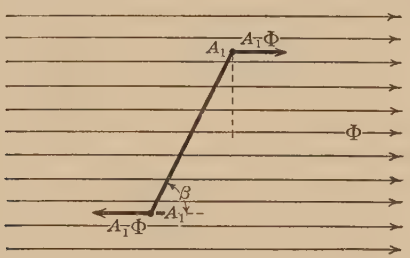


Fig. 83.

torque-producing effect, and the coupling angle θ roughly to the mechanical phase difference (measured in electrical degrees) between the revolving parts of the generator and motor.

The electrical angular velocity of the motor is the same as that of the generator, but their mechanical angular velocities are inversely as their numbers of poles; while in the mechanical analogue the angular velocities of the two parts of the coupling must be the same.

In order to see what determines the stiffness of the electromagnetic coupling between alternator and synchronous motor, it is necessary to remember that the torque is strictly proportional to the product of the flux Φ and that component of the armature M.M.F. in quadrature with Φ, and that for a given value of Φ the necessary M.M.F. R will depend upon the reluctance of the magnetic circuit; e.g., if a synchronous motor has its pole faces bored back so as to increase the reluctance of the magnetic circuit, a larger resultant

excitation R must be supplied in order to produce the same flux Φ made necessary by the same impressed E.M.F.; but with this same flux, the same load will require the same torque component $\overline{OB}_0$ of the armature M.M.F. A. Thus although R and F have increased, $\overline{OB}_0$ is unchanged, and the angle θ reduced, for the same ratio of F to R. This decrease of the angle θ for a given torque corresponds to a stiffening of the elastic band in the mechanical coupling, and increases the break-down torque (when $\theta = 90°$). In fact, the above-described change in the synchronous motor is commonly called a stiffening of the field, in that the latter is then less distorted by a given armature M.M.F.

It will be observed in this connection that every synchronous motor has a natural period of oscillation about its mean running position, which depends upon the moment of inertia of the revolving part and the stiffness of the electromagnetic coupling. These oscillations correspond to a variation of the coupling angle θ about its mean value, and a pulsation of power-flow. If the angular velocity of the supply alternator should oscillate about a mean value, the resulting oscillation transmitted to the synchronous motor through the electromagnetic coupling would depend upon the amplitude and frequency of the impressed oscillation, upon the moment of inertia of the revolving part of the motor, and upon the stiffness of the coupling. If the impressed oscillation should have the same frequency as that of the freely oscillating motor, the latter would tend to increase its amplitude of oscillation until it would swing past the point of maximum torque ($\theta = 90°$) and break down. If, on the other hand, the frequencies were widely different, the amplitude of the transmitted oscillation would be greater, the stiffer the coupling and the less the moment of inertia of the revolving part.

These phenomena are usually covered by the term "hunting" or "phase swinging," a quantitative analysis of which is given in Chapter IV.

Length of Air-Gap

It will be interesting here to review the relative merits of high and low synchronous impedance in a synchronous motor; i.e., of a soft and a stiff electromagnetic coupling, or what is equivalent, the relative merits of a short and a long air-gap.

Take first a system in which there is a considerable pulsation of

frequency due to the non-uniform crank effort of a single cylinder steam-engine which drives the supply alternator.

If the impedance (including generator, motor and line) be very low, i.e., if the air-gap be long and the coupling stiff, the motor speed will follow closely the frequency pulsation. There is thus a pulsation strain on the coupling accompanied by a large pulsation of power and current. If the motor is large enough as compared with the alternator, the stiff coupling will have the effect of adding fly-wheel capacity to the alternator and will tend to reduce the frequency pulsation, but at the expense of heavy current and power pulsations. If, on the other hand, the impedance be large, the frequency pulsations will be partly absorbed in the soft coupling, the motor will not follow closely the frequency pulsations, and there will be much less pulsation of power and of current. There is, however, an obvious limit to the desirable softness of coupling, namely, that beyond which there is danger of a breakdown of the motor *because* of the soft coupling.

There is also another objection to the soft coupling, which will appear from an inspection of the E.M.F. diagram (Fig. 27). If the reactance be high, the coupling angle will be relatively large for a given power, and there will be a large variation in the power factor of the motor from no load to full load under constant excitation, or a frequent adjustment of excitation will be demanded in order to maintain a constant power factor.

When polyphase synchronous motors or synchronous converters are started by the induction motor action of the damping coils, or by hysteresis and eddy current torque, it is desirable to have as short an air-gap as possible in order to keep the necessary starting current within limits.

The choice of air-gap is thus a compromise and depends somewhat upon the particular service demanded of the motor in question, and upon the nature of the system on which it operates. The gap should preferably be not as long as demanded by good regulation in the synchronous alternator, nor as short as demanded by high-power factor in the induction motor or induction generator. On this point there is considerable difference in practice, particularly in the case of synchronous converters.

CHAPTER VIII

GENERALIZATION OF DIAGRAM FOR COUPLED SYNCHRONOUS MACHINES

THE diagram represented in Fig. 27 (page 44), which is based upon Joubert's theory that the armature-reaction of the alternator employed can be adequately explained by supposing the presence in the armature of a simple mean synchronous reactance, can be retained to advantage for certain purposes even in the case of the theory of "two reactions," developed by the author in Part III, Chapter I, for alternators with saturated magnetic fields. It may, indeed, be noted that the impedance Z, which enters into the expression for the vector, $\overline{A_1A_2}$, is still a constant of the same character as before, though it now represents the impedance of the transverse reaction, which may be symbolized by Z_t, when we take

$$Z_t = \sqrt{r^2 + \omega^2 L_t^2}.$$

We will first suppose the case of two alternators which comply with Joubert's law (i.e., which have a constant synchronous reactance). The clearest form of electrical transmission diagram will be obtained by first combining the diagrams shown in Fig. 27 (page 44) and in Fig. 43 (page 86). All that is necessary is to point off along OA (Fig. 84) a distance proportional to the voltage (U) at the terminals of the two alternators, these being assumed to be near each other. (If they are far apart the impedance of each one is to be increased fictitiously by an amount equal to half the impedance of the line by which they are coupled together). The vectors of the e.m.f.'s of these alternators are then drawn in proper magnitude and phase. (The alternators may be unlike provided they both have the same characteristic ratio $\frac{\omega L_t}{r} = \tan \gamma$). We then draw the line of zero

phase-angle, AN, lagging by the angle, γ, with respect to U, and point off, at N, the center of the circles of constant internal power, which will be the same for the generator as for the motor. The three points A_1AA_2 are then joined by a straight line, A_1A_2, which represents twice the product Z_tI, of the impedance Z_t by the current, I, that is to say twice the impedance e.m.f. of the circuit. In reality the e.m.f. U may be considered as being that of a " line " absorbing the energy of the generator and supplying the

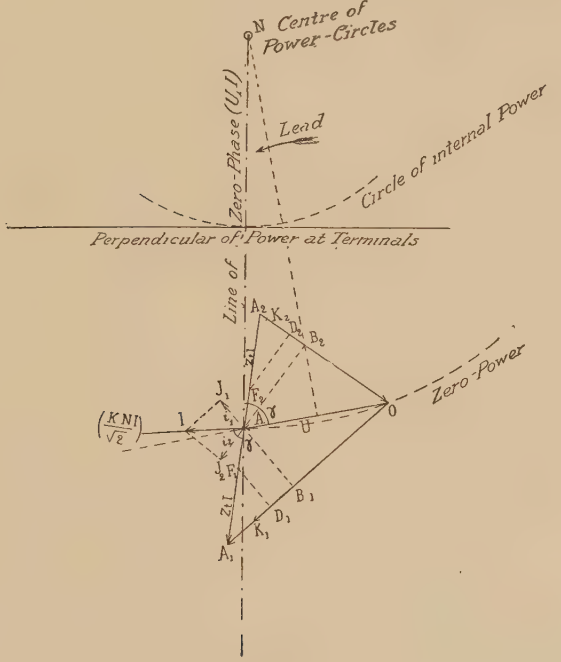


FIG. 84.

same energy to the motor. We thus introduce a " fictitious " line which does not change the phenomena in any way, but which has the advantage of enabling the diagram to be generalized completely. The current I passing between the two alternators is equal to $\frac{AA_1}{Z_t}$ and it lags behind AA_1 by the angle γ.

In the " two-reactions " theory, this diagram is modified in the manner now to be explained. The phases of the currents are

still deduced from those of the e.m.f.'s by means of the internal resistance r and an inductance; but that inductance is then the total transverse inductance $L_t = l_t + s$. The sides, A_2A and AA_1, of the triangles will therefore represent Z_tI, where Z_t symbolizes the impedance which corresponds to the transverse inductance, where I represents the current, and where $\tan \gamma = \frac{\omega L_t}{r}$. But OA_1 and OA_2 no longer represent the internal e.m.f.'s except in regard to their phases. To represent them in magnitude as well as in phase, the diagrams shown in Fig. 85 must be constructed by reference to the characteristic curves for these two alternators, which curves, for the sake of simplicity, are here supposed to be the same for both machines.

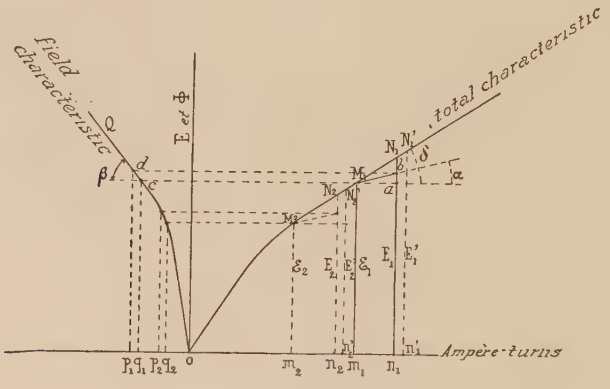


FIG. 85.

Let OM_1N_1 be the total excitation characteristic or saturation curve obtained by plotting the excitation ampere-turns as abscissas and the induced e.m.f.'s at no load as ordinates. Let $\frac{KNI}{\sqrt{2}}$ = the armature ampere-turns produced by an effective current I in the armature (with N peripheral conductors per pair of poles; and let us select, for Fig. 84, a scale such that the current-vector AI will also represent these armature ampere-turns according to the scale of abscissas in Fig. 85. We then draw AB_1 and AB_2, perpendicular respectively to OA_1 and OA_2 (in Fig. 84); and we point off AF_1 and AF_2 both equal to ωsI, and also draw F_1D_1 and F_2D_2, perpendicular respectively to OA_1 and OA_2. The vectors OD_1 and OD_2 represent the

internal e.m.f.'s generated in the two alternators by the *direct* flux, i.e., the flux along the field axis. Let $\mathcal{E}_1$ and $\mathcal{E}_2$ represent these two e.m.f.'s, and let them be drawn as ordinates on the total characteristic curve (Fig. 85). The corresponding abscissas om_1, and om_2, represent the excitation ampere-turns necessary to generate these e.m.f.'s at no load. We add to them, respectively, the counter ampere-turns of the armature, m_1n_1 and m_2n_2, obtained, respectively, by projecting (Fig. 84) the vector AI on the perpendiculars to OA_1 and OA_2, since these projections represent the reactive components of the armature ampere-turns. They are

$$\frac{KN_1J_1}{\sqrt{2}} \quad \text{and} \quad \frac{KN_2J_2}{\sqrt{2}},$$

in which N_1 and N_2 represent the numbers of peripheral armature-conductors per pair of poles in the two machines, and K is the utilization-coefficient for the particular winding. We thus obtain the e.m.f.'s E_1 and E_2; but these are not yet altogether exact, because we have neglected the small increases of excitation which are necessary to compensate for the increase in saturation of the field magnet cores resulting from the increase in magnetic leakage between the pole-pieces. The correction necessary is very easily made, if the permeance B_l, of the leakage path is known and if the excitation-characteristic of the magnetic field cores alone (OQ) has been drawn, as shown in Fig. 85, by plotting it reversed, to the left of the axis of ordinates, taking the excitation ampere-turns as abscissas and the magnetic flux through the magnets and yoke as ordinates. This characteristic curve should be drawn according to a scale of ordinates such that the magnetic fluxes may be represented by the electromotive forces which they would generate in the armature if they were to thread through it. Let β be the angle of the tangent to the curve OQ at the point c. Let us now draw at M_1 an angle $bM_1a = \alpha$, whose tangent represents the permeance B_l. of the magnetic leakage path. Then the vertical segment ab, intercepted on the line N_1n_1, represents the additional magnetic leakage flux. Let us draw two horizontal lines ac and bd, and take their intersection with the field-characteristic (at the left in Fig. 85), and let us then find the corresponding abscissas, p_1q_1. The segment $p_1q_1 = \frac{\tan \alpha}{\tan \beta} m_1n_1$ represents and measures the supplementary ampere-turns required. Let us

take $n_1 n_1' = p_1 q_1$; and $m_1 n_1'$ represents and measures the total ampere-turns made necessary by the armature-current. In like manner, $m_2 n_2'$ represents the corresponding ampere-turns required for the motor OA_2 at the output indicated.

To the two values on'_1, and on'_2, of the total ampere-turns, correspond electromotive forces E', and equal, respectively, to $N_1' n_1'$ and $N_2' n_2'$, on the total characteristic. These e.m.f.'s (which are those that would appear if the current I ceased suddenly to pass through the alternators), correspond, in the present case, to what we term *internal* electromotive forces in the case of Joubert's theory. By measuring off (Fig. 84) from O, along OA_2 and OA_1, distances OK_1 and OK_2, respectively equal to these e.m.f.'s, we obtain two vectors which replace, in the "two-reactions" theory, the vectors OA_1 and OA_2 in the diagram obtained with Joubert's theory.

It is seen that, in consequence of saturation, these vectors are, in general, shorter than OA_1 and OA_2; but this is not a necessary result, because it is possible to design an alternator in which the transverse reaction would be very small, by reducing the width of the pole-pieces, and which could, consequently, lead to a length for OA_1 less than OK_1, at least so long as the saturation of the field-magnets is low.

For the sake of simplicity L_t has been assumed constant at all loads, because the influence of the air-path is generally preponderating in the transverse magnetic circuit, contrary to what is true for the principal circuit; but those who may wish greater refinement can take into account the slight variations of L_t with the load-conditions of the alternator, and they can also replace the perpendicular straight lines AB_1 and AB_2 by curves comprised between these straight lines and segments of circles. In this way allowance can be made for the fact that, in certain cases, the transverse reaction may not raise the voltage at the terminals as much as when the pole-pieces are wide. This complication of the diagram does not materially increase its precision and it is therefore preferable to use the simpler theoretical diagram.

The circles of constant power of the old diagram are to be replaced here by lines, which are drawn by points. To obtain these lines, the values of E'_2 are drawn or determined for a large number of positions of A_2 and for the corresponding power-outputs, and each power-value is indicated near the point A_2 corresponding to it. When the entire diagram has been in this way covered by a sufficiently

numerous series of points, these may be considered as being " levels " on a topographical plan, and lines connecting together the points at the same " levels " can be drawn like the " contour " lines on topographical plans. Between any two such lines of constant power, other lines of constant power of intermediate value can be drawn by interpolation in the well-known way.

PART II

GENERAL DIAGRAMS DEDUCED FROM THE DIAGRAM FOR SYNCHRONOUS MOTORS

CHAPTER I

GENERAL DIAGRAMS DEDUCED FROM THE DIAGRAM FOR SYNCHRONOUS MOTORS

Introduction. The author presented, at the Electrical Congress in Paris, in 1900, a paper on *The Graphical Theory of Rotary Converter Regulation*, the object of which was to set forth a purely graphical, but nevertheless complete, method of treating this complicated though interesting question, and to show, at the same time, that the solution of the problems involved could be reduced to the solution previously worked out for synchronous motors, by the author, and could be reconciled with his "theory of two armature-reactions in alternators."

In accordance with that theory, the rotary converter can, in effect, be treated as a simple particular case of the synchronous motor, namely, *as a synchronous motor having no transverse reactions;* and the vectors obtained for synchronous motors are rendered applicable to rotary converters by merely suppressing the transverse reaction vector.

It is thus seen that this theory contains nothing artificial, but that, on the contrary, it is both general and homogeneous to a high degree.

Notation. The notation used may be substantially the same as for synchronous motors; but the meaning of some of the subscripts must be changed somewhat to take into account the fact that the armature of the rotary converter receives currents of two kinds, primary

and secondary. (The primary and secondary windings can be different.) The notation used will be the following:

T = periodic time (duration of one period), in seconds;

$\omega = \frac{2\pi}{T}$ = frequency, or speed of pulsation of the current;

R = total resistance of circuit (generator, line, and primary winding of converter); [1]

l = cumulative self-induction of generator and line;

s = self-induction of converter due to magnetic leakage of armature;

L = self-induction added in the form of reactance-coils;

$X = \omega(L+l+s)$ = total reactance of alternating current circuit;

$Z = \sqrt{R^2+X^2}$ = total impedance corresponding;

E = effective E.M.F. (being the E.M.F. supplied by the line or that induced in the generator, in the case of a simple transmission circuit). This E.M.F. is to be considered always constant unless otherwise stated;

q = number of phases of the primary current;

ε = effective primary (alternating) E.M.F. of the converter;

$\frac{\varepsilon}{\xi} = E_2$ = secondary or induced (D.C.) E.M.F.;

ξ = ratio of transformation of the converter;

I = effective value of one of the primary (alternating) currents;

I_w, or J, = active component of that current;

I_d = reactive component of that current;

ϕ = phase-angle of lag corresponding to $\left(\tan\phi = \frac{I_d}{I_w}\right)$;

θ = phase-angle between the E.M.F.'s ε and E;

I_2 = secondary (direct) current, proportional to J less the losses;

N = number of peripheral primary conductors per magnetic field;

K = reduction-coefficient for primary winding;

N_2 = number of peripheral secondary conductors per magnetic field;

A = ampere-turns of (shunt) field-coils;

n = number of exciting turns of series-winding;

r = resistance per turn of shunt-winding;

j_0 = active current when the load is zero;

i_0 = reactive current when the load is zero.

[1] The resistance of the line and generator are negligible in the case of a rotary converter supplied with current from a large distribution-system.

Generalities. Reduction of all Armature-Reactions to the Single Direct Reaction. The inducing field is the resultant of three magnetomotive forces (Fig. 1):

1. The inducing ampere-turns of the exciting current;
2. The ampere-turns which are reactive with respect to the E.M.F. and which act in the same way as the preceding;
3. The difference between the active ampere-turns and the secondary ampere-turns produced by the direct current. These two kinds of ampere-turns tend to produce fields which are diametrically opposed to each other and which make a phase-angle of $\frac{\pi}{2}$ with respect to the exciting field.

Practically, the theory is greatly simplified by the fact that the active ampere-turns and the direct-current secondary ampere-turns are substantially equal to each other at all loads.

On the one hand, the winding being a direct-current winding, the average number of active ampere-turns will be given by the formula,

$$K\frac{N}{2}I_w\sqrt{2}, \quad \ldots\ldots\ldots \quad (a)$$

in which N = number of *wires;*

I_w = active component of primary alternating current;

K = reduction-coefficient due to the overlapping of the windings, for an infinitely large number of slots.

The author has published some average values for this coefficient. It should, preferably, be determined from existing machines.

On the other hand, the average number of ampere-turns due to the direct current, I_2, produced by the converter, is equal to the mean value,

$$\frac{1}{2}\frac{N}{2}\left(\frac{I_2}{2}\right). \quad \ldots\ldots\ldots \quad (b)$$

Dividing (b) by (a) we have the ratio of the secondary and primary ampere-turns:

$$\zeta = \frac{1}{4K\sqrt{2}}\frac{I_2}{I_w}. \quad \ldots\ldots\ldots \quad (c)$$

Whatever may be the number of phases, q, if the efficiency be assumed equal to unity, which makes the primary and secondary

watts equal, the induced E.M.F., as a function of the reduction-coefficient k, due to overlapping, may be expressed thus:

$$I_w \frac{2\pi k}{2\sqrt{2}} \frac{N\Phi}{T} = I_2 \frac{N\Phi}{T}, \quad \ldots \ldots \ldots \quad (d)$$

in which T = the periodic time,
and Φ = the magnetic flux.

From this, we have

$$\frac{I_2}{I_w} = \frac{\pi k}{\sqrt{2}}. \quad \ldots \ldots \ldots \quad (e)$$

Substituting in (c) we have

$$\zeta = \frac{1}{4K\sqrt{2}} \frac{\pi k}{\sqrt{2}} = \frac{\pi k}{8K}. \quad \ldots \ldots \ldots \quad (f)$$

I have shown elsewhere (see *Properties of Revolving Magnetic Fields*, Eclairage Electrique, 1895) that the ratio $\frac{k}{K}$ is substantially equal to $\left(\frac{\pi}{2}\right)^2$. We therefore have

$$\zeta = \frac{\pi}{8}\left(\frac{\pi}{2}\right)^2 = \frac{\pi^3}{32} = \frac{30.935}{32} = 0.969.$$

This close approach of the value of ζ to unity shows that the transverse reaction is practically negligible in comparison with the direct reaction. Hence rotary converters may be treated, practically, as if there were no transverse reaction in them.

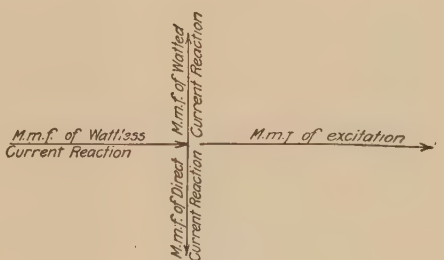


FIG. 1.

If the internal losses are taken into account, the secondary (output) watts will be from 10 to 15 per cent less than the primary (input) watts; and the ratio ζ will be still less near unity. Nevertheless it is found that:

1. The direct-current E.M.F. undergoes no modification in consequence of the transverse reaction, because the brushes always remain

absolutely at the neutral point, and, consequently, only the direct flux is cut by the armature conductors;

2. The alternating current E.M.F. is affected only to a slight extent, because the field resulting from the active current I_w, and from a direct current I_2, which is 10 to 15 per cent weaker, constitutes only 10 to 15 per cent of the transverse reaction field of the direct-current armature, which field, as a rule, would not exceed half the direct field. But, even assuming that this resultant transverse field may amount to 20 per cent of the direct field, the alternating E.M.F. would be thereby increased only in the ratio,

$$\frac{\sqrt{1+(0.20)^2}}{1}=1.02,$$

or, only about two per cent, which is negligible. We are therefore warranted in treating the rotary converter as a synchronous motor having no transverse reaction, as has been done by other authors. There might possibly be an exception in the case of rotary converters having no field-excitation; but these are used very little, and their theory, in any case, can be established much more easily by considering them as transformers.

As in the case of every alternator, the armature, in a rotary converter, is subject to magnetic leakage, that is to say a portion of the magnetic flux produced by the alternating currents in the armature winding is shunted by the air or by the armature-teeth. This leakage is offset in part by the leakage which the secondary current tends to produce, in the contrary direction (and which offsets, especially, the effect of the active current); hence, the resultant leakage is much smaller than in a synchronous motor. Nevertheless, in order to make the theory general, we will suppose that this magnetic leakage exists; and it will be represented, as usual, by an inductance, s, giving rise to a reactance, ωs.

Factors Determining the Practical Conditions of Operations. It is very important to note that the operation of rotary converters does not depend on the characteristics of the machine only, but also depends on the constants of the electric supply-circuit. It is, in fact, the self-induction of the supply-circuit and the difference between its E.M.F. and the E.M.F. induced in the converter, which govern the ratio of reactive to active currents.

Every active current lagging behind the E.M.F. produces a demagnetizing effect, whereas every reactive current leading the

E.M.F. produces a magnetizing effect. The variations in the conditions of the external circuit, therefore, constitute an indirect process for making the field-excitation of the rotary vary, though this variation is often contrary to what is wanted.

I. CONDITIONS OF ELECTRIC-CURRENT SUPPLY TO ROTARY CONVERTERS

From what precedes, it is important to know, quite precisely and quantitatively, the manner in which the reactive current varies with the conditions of the current-supply to the converters, especially with

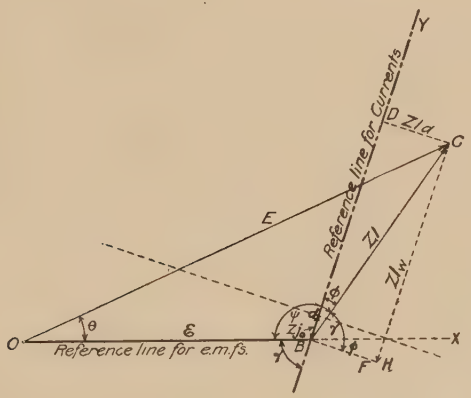


Fig. 2.

the constants of the circuit and with the impressed E.M.F. This question will now be discussed.

Fundamental Diagram. We can start from the usual diagrams for alternators, made with two scales, one for currents, and the other for E.M.F.'s.

Assuming E_2 = the secondary (D.C.) voltage at the terminals, let $\varepsilon = OB$ (in Fig. 2) represent the primary voltage, which is *substantially* proportional to E_2, owing to the low internal resistance of the armature.

Let $E = OC$ = the E.M.F. of the alternator supplying current to the converter (or the voltage of the distribution-system, according to the case). Unless otherwise stated OC will be assumed constant at all loads.

Let R = resistance of primary *circuit* of converter;

$X = \omega(L + l + s)$ = reactance of same;

$Z = \sqrt{R^2 + X^2}$ = impedance of same;

$\tan \gamma = \frac{X}{R}$ = reactance-coefficient;

$\omega = \frac{2\pi}{T}$ = frequency.

The "primary" circuit in this case *includes the armature.* The reactance constant X must therefore include the following: The reactance (ωL) of the reactance-coils connected in series with the converter; also the reactance (ωl) of the alternator and of the line if the current is supplied directly by an alternator, and not by a distribution-system; and, finally, the reactance (ωs) corresponding to the effect of the *magnetic leakage* at the armature itself.

For each value, θ, of the phase-angle between the E.M.F.'s E and ε, the third side, BC, of the triangle COB, represents the resultant E.M.F. of E and ε, which is equal to the product of the impedance Z by the primary alternating current I corresponding to and passing through this impedance. Constructing the triangle CBH, on BC as the hypothenuse, with the angle γ at B we will have

$$BH = XI \quad \text{and} \quad HC = RI.$$

In the diagrams ϕ will always represent the angle of lag of the current behind the voltage at the terminals.

In order that the vector BC may measure the current I itself, both in magnitude and in phase, it is only necessary to take, for the amperes, a scale which is Z times greater than for the volts, and to take, as the *origin of phase-angles* (ϕ), a reference line BDY, making, with the line OX, the angle γ defined hereinabove. The angle DBC then measures a phase-angle (ϕ), representing a "*lag*" if it be to the *right* of BD, and a "*lead*" if it be to the *left.* We therefore count the reactive current, I_d, as *positive* on the right-hand side of D and as *negative* on the left-hand side.

If, from C (Fig. 3) a perpendicular CD be drawn to the line of reference BY, the segments CD and BD will measure the reactive and active currents for each position of C. The active current I_w can, itself, be considered as equal to the sum of two parts. One of these parts (i_0), represented by the segment Bd, is practically constant and

corresponds to the approximately constant loss produced by the "no-load" active current, in consequence of friction, resistance, hysteresis, and eddy currents. The part i_0 will be called the "active current for zero-load." The other part, represented by Dd, corresponds to the energy of the useful secondary current, and which is proportional to the said current.

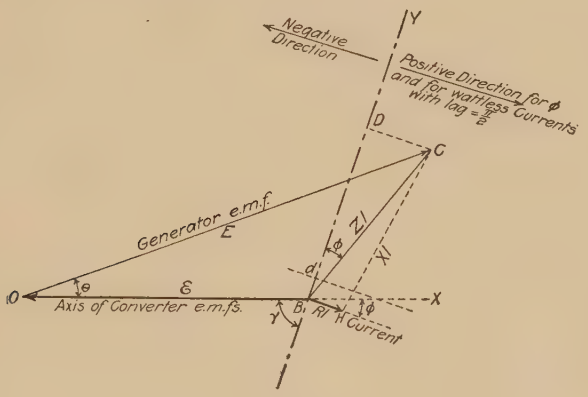


FIG. 3.

Fundamental Equation. The fundamental equation connecting ε, E, and the active and reactive currents can be written directly from Fig. 2. The triangle OBC gives, in fact,

$$\left.\begin{aligned} E^2 &= \varepsilon^2 + (ZI)^2 - 2\varepsilon(ZI)\cos\psi \\ &= \varepsilon^2 + (ZI)^2 + 2\varepsilon(-ZI\cos\psi) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (a)$$

In this case we have

$$\begin{aligned} -(ZI)\cos\psi &= \text{projection of } BC \text{ on } OX \\ &= (\text{proj. of } BD + \text{proj. of } DC) \text{ on } OX \\ &= (\text{proj. of } CH + \text{proj. of } BH) \text{ on } OX \\ &= ZI_w\cos\gamma + ZI_d\cos\phi \\ &= ZI_w\cos\gamma + ZI_d\sin\gamma. \end{aligned}$$

Substituting this value, and remembering that $I^2 = I_w^2 + I_d^2$, we have, from (a),

$$E^2 = \varepsilon^2 + Z^2(I_w^2 + I_d^2) + 2\varepsilon Z(I_w\cos\gamma + I_d\sin\gamma). \quad . \quad . \quad (1)$$

We also find (from the triangle CBH, in Fig. 3),

$$Z\cos\gamma = R = \text{resistance},$$
$$Z\sin\gamma = X = \text{reactance}.$$

Substituting these values in the last term, in (1), we have

$$E^2 = \varepsilon^2 + Z^2(I_d^2 + I_w^2) + 2\varepsilon(RI_w + XI_d). \quad . \quad . \quad . \quad . \quad (2)$$

This equation enables the reactive current, I_d, to be calculated, as a function of the load, i.e., of the active current I_w; because, on solving with respect to ZI_d, we have

$$Z^2I_d^2 + 2\varepsilon\sin\gamma ZI_d + \varepsilon^2 - E^2 + Z^2I_w^2 + 2\varepsilon ZI_w\cos\gamma = 0.$$

Solving this quadratic equation for I_d, we have

$$I_d = \frac{1}{Z}\left[-\varepsilon\sin\gamma + \sqrt{\varepsilon^2\sin^2\gamma + E^2 - Z^2I_w^2 - 2\varepsilon ZI\cos\gamma}\right]. \quad . \quad (3)$$

This expression will be put into another form later.

Application of the Diagram. Representation of Converter Operation with Constant Potential at Primary Terminals and at Brushes. If all that is desired is to maintain the potential constant at the terminals of the converter, the segment $OB = \varepsilon$ remains constant, and, in order to predetermine the different conditions of operation it is sufficient to let the point C, which defines the conditions of operation, describe a circle around O as a center, with the constant radius $OC = E$. For each position of C the load is measured by dD and the reactive current by CD. It is thus seen, in Fig. 4, that the reactive current, which is positive for light loads (since the characteristic point C is to the right of the line BY), diminishes as the load increases from the maximum value F (secondary load alone) to zero, at G; and it then changes in sign, and begins again to increase.

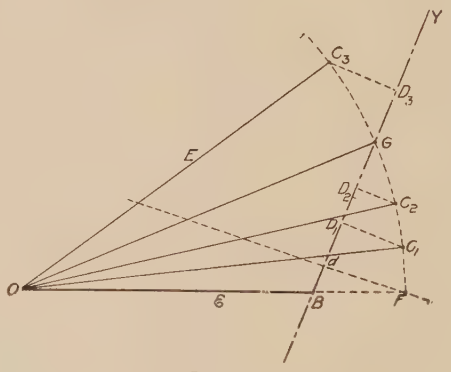


FIG. 4.

If the supply E.M.F., E, is varied, which amounts to the same thing as changing the radius OC of the circle, the particular load for which the reactive current vanishes may be varied at will. For example, Fig. 5 (in which four circles of different radii are drawn), shows that it is possible to have the current in phase (power-factor equal to unity) with a certain useful load dC_2 or with zero load, instead of the load dG, and that it is possible even to have negative lag (or a leading current) at all loads, by suitable variation of the supply E.M.F. E.

It is seen that in any case it is impossible to prevent the reactive current from varying when the load changes. Since this current produces a magnetizing or a demagnetizing action equal to that of

$$K\frac{N}{2}I_d\sqrt{2} \text{ inductive ampere-turns,}$$

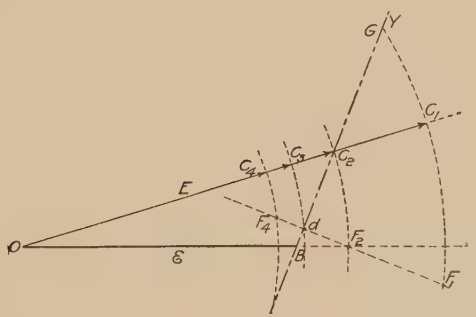


Fig. 5.

it is not possible to maintain constant potential by means of constant excitation if the impedance of the circuit is not negligible, but that it will be necessary to increase or decrease the excitation ampere-turns by an amount equal and contrary to that of the ampere-turns of the reactive current in order that the total inducing flux may remain constant. (The variation of the magnetic leakage may introduce a slight complication, as will be seen later.)

With a given value of γ, if the impedance Z is varied, we see that, the greater its value, the larger the *scale* of amperes will be, and, consequently, the smaller will be the *variation* of load which corresponds to a given angular displacement of the vector OC. The variations of the active current with the load will therefore increase in proportion with the impedance.

If the impedance is varied by changing only the reactance, X, without changing the resistance, R, the result is less clearly seen, because the direction of the reference line OY then changes at the same time as the scale of amperes. This point will be discussed later under the general case.

General Case. Reactive Current Values for a Given Voltage Variation as a Function of the Load. In the most frequent case the voltage is raised slightly when the load increases, to compensate for the line-losses.

In Fig. 6, let E=the E.M.F. corresponding to zero-load, and let E', E'', E''', etc., represent the successive values which E must have when the load corresponds to the active currents I_w', I_w'', I_w''',

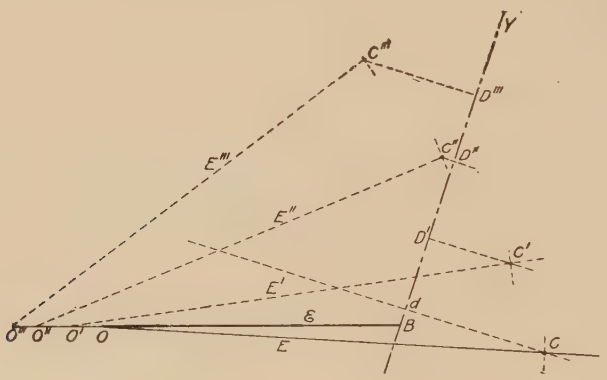


Fig. 6.

etc. These current-values are to be measured off on a reference-line BY, starting from the point B, giving the distances BD', BD'', BD''', etc., and the E.M.F.'s are to be measured off, along the horizontal line BO starting from B toward the left, giving the distances BO, BO', BO'', BO''', etc.

From each of the "O" points thus determined, as a center, let a circular arc be drawn, with the same radius, equal to the external E.M.F. E, which is a constant, assumed to be known. (In Fig. 6, in order to make the case more general, it is assumed that the E.M.F.'s may be unequal.) The respective points of intersection of the circular arcs with the lines dC, $D'C'$, $D''C''$, $D'''C'''$, drawn perpendicular to the reference-line BY, give the conditions corresponding to the various loads.

The reactive currents corresponding to these loads are therefore entirely determined.

It will be seen that there is an infinite number of possible solutions, according to the value taken for the constant E.M.F. E. This E.M.F. can be made high or low, as desired. It depends on the ratio of transformation of the static transformers used for supplying current to the converters.

Moreover, the impedance Z can also be regulated, within certain limits, at least, by the introduction of reactance in the current supply-circuit. (It is obviously desirable to reduce the ohmic resistance to the lowest possible value, to avoid lowering the efficiency.) It remains to be seen what are the most suitable values for E and X. This point will be discussed in the next two sections.

Most Suitable Value of Current-Supply Voltage. This value is evidently that which gives zero-lag (unity-power-factor) for the most usual or frequent load, because we then have, for that load, a minimum current-value, consequently better line-efficiency, and better "over-all" efficiency. This condition of operation can be determined from the known or expected conditions of service, for the particular apparatus.

One might be tempted, on general principles, to take the maximum load as the one for which the supply-voltage is to be specially adjusted, so as to give the best results with that load, because it is precisely then that it is most important to reduce the losses; but since, in practice, it is necessary to consider the *mean* daily heating and efficiency of the apparatus used, it is for the *average* or *normal* load, and not for excessive loads of momentary duration only, that the supply-voltage should be adjusted, with the object of obtaining unity power-factor.

The range of voltage-regulation desired in the rotary converter can be obtained with more ease and precision when the point of maximum power-factor ("no lag") corresponds to the average load. In fact, as will be seen later, the only converter-voltages (D.C.) which can be obtained exactly, by practical methods of excitation, are those corresponding to zero-lag and zero-load.

Most Suitable Value of Reactance. It is important to note, at the outset, that a certain minimum reactance is necessary in order to obtain, with a constant (A.C.) E.M.F. E, a (D.C.) voltage which will increase, or even remain constant, with the load.

Let us suppose that there is no reactance in the circuit. This makes $\gamma = 0$; the reference-line BY (Fig. 7), will then coincide with

the axis OX; and the currents corresponding to increasing loads will be represented by the distances Bd, BD', BD'', etc. Let D'', for example, represent the normal load condition for which there is no lag ($\gamma=0$). The vectors Cd, $C'D'$, $C''D''$, which measure the corresponding reactive currents, increase extremely fast at light loads; and, since the scale of current-values is very small, owing to the fact that Z is very small, it is seen that the reactive currents at light loads would be very large. On the other hand, in the case represented by Fig. 6, owing to the fact that the load-lines are no longer perpendicular to OB, it is easy to find, with a constant E.M.F. E, and with small active currents, load-points, C, C', C'', corresponding to points

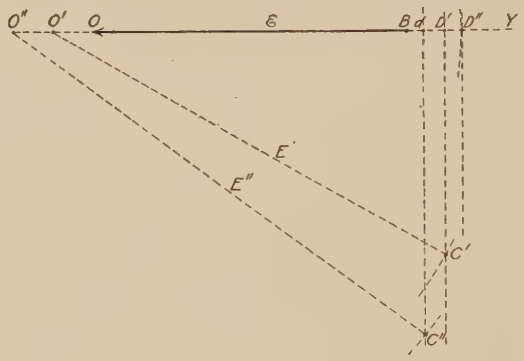


Fig. 7.

O, O', O'', which are displaced to the left, and, consequently cause an increase in the voltage $\left(\frac{\varepsilon}{\xi}\right)$ at the brushes.

It is seen, thus, that it is possible to obtain economically a voltage-regulation giving constant or rising voltage, as the load increases, only when the circuit contains a certain amount of reactance.

This point being established, it is easy to note the influence of reactance, by supposing that the "no lag" condition is made to correspond always to the same load, i.e., to the same value of the active current, I_w. In fact (Fig. 8), all the load-points corresponding to that condition are found to be located at distances, ZI_w, from the point B, which are proportional to the impedances $Z=\sqrt{R^2-X^2}$; hence, if we measure off, on OB prolonged, a distance BK, equal to RI_w, all the points G, corresponding to different reactances X, will be located on the right line KP perpendicular to OK. Likewise,

the point of zero-load, d, will be on the perpendicular kd corresponding to a segment $Bk = Ri_0$. We will compare, for various cases, the values of the reactive current i_0 corresponding to this no-load condition.

This reactive current is obtained by drawing through d the right line dF, perpendicular to BG, until it crosses the circle of radius E. In order to make this current-value comparable with BK, it is only necessary to reduce also the segment dF in the proportion $\frac{R}{Z}$, which can be done by projecting it on KG. The segment $d'F'$ then measures the value Ri_0, while Bk measures RI_w. By this process all the current values are referred to one and the same scale, i_0, whatever may be the value of Z. On making the diagram anew for cases corresponding to various angles of the line of reference BG, it is found that the reactive current, which is very large for very small "lags" diminishes, rapidly at first, then more slowly.

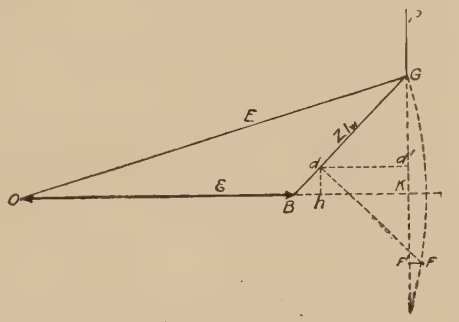


Fig. 8.

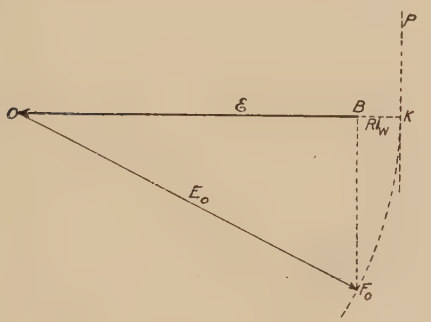


Fig. 9.

In order to follow more easily the law of variation, let us suppose the active current to vanish at zero-load, i.e., let us assume $i_0 = 0$, which makes d coincident with D (Fig. 10). The reactive current i_0 is then proportional simply to the distance FH.

If the point G be raised gradually on the vertical line KG, starting from the point K, the reactive current is at first equal to $\frac{BF_0}{R}$ (Fig. 9), and it will then decrease; for example, for the point G (Fig. 10), it will decrease to FH. When G is very high, it is seen that an additional rise, $G'G''$, for example, will increase almost proportionally the

radius E without altering materially the inclination of the segment BF'. Hence, if the reactance is increased beyond a certain point the reactive current i_0, corresponding to zero-load, will increase with it.

This reactive current must therefore have a minimum; and to every value higher than this minimum there will correspond two reactance values, and two inclinations of the line BG, one greater, the other smaller.

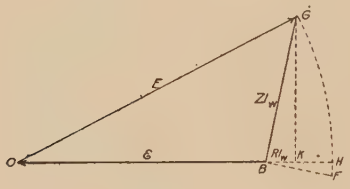


FIG. 10.

We can learn more about this minimum by calculations based on the fact that the same value of the supply-voltage E enables both the "no-lag" point G and the "no-load" point F to be obtained.

To make the case more general, let us suppose a condition wherein i_0 does not vanish and wherein the voltage with load, ε', is different from the voltage ε without load. We will have the two following relations:

$$\text{At the point } G, \quad E^2=\varepsilon'^2+Z^2I_w^2+2\varepsilon' RI_w. \quad . \quad . \quad . \quad (a)$$

$$\text{At the point } F, \quad E^2=\varepsilon^2+Z^2i_0^2+2\varepsilon Xi_0. \quad . \quad . \quad . \quad . \quad (b)$$

From these, eliminating E, we have

$$\varepsilon'^2+Z^2I_w^2+2\varepsilon' RI_w-\varepsilon^2-Z^2i_0-2\varepsilon Xi_0=0. \quad . \quad . \quad . \quad (4)$$

This is the condition required. Solving for i_0 and then for X, we obtain the two following formulas:

$$i_0=\frac{-\varepsilon X+\sqrt{\varepsilon^2X^2+Z^2(\varepsilon'^2-\varepsilon^2+Z^2I^2{}_w+2\varepsilon' RI_w)}}{Z^2}; \quad . \quad . \quad . \quad (5)$$

$$X=\frac{\varepsilon i_0\pm\sqrt{\varepsilon^2i_0^2-(I_w^2-i_0^2)[\varepsilon'^2-\varepsilon^2+R^2(I_w^2-i_0^2)+2\varepsilon' RI_w]}}{I_w^2-i_0^2}. \quad (6)$$

In (5) the expression under the radical sign is taken with the + sign only, because, for each value of X, i_0 can have only one value, which, necessarily, is positive. On the other hand it has been seen that X has two values for each value of i_0; hence it is necessary to retain both signs ($\pm$) before the expression under the radical sign, in Eq. (6).

The minimum value of i_0 could be obtained by making the derivative of Eq. (5) equal to zero and solving; but it is simpler to equate to zero the expression under the radical sign in Eq. (6), because the minimum value of i_0 is evidently that *below which* the value of X would become imaginary. Hence, the positive and negative quantities under the radical sign must offset each other, when there is a minimum.

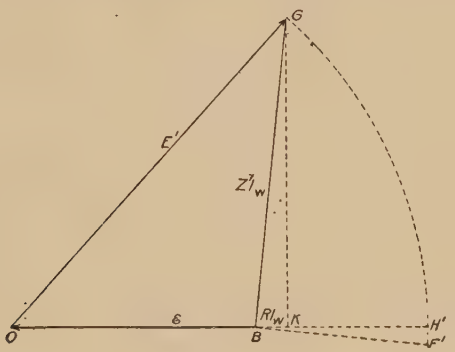


FIG. 11.

Equating the quantity under the radical sign in (6) to zero, and simplifying with respect to $(I_w^2 - i_0^2)$, instead of i_0^2, we have

$$\varepsilon^2 I_w^2 - (I_w^2 - i_0^2)[\varepsilon'^2 + 2\varepsilon' R I_w - R^2(I_w^2 - i_0^2)^2] = 0. \quad . \quad . \quad (7)$$

Since the ohmic drop of voltage should be only a small fraction of E, the last term, $R^2(I_w^2 i_0)$ may be neglected, and we thus obtain the approximate value

$$(I_w^2 - i_0^2) = \frac{I_w E_1^2}{\varepsilon'^2 - \varepsilon^2 + 2\varepsilon' R I_w},$$

and the minimum value of i_0^2 is

$$i_0^2 \text{ minimum} = \frac{\varepsilon'^2 - \varepsilon^2 + 2\varepsilon' R I_w}{\varepsilon'^2 + 2\varepsilon' R I_w} I_w^2.$$

Since the quantity under the radical sign in (6) practically vanishes for this minimum value of i_0^2, the expression for X which corresponds to the minimum reactive current i_0 is, simply, by approximation,

$$X_0 = \frac{\varepsilon i_0}{I_w^2 - i_0^2},$$

or, substituting for i_0,

$$X_0=\frac{\sqrt{(\varepsilon'^2+2\varepsilon' RI_w)(\varepsilon'^2+2\varepsilon' RI_w-\varepsilon^2)}}{\varepsilon I_w}.$$

This is the most suitable reactance-value for reducing i_0 to a minimum. Inasmuch as, on the other hand, the supply-voltage required increases with the reactance X, and inasmuch as this voltage must be kept as low as possible in order to utilize the apparatus to the greatest advantage (i.e., to obviate the necessity of making it too large and, consequently, more expensive), for the effective output, we may conclude that, *in general, the reactance value should be comprised* between zero and the value (X_0) corresponding to the minimum zero-load reactive current.

NOTE 1.—Formulas 5, 6, 7, 8 can be put into a form which is often more convenient practically, by expressing the values of i_0 in terms of I_w, the values of RI in terms of ε, the values of ε' in terms of ε, and the values of X in terms of R.

Let us take

$$\frac{\varepsilon'}{\varepsilon}=(1+\varepsilon);\quad \frac{RI_w}{\varepsilon}=\alpha;\quad \frac{X}{R}=m;\quad \frac{i_0}{I_w}=\tan\phi;$$

$\cos\phi$ being the power-factor, and m being the value of $\tan\gamma$.

From these equations, we deduce

$$\frac{XI_w}{\varepsilon}=\alpha m,$$

$$\frac{\varepsilon}{Z}=\frac{I_w}{\alpha\sqrt{1+m^2}},$$

and also the equivalent formulas:

$$i_0=\frac{I_w}{\alpha(1+m^2)}\{-m+\sqrt{m^2+\alpha^2(1+m^2)^2+(1+m^2)(\varepsilon^2+2\varepsilon+2\alpha(1+\varepsilon)}\}. \quad (5')$$

$$X=\frac{R\tan\phi}{\alpha(1-\tan^2\phi)}\pm R\sqrt{\frac{\tan^2\phi}{\alpha^2(1-\tan^2\phi)}-\frac{\frac{2}{\alpha}(i+\varepsilon)+\frac{\varepsilon^2+2\varepsilon}{\alpha^2}}{1-\tan^2\phi}-1}, \quad (6')$$

minimum of $$i_0^2=I_w^2\frac{(1+\varepsilon)^2-1+2\alpha(1+\varepsilon)}{(1+\varepsilon)^2+2\alpha(1+\varepsilon)}, \quad (7')$$

$$X_0=\frac{R}{\alpha}\sqrt{[(1+\varepsilon)^2+2\alpha(1+\varepsilon)][\varepsilon^2+2\varepsilon+2\alpha(1+\varepsilon)]}. \quad (8')$$

In the particular case where the voltage must be constant, we must have $\varepsilon' = \varepsilon$, or $\varepsilon = 0$, and the expression for i_0^2 reduces to the simple form

$$i_0^2 = I_w^2 \frac{2\alpha}{1+2\alpha},$$

which gives, substantially,

$$i_0 = I_w\sqrt{2\alpha} \quad \text{and} \quad X_0 = \frac{R}{\alpha}\sqrt{2\alpha(1+2\alpha)},$$

or, practically,

$$X_0 = R\sqrt{\frac{2}{\alpha}}.$$

NOTE 2.—In the particular case where $i_0 = I_w$ and $\tan\phi = 1$, formulas (7) and (7′) will no longer apply, and they must be replaced by the following formulas, deduced from Eq. (4):

$$X = \frac{\varepsilon'}{\varepsilon}\frac{RI_w}{i_0} - \frac{\varepsilon'^2 - \varepsilon^2}{2\varepsilon i_0}, \quad \ldots\ldots\ldots \quad (7a)$$

$$X = \frac{R}{\tan\phi}\left[(1+\varepsilon) - \frac{\varepsilon^2 + 2\varepsilon}{2\alpha}\right]. \quad \ldots\ldots\ldots \quad (7a')$$

Regulation of Voltage at Terminals by Variation of the Supply E.M.F. In what precedes it has been supposed that the E.M.F. E of the source of current is constant at all loads. This necessitates a reactive current at light loads.

It is possible to compound or to regulate automatically the excitation of the alternator in such manner that its E.M.F., E, will increase with the load. This compounding will obviate all necessity for any reactance; and the line of reference in the diagram can be the line OB prolonged, if there is no other loss of voltage than that due to ohmic drop. This solution is obviously the most perfect one, theoretically, but it cannot be realized exactly otherwise than in the exceptional case where the rotary converters are supplied individually by separate alternators. It is not possible otherwise to establish a correlation between the load of the converters and that of the lines when they supply current at the same time to other apparatus.

An approximation to this method of regulation can be made by supplying the converters with current through transformers whose

ratio of transformation is made variable by means of "taps" on the the secondary winding connected with suitable switching apparatus.

The fundamental diagram (Fig. 12), or the formula (*a*), will determine the values which should be given to E for each value of the E.M.F. ε, and of the load ZI_w, measured off along the line BY corresponding to the fixed reactance of the circuit. The most suitable value for the reactance is, in this case, evidently, the lowest value, $X=0$, when possible; because this value reduces to a minimum the values of the E.M.F. E required. Therefore, no supplementary reactance should be included in the circuit. Under these circumstances BY becomes the prolongation of OB in the diagram.

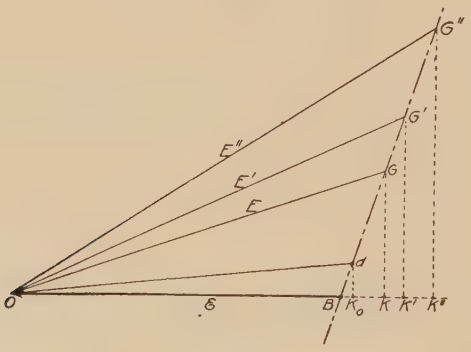


FIG. 12.

Regulation of Voltage at Terminals by Variation of Reactance. It is also possible to neutralize the lag at all loads more simply by keeping the supply E.M.F. E constant (and equal to that required to insure the necessary voltage at the terminals with full load without reactive current), and then reduce this voltage at lighter loads by the introduction of suitable reactances in the circuit.

Fig. 13 explains this method of regulation in the case where the E.M.F. E is constant; and it enables the characteristic features of this case to be studied.

Let $BK = RI_w$, represent the loss of voltage by ohmic drop in the circuit, with full load. The segment BK can serve to represent the acti e current.

Let $BK,'$ $BK,''$ etc., represent intermediate current values; and let the corresponding perpendiculars KG, $K'G,'$ $K''G''$ be drawn. Their points of intersection G, $G,'$ G'', with the circle of radius E,

described around the center O, will define the load-conditions corresponding to these currents.

In order to obviate reactive currents it is sufficient, for each case, that the reactance should be adjusted to a value exactly equal to that

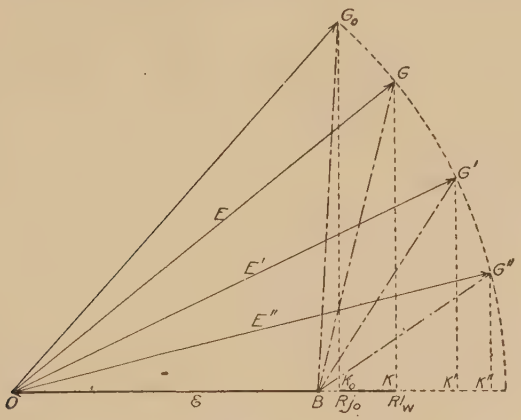


FIG. 13.

which makes the reference line pass through G. This means that the reactance X must be made successively equal to the following values:

$$\frac{GK}{BK}R; \quad \frac{G'K'}{BK'}R; \quad \frac{G''K''}{BK''}R; \text{ etc.}$$

These values of X can be calculated for each value of I_w, by the Eq. (a) given herein above. From this equation

$$E^2 = \varepsilon'^2 + Z^2 I^2 w + 2\varepsilon' R I_w, \quad \ldots\ldots \quad (a)$$

on substituting for Z^2 its value ($Z^2 = R^2 + X^2$), and solving for X^2, we have

$$X^2 = -R^2 + \frac{E^2 - \varepsilon'^2 - 2\varepsilon' R I_w}{I_w^2}.$$

Practically, this regulation can be worked by hand or even automatically, by making use of reactance-coils having movable cores, which are pushed all the way in when the load is zero, and are gradually

pulled out as the load increases, in such a way as to bring back to zero a direct-reading phase-indicator, connected at the terminals of the converters.

This method is applicable whatever may be the value of E (i.e., the radius of the circle); but the most suitable value is, obviously, the smallest value, which corresponds to $E=\varepsilon'+RI$.

The diagram presupposes that the locus of the point G is a circle; but if the E.M.F. ε varies the center O of that circle is displaced and the locus of G is a curve, which can be drawn by points determined from the data. This does not, in any way, necessitate modifications in the method.

Power-Factor of the Generator. In what precedes, we have seen the necessity of keeping down the value of the E.M.F., ε. It is customary to express the disadvantages of the useless increase of this E.M.F. by stating the value of the *power-factor*, i.e., by the value of the cosine of the phase-angle, usually termed the "*angle of lag*," between the E.M.F. and the current.

Let ψ denote this phase-angle, and let θ denote the phase-angle between ε and E; and let, as before, ϕ denote the phase-angle between the current and the E.M.F. ε. We have

$$\tan\phi=\frac{I_d}{I_w},$$

and this angle of lag, ϕ, is always in the same direction as θ (see Fig. 1), i.e., in the direction of the hands of a watch (in these diagrams). We therefore have

$$\psi=\theta+\phi.$$

For the condition of average load, that is to say, in the cases where there is no lag, we have $\phi=0$, and, therefore, ψ reduces to the same value as θ itself.

In order to increase the power-factor of the generator as much as possible at average loads, we should, therefore, reduce θ, and, consequently, ε, as much as possible, as already stated. It is much more important to do this than to reduce to zero the "no-load" lag between the current and the converter E.M.F. Therefore, the reactance used should have a value comprised between zero and the value which renders the no-load lag a minimum. This is all the more desirable

because the lag θ then becomes negative at light loads, as indicated in Fig. 14, and the value of ψ is then a difference $\psi=\phi-\theta'$, which increases but slowly with ϕ.

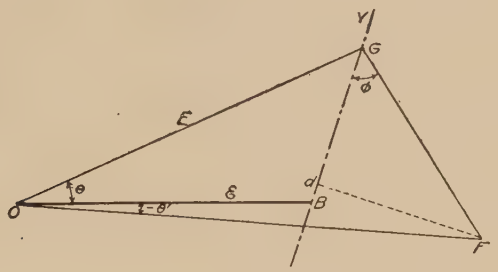


FIG. 14.

It is logical to endeavor to make θ and $\phi-\theta'$ equal to each other, in such a way as to render the two extreme values of ψ as equal and as small as possible.

CHAPTER II

PREDETERMINATION OF THE FIELD-EXCITATION OF ROTARY CONVERTERS

In the preceding chapter we discussed the conditions attending the supply of electric current to rotary converters, and, in particular, the effects of reactance in the supply-circuit, and the various ways of using it to obtain a given range of impressed voltage for a given range of load without exceeding a certain maximum reactive current, and even while making this current approach a minimum value. There still remains the second portion of the problem to be solved, namely, the determination of the field-exciting ampere-turns, due to either the series or the shunt-winding coils, which enable the rotary converter to follow approximately this law of variation, in consequence of the effects of the reactive currents.

In what follows it will be assumed that we are dealing with the ordinary case of a converter which is operating without lag, at its average load, and which consequently, has, at lighter loads, a positive reactive current $\left(\text{lagging } \frac{\pi}{2} \text{ behind the E.M.F.}\right)$ whereby there is produced a magnetizing effect on the field.[1] This is the case which was discussed in the preceding chapter.

Characteristic Features of the Rotary Converter. We will begin by considering the magnetic features of the rotary converter, which involve two new important elements:

1. The excitation-curve, or the curve showing the variation of induced E.M.F. as a function of the field-exciting ampere-turns due to the field-windings. This curve is supposed to be known from the shop-tests made of the machine at the time it was finished.

In the excitation-curve (Fig. 15) the ordinates represent the E.M.F.'s

[1] It is easy to see that any reactive current which lags behind the impressed E.M.F. E_1 is leading with respect to the internal E.M.F. ε, since E and ε are practically in opposition.

(volts) induced, and the abscissas represent the corresponding ampere-turns of excitation.

2. The equivalent magneto-motive force of the armature, i.e., the number of ampere-turns which, if acting upon the field-cores (same as the ampere-turns of the regular field-winding) would produce in the magnetic circuit the same magnetomotive force as is produced by the reactive current I_d circulating through the armature.

These equivalent average ampere-turns can be represented, as in an alternator, by the expression,

$$K\frac{N'}{2}I_d\sqrt{2},$$

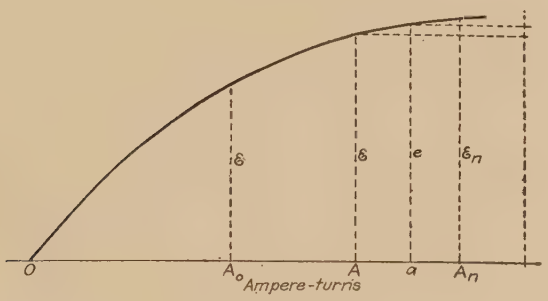


FIG. 15.

in which N'= the number of peripheral *wires* of the armature-winding per double field (i.e., per magnetic circuit); K being a coefficient of reduction which depends upon the number of phases and of armature slots, and on the width of the poles; and I_d being the *effective* value of the reactive current.

K is determined by calculation, by taking I_d=1 ampere, with zero-lag, and calculating the mean magnetic potential produced under one pole.

The value of K can be obtained by experimental measurement. This can be done with an approximation which is generally sufficient for all purposes (since the leakage-reactance, ws, is practically negligible), by running the converter at normal speed by power applied mechanically, and then making it supply (on the A.-C. side) with normal voltage, a purely inductive external circuit (composed of a reactance giving a power-factor—cos ϕ—equal, at most, to 0.20); the difference between

the voltage (ε) at the terminals, with this load, and the voltage (e) obtained with open circuit, with the same excitation, is the measure, in volts, of the armature-reaction corresponding to the current supplied to the external circuit by the converter.

Taking the values ε and e thus obtained and referring them to the excitation-curve (Fig. 15), they will correspond to different points on this curve. The difference between the *abscissas*, A and a, corresponding to these two points will be a measure of the equivalent ampere-turns of the armature (neglecting, as before, the effects due to magnetic leakage). It is only necessary, in order to obtain K, to divide $a-A$ by the product of the effective reactive current I_d measured in the above-mentioned experimental test, by half the number of turns of wire, $\frac{N'}{2}$, and by the constant $\sqrt{2}$. We will have

$$K=\frac{a-A}{\frac{N'}{2}I_d\sqrt{2}}. \qquad (16)$$

Compound-Excitation. Different Factors of this Excitation. Rotary converters are excited by current taken from their secondary (D.-C.) side. The field-excitation may, obviously, be either shunt, series, or compound.

Let E_2 = secondary voltage at the terminals;

I_2 = secondary or output current delivered by the rotary converter;

n = number of turns of the series-winding;

r = resistance of one turn of the shunt-winding.

The shunt ampere-turns and the series ampere-turns will be, respectively, equal to

$$\frac{E_2}{I} \quad \text{and} \quad nI_2.$$

It is these ampere turns which, conjointly with the equivalent ampere-turns of the armature, determine the total excitation.

In the most general case, that of a compound-wound converter, the question is, therefore, quite complex, since there are three variable factors, the first of which varies with the voltage, the second with the output (direct) current, and the third with the reactive (alternating) current.

As already seen, the secondary ampere-turns are proportional to the ampere-turns due to the useful primary current, I_w-j_0.

Let ξ represent the ratio of transformation, i.e., the ratio of the primary E.M.F. to the secondary E.M.F. Since the armature-resistance is already, by hypothesis, counted once in that of the supply-circuit, it need not be counted again here. We can, therefore, write:

Ampere-turns of shunt excitation,

$$A_d=\left(\frac{1}{\xi r}\right)\varepsilon; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Ampere-turns of series-excitation,[1]

$$A_s=nI_2=\left(\frac{\pi kn}{\sqrt{2}}\right)(I_w-j_0); \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Ampere-turns of reactive currents, produced by the armature,

$$A\alpha=\left(\frac{KN'}{\sqrt{2}}\right)I_d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The magnetomotive forces are thus expressed directly in terms of the elements of the primary currents. This will enable the excitations for each load-condition to be calculated.

If we count as positive the lagging reactive currents (which are magnetizing currents) the total number, A, of exciting ampere-turns corresponding to the load-condition εI_w will therefore be

$$A=\frac{\varepsilon}{\xi r}+\frac{\pi kn}{\sqrt{2}}(I_w-j_0)+\frac{KN'}{\sqrt{2}}I_d. \quad . \quad . \quad . \quad . \quad (20)$$

The +sign is placed before the reactive ampere-turns because, under the normal conditions of current-supply previously considered, the reactive current is directed to the right of D, i.e., it is *positive* and *magnetizing* between zero-load and normal load, and it is demagnetizing beyond normal load. The +sign is placed before the series ampere-

[1] From the relation found at the beginning of Chapter I, namely, $\frac{I_2}{I_w}=\frac{\pi k}{\sqrt{2}}$, we get $I_2=\frac{\pi K}{\sqrt{2}}I_w$. In Eq. (18), for the sake of greater precision, I_w is replaced by I_w-j_0. If the armature had two different windings, the ratio would have to be multiplied by that of the numbers of turns of these two windings, $\frac{N}{N_2}$.

turns because, in general, the series-winding must work concordantly (or cumulatively) with the shunt-winding, as will be seen later.

The unknown constants, r and n, of the windings, are easily determined when the E.M.F., ε_0 for zero-load, and the E.M.F. ε_m for the mean load (corresponding to the current I_m, with zero-lag) are known.

The excitation-curve (Fig. 15) gives the ampere-turns, A_0, A_m, corresponding, respectively, to the E.M.F.'s ε_0 and ε_m. For the normal load-condition we will have

$$A_m = \frac{1}{\xi r}\varepsilon_m + \frac{\pi k n}{\sqrt{2}}(I_m - j_0). \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

For the no-load condition we will have

$$A_0 = \frac{1}{\xi r}\varepsilon_0 + \frac{KN'}{\sqrt{2}}i_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

As already seen, the value of the reactive current corresponding to zero-load cannot fall below a certain minimum. If a minimum value be assumed for this current, Eq. (22) can be used to determine the conductance of one turn of the shunt winding. From Eq. (22), solving for $\frac{1}{r}$, we have

$$\frac{1}{r} = \frac{\xi}{\varepsilon_0}\left(A_0 - \frac{KN'i_0}{\sqrt{2}}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

From Eq. (21), solving for n, and substituting for $\frac{1}{r}$ the value given in Eq. (23), we have

$$n = \frac{A_m - \frac{\varepsilon_m}{\xi r}}{\pi k (I_m - j_0)}\sqrt{2} = \frac{\left(A_m - \frac{\varepsilon_m}{\varepsilon_0}A_0\right)\sqrt{2} + \frac{\varepsilon_m}{\varepsilon_0}KN'i_0}{\pi k (I_m - j_0)}. \quad . \quad . \quad (24)$$

It is seen that the shunt ampere-turns, which are proportional to $\frac{\varepsilon_m}{r}$, are less than the total ampere-turns A_0 (on the assumption that $\varepsilon_m > \varepsilon_0$), and that they decrease when the reactive current i_0 increases; whereas the series ampere-turns, which are proportional to $n(I_m - j_0)$ increase with the reactive current i_0.

Determination of Reactive Current as a Function of the Excitation, when the Active Current is Constant, then when the Power

is Constant, the Generator E.M.F. being always Constant. Let us suppose, as in the case of synchronous motors, that the converter is supplied by a constant E.M.F. E, and excited by an exciting current which can be regulated at will.

The fundamental diagram (Fig. 2), combined with the preceding curve (Fig. 16), will enable us to find readily, for each value of the active current, I_w, the values of reactive current corresponding to each value of the excitation, expressed in ampere-turns.

Let, for example, I_w be the given active current, and BD the line of reference, which is drawn after the value of γ has been obtained. The locus of the point C, characteristic of the load-conditions, for that constant value of I_w, is the right line DQ, drawn perpendicular to BD, at the distance $BD = ZI_w$, whereas the locus of the point O the second end of the vector E, is the line OBX.

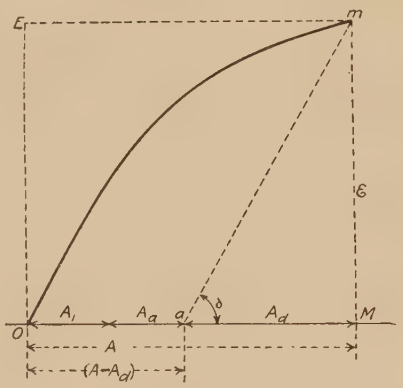


FIG. 16.

All the load-conditions corresponding to the active current I_w will therefore be obtained by drawing, from the first points C, C'', C''', ..., on the first right line, circular arcs of radius E, intersecting the second line at the points O, O', O'', ... The segments BO, BO', BO'', ... thus formed measure the values of the corresponding internal E.M.F.'s, ε, ε', ε'', ..., and the segments DC, DC', DC'', ... divided by Z, measure the values of the reactive current I_d.

For each pair of values of ε and I_d, thus determined, the corresponding ampere-turns of excitation, A_e, are obtained, from the excitation-curve (Fig. 16), by subtracting the counter ampere-turns of the armature from the resultant ampere turns (A) corresponding to the ordinate (ε) of this curve; whence we have

$$A_e = A \mp \frac{KN'}{\sqrt{2}} I_d.$$

It should be remembered that the reactive current I_d is demagnetizing and positive if it is counted to the right of the axis BD, and that

it is magnetizing and negative if counted to the left. The upper sign applies in the first case, and the lower sign in the second case. Under those conditions the diagram can now be used for determining the locus of the points C and the variations of E, when the power is constant, if the generator E.M.F. be maintained constant as in the case of synchronous motors. We have in this case, simply,

$$P = \varepsilon I_w,$$

so that, for each value of I_w, and, consequently, for each position of the right line DQ, the value of ε, and, therefore, the position of the point O, are known; and, from these points, circles of radius E can be drawn,

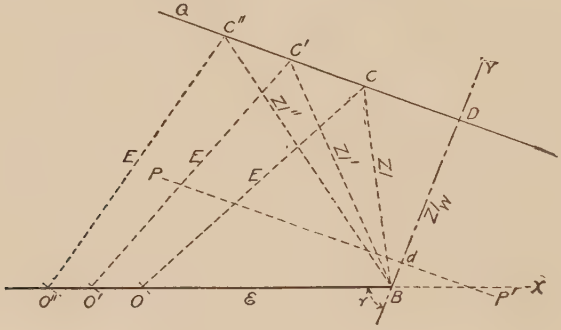


Fig. 17.

whose respective intersections with the right lines DQ give the C points required. It is possible, thus, to draw the " V " curves analogous to those of synchronous motors, and the curves of the corresponding values of ε. But these load-conditions for constant power no longer present in the case of rotary converters the same practical interest as in the case of synchronous motors, because the variation of ε would be very troublesome.

Different Values of the Excitation, with Constant Power and Constant Potential. V-Curves for Constant Potential. The preceding diagram (Figs. 16 and 17) also gives the immediate solution of the more practical problem [1] which consists in determining the current-values, I, corresponding to different excitations, with constant

[1] The case considered is, as yet, more of theoretical than of practical interest, because it assumes that the E.M.F. is maintained absolutely constant, a condition which, as will be seen later, can be realized only approximately. In the general case therefore both ε and E will vary.

power, *and with the secondary voltage constant at the brushes.* It is the same problem as the preceding one except that it is not the difference of potential at the terminals, but the induced E.M.F. ε, which must be supposed constant, in this case.

The locus of the point C is again the right line CD itself. But, as we have added the condition that the segment OB remains constant, the point O will, therefore, remain fixed (Fig. 17*a*).

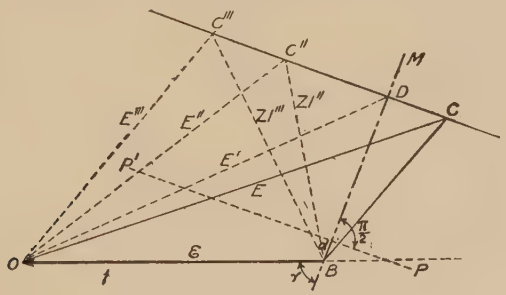


FIG. 17*a*.

For each position of the point C the corresponding segment CB indicates the current, and OC indicates the E.M.F. required at the generator. Taking the E.M.F. values as abscissas and the current values as ordinates, we obtain V-shaped curves (Fig. 18), which, in

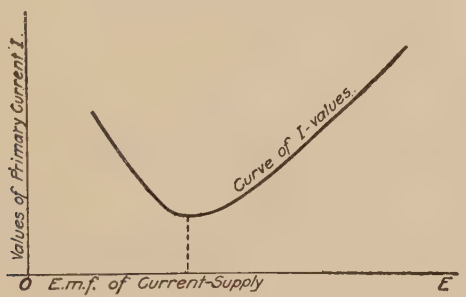


FIG. 18.

consequence of making ε constant, are independent of the field-winding and of the law of saturation.

It is more interesting, however, to take the excitation ampere-turns as abscissas, as in the case of synchronous motors, without distinguishing those due to the series and the shunt windings. This amounts to the same thing as assuming the machine to be separately excited.

These ampere-turns are determined for each value of the alternating current I by measuring on the diagram (Fig. 13), the corresponding segment CD, which indicates the reactive current I_d, and subtracting from this value the ampere-turns corresponding to zero-load which can give the E.M.F. ε and the magnetizing ampere-turns produced by the armature, namely,

$$A_a = \left(\frac{KN'}{\sqrt{2}}\right) I_d,$$

using the + or the − sign according to whether I_d is positive or negative, i.e., according to whether the point C be at the right or at the left of D.

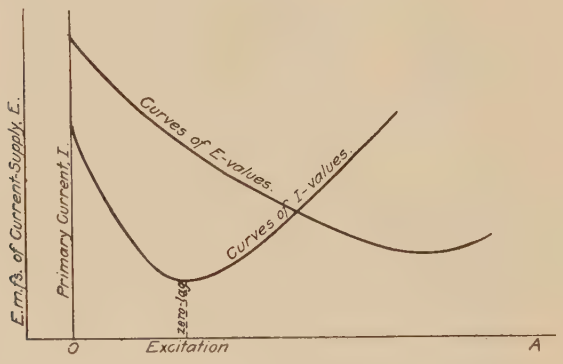


FIG. 19.

Taking the values of the excitation A, thus determined, as abscissas, and the corresponding values of I as ordinates, we obtain a V-curve such as shown in Fig. 19, and of which the "bend" corresponds to the load-condition for which there is no lag of the current. If the corresponding values of E are also plotted as ordinates, a second V-curve will be obtained, whose bend is much farther to the right, so that, in the useful portion of the diagram, it consists only of the descending portion of the curve. These curves would be prolonged to the left if we were to consider negative excitations.

A similar pair of V-curves will be obtained for each value of the power-output of the converter.

Upper Limit of Reactive Current. It has already been seen that there is an inferior limit to the reactive current. It also has an upper limit, according to Eq. (23), namely, that value which makes the shunt-

excitation vanish (a negative shunt-excitation being out of the question). This upper limit value

$$i_0 = \frac{A_0\sqrt{2}}{KN'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

must be greater than the minimum limit, unless the number of turns on the armature is excessive. There is no actual need of an upper limit to the reactive current, because the aim is to reduce it as much as possible, on account of the undesirable effects produced by it on the distribution-system, namely, its increasing the ohmic losses, and its necessitating a higher E.M.F. E. What should determine the value which is to be selected for i_0 is the condition that, between zero load and normal load, the normal variation of voltage (which determines the range of effect produced by the field-excitation windings), should be that which is suitable for the practical purpose for which the machine is intended, and that which is represented by the current supply-characteristic which has already been defined.

The investigation of this point is facilitated by a new diagram, which will now be considered, and which indicates *the lag characteristics of the converter at constant potential.*

Lag-Characteristics of Rotary Converters at Constant Potential. Let us suppose that, in some way, the potential difference at the brushes of the converter is maintained constant. The shunt-excitation, which is proportional to this voltage, will also remain constant; and the total ampere-turns which produce this voltage will also remain constant. We will therefore have

$$A = \text{constant}; \quad A_d = \text{constant}.$$

Hence
$$A_s + A_a = \text{constant } (A - A_d) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

or
$$\frac{\pi kn}{\sqrt{2}}(I_w - j_0) + \frac{KN'}{\sqrt{2}}I_d = (A - A_d). \quad . \quad . \quad . \quad . \quad (29)$$

This formula simply states that, with constant potential, the converter can furnish any secondary current whatever (proportional to I_w) which passes through its series-winding, owing to the automatic decrease of the reactive current, I_d, as the latter, of itself, takes the value necessary to make up the same total magnetomotive force A.

The relation between $I_w - j_0$ and I_d is simply linear. If, therefore, in Fig. 20, the load-values be represented, as previously, by distances

proportional to $dD = ZI_d$ and measured along the reference-line BY; and if the corresponding reactive currents, I_d, be represented by other segments proportional to $DC = ZI_d$; then the locus of the point C, at the end of the segment DC, is a right line, QP, whose angular coefficient with respect to the point Q as a pole is

$$\tan \beta = \frac{DC}{DQ} = \frac{\pi kn}{KN'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

This coefficient depends only on the field and armature windings. The right line QP is easily drawn, in practice by determining the points

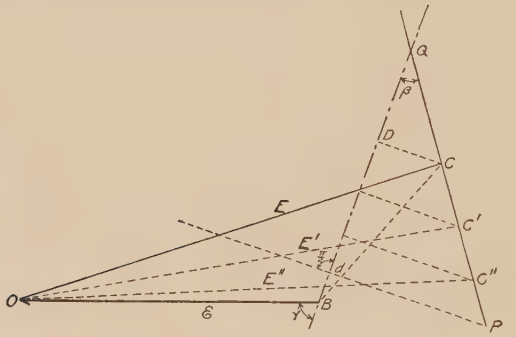


FIG. 20.

Q and P which correspond, respectively, to zero-lag and zero-load: we have

$$\text{For the point } P, \quad \begin{cases} I_w - j_0 = 0 \\ P_d = i_0 = \dfrac{A - A_d}{KN'}\sqrt{2}. \end{cases} \quad . \quad . \quad . \quad . \quad (31)$$

$$\text{For the point } Q, \quad \begin{cases} I_d = 0 \\ Q_d = I_w - j_0 = \dfrac{\sqrt{2}}{\pi kn}(A - A_d). \end{cases} \quad . \quad . \quad (32)$$

For each value of ε there exists a value of $A - A_d$. This value is obtained from the characteristic curve (Fig. 16), by drawing, from the point m, corresponding to this coordinate, the right line ma, making the angle δ with respect to the axis of A values, the angular coefficient of this line being the resistance of an average turn of the shunt winding,

$$\tan \delta = r.$$

Therefore, to each value of ε there corresponds a right line PQ, which is more or less distant from the point d; *but all these right lines are parallel to each other*, having the same angular coefficient, ρ, which depends on the windings only. The smaller the number of turns of the series-winding, the farther, evidently, the point Q will be from the point d, on the right line BY. When there is no series-winding (in the case of a shunt-wound rotary converter), the right line PQ becomes a right line parallel to QY, drawn through P. Conversely, the decrease of the number of turns in the armature displaces the point P toward the right, on the perpendicular dP.

Finally, increasing or decreasing the shunt ampere-turns decreases or increases the difference $A - Ad$, and, consequently, brings the right line PQ nearer to or farther from the point d. This line passes through the point d when $Ad = A$.

Effective Characteristic of Rotary Converter under Load. Following the methods just described, it is easy to study, on the diagram,

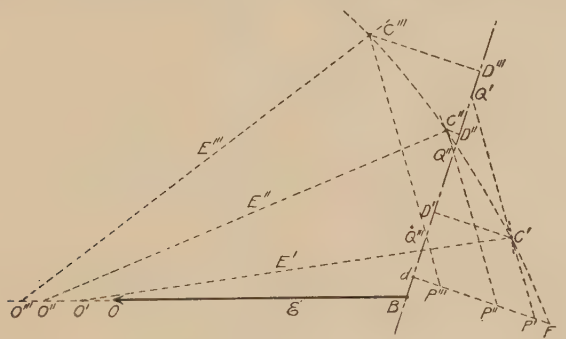


FIG. 21.

the variations of voltage at the terminals of the converter, as a function of its current-output. It is only necessary to reverse the problem and to find the output corresponding to each voltage, within the limits of variation prescribed or allowed for the voltage.

Let ε, ε', ε'', . . . represent a series of values of the voltage ε. Let these values be represented, in Fig. 21, by the corresponding distances BO, BO', BO''. . . . From the points O, O', O'', . . . let circular arcs be described, with a radius equal to E, if the supply E.M.F. is constant, or equal to E, E', E''. . . respectively, if the supply E.M.F. is variable. Let the points of intersection C, C', C'', . . . of these circles with the lag-characteristic line QP, $Q'P'$, $Q''P''$. . . correspond, respectively, to the E.M.F. values ε, ε', ε'', ... These points will define the cor-

responding current values, I_w and I_d, of the converter, and, in particular, the loads dD, dD', dD''. . . . If the *required* characteristic has already been drawn by points, for the same values of ε, ε', ε'', . . ., the comparison of the two curves will show at once the discrepancies between the values wanted and those which the apparatus can give.

The objection to this diagram is that the circles, in practice, cut the QP right lines at very sharp angles, and that the points of intersection are somewhat uncertain, i.e., it is difficult to locate them accurately.

In order to make the comparison with greater precision by calculation, the value of I_w is calculated, as a function of ε, by Eqs. (2) and (29), namely,

$$\begin{cases} Z^2(I_d^2+I_w^2)+2\varepsilon(RI_w+XI_d)=E^2-\varepsilon^2, \quad . \quad . \quad . \quad (2) \\ \left(\frac{\pi kn}{\sqrt{2}}\right)(I_w-j_0)+\frac{KN'}{\sqrt{2}}I=A-A_d. \quad . \quad . \quad . \quad . \quad . \quad (29) \end{cases}$$

These two equations, containing two unknown quantities, I_w and I_d, enable I_d to be eliminated, and a solution to be obtained for I_w in terms of known quantities, ε being supposed to have been previously determined. This calculation, though a trifle tedious, presents no difficulties.

The conditions of operation in various cases can be easily predetermined without drawing the curve accurately. Since, from what precedes, the angle of the PQ phase-lines depends on the reactive current i_0 permitted at zero load, it will be apparent that the choice of values for this current should be made with the idea of making the *actual* curve agree as closely as possible with the *desired* curve. The principal practical cases will be investigated from this standpoint.

Application in the Case of Separate Excitation. Let us suppose, at first, for the sake of greater simplicity, that the shunt-excitation is replaced by a separate excitation which is constant. A_d is then constant. It is seen, immediately, that if no series-winding is added, the converter can only give a decreasing voltage as the load increases. The lag-characteristic for all loads is, in fact, a right line, PQ, parallel to BY (Fig. 22); and if we take, on that line, points $C'C''$, whose distance from P increases, the circles of constant radius, E, cut the axis BO at points O, O', O'', which come nearer to B; hence, the voltages proportional to OB are decreasing.

Therefore, whatever may be the reactance and the value selected for the external E.M.F. E, it is not possible to maintain the voltage constant at the brushes on the direct-current side, with a separate excitation, and still less so with a shunt-excitation. It goes without saying

that the best excitation under these conditions of operation with falling voltage would be one which neutralizes the reactive current dP at all loads; but this mode of operation is not of great practical interest.

It has been proposed to use converters with a falling voltage-characteristic in connection with storage batteries, so as to make the batteries discharge when the load is heavy, and make them charge when the load is light. With the usual method of current-supply this mode of operation would cause variations of 10, 20 per cent, or more, in the supply voltage. Dr. F. B. Crocker [1] has devised an ingenious method of regulating the supply-voltage which enables a wide range of E.M.F. regulation to be obtained in a rotary converter, either with a falling or a rising

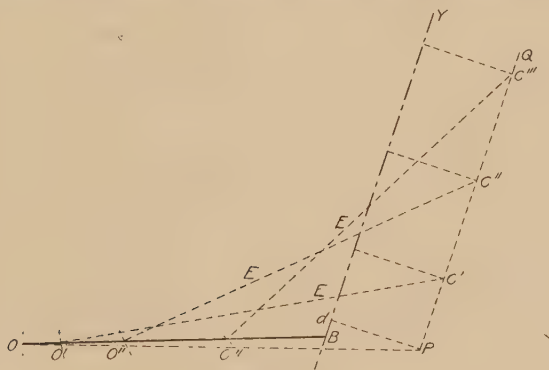


FIG. 22.

characteristic, without affecting the E.M.F. of the source of supply. The same result is also accomplished by the so-called "split-pole" rotary converter, in another way.

If we add a series-winding which causes the right line PQ to incline toward the left, the variations of ε will evidently become smaller. We proceed to find the condition which reduces these variations of ε to a minimum, i.e., which makes the converter give a substantially constant voltage.

It is evidently necessary that the lag-characteristic PQ should coincide as far as possible, within the practical load limits, with a circular arc described from the point O as a center situated at the end of the line $BO = E$. Let F (23) represent the zero-load condition; let G represent the average load, and M the maximum load. A right line

[1] See U. S. Patents to F. B. Crocker, on "Automatic Regulation of Rectifiers and Rotary Converters," No. 891,797, June 23, 1908, and No. 1,012,524, Dec. 19, 1911.

sab can be drawn which cuts the circle at two intermediate load-points *ab* between the extreme points *F* and *M*, in such a way that this line will not be far distant from the circular arc of any point in its useful portion, *PM*. The segments *dP* and *dQ*, read off from the diagram (by the proper scale) will determine the currents i_0 and I_w corresponding to zero-load and normal load; and Eqs. (31) and (32) can be used with these values to deduce the number of constant ampere-turns, *Ad*, which are necessary, and also the number of turns *n* of the series-windings.

The slight residual differences in the E.M.F. ε will be obtained by drawing circles of radius *E* from every point on the right line *PS* and

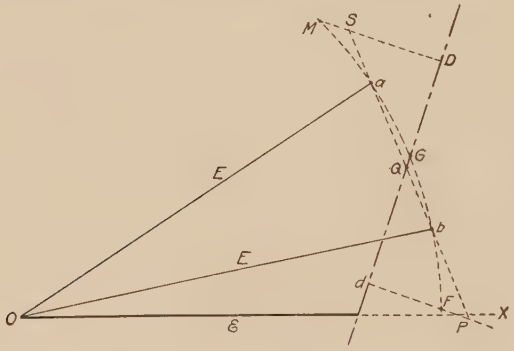


FIG. 23

taking their intersections with the axis of voltages *BO*. The value of the reactance *X* affects the precision of regulation, because, the greater the length of the arc *FM*, corresponding to a given variation of load, the more difficult it is to make it coincide with a right line such as *PS*. To improve the regulation, therefore, the segment $dD = ZI_w$, which measures the maximum load I_w, ought to be decreased to a minimum; and, to this end, the reactance (which is the more important factor in the value of *Z*) ought to be decreased to the lowest value possible. However, as already seen, the reactance cannot be decreased below a certain value X_0 without causing the zero-load reactive current i_0 to increase. It is therefore necessary to take a suitable mean value between X_0 and zero, which will depend on the circumstances and the conditions of the case. In general, the more the load of the converter is uniform and the less often it runs without load, the more it will be possible to reduce *X*.

To overcompound the converter the right line PQ should be straightened more, by raising the point Q higher on the right line BY; but the principle always remains the same, namely, to draw the right line PQ in such a way that it shall come as close as possible, at every point, to the desired characteristic, which should itself be drawn first.

Application to the General Case of Self-Excitation. In the general case of self-excited converters, the solution is nearly the same, but it is complicated by the fact that the shunt-excitation varies with the direct-current voltage. Hence, instead of only one right line PQ to indicate the lags, there will be a series of parallel right lines corresponding to various values of ε.

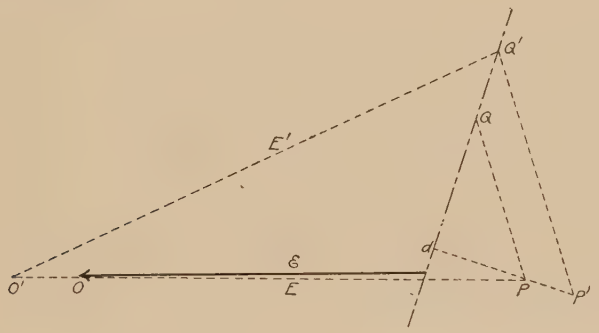


FIG. 24.

In the case of constant potential regulation, the method of determining the excitation-windings and the reactance of the circuit remains exactly the same as given in the preceding paragraph, since the shunt-winding must, by hypothesis, produce constant excitation. But the small variations of A_d have to be considered in the final calculation of the differences of voltage due to the imperfection of the regulation.

For values of ε lower than the normal value, the shunt-excitation decreases of itself and, consequently, the right line PQ comes closer to d, remaining parallel to itself. On the other hand, this line recedes from d when ε increases. It is easy to see, in Fig. 24, that this effect tends to increase the voltage-differences as compared with these differences when the shunt-excitation is constant.

When the machine is to be overcompounded the zero-load and the normal load no-lag conditions determine, as before, the reactive

ampere-turns, for zero-load, A_0, and, therefore, the shunt and series windings

Let ε and ε' represent the voltage-values corresponding to these two load-conditions, according to the excitation-curve. The lag-lines corresponding to all the intermediate values are parallel, and are comprised between two lines which are determined by Eq. (29) when the right-hand member is made equal to $A_1 - A_0$ and to A', respectively. These lines pass through P and P' respectively.

In order to obtain effective overcompounding, i.e., to have $\varepsilon' > \varepsilon$, it is necessary that the point O should not be very far distant from P on the right line of reference BY. This means, simply, that it is necessary to have either a powerful series-winding, or else a large reactive current with zero-load. The first of these two ways appears to be the

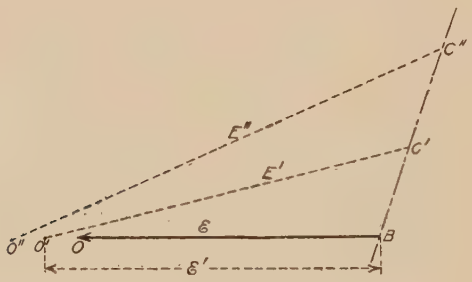


Fig. 25.

preferable one; but inasmuch as it is always desirable to increase the inclination of the right line BY to the right in order to decrease ε, it is necessary to resort to both plans to obtain overcompounding.

Regulation of Supply E.M.F. by Compounding of the Generator. As already stated, it is possible, theoretically, to do away with the reactive current at all loads by using, as the source of current-supply for the converter, a suitably compounded alternator, giving an E.M.F. E, which increases with the load in such a way that the point C is always displaced on the right line of reference BY, while ε remains constant or else increases, according to the results desired. It is interesting to know how the excitation should be effected in such a case.

If it be required to have the direct-current voltage, ε, constant, the excitation will evidently be that of a simple shunt-winding; and its value is easy to determine by the condition that it must give exactly the voltage ε, with zero-load, since there is not to be any reactive current.

If, on the other hand, it be required to have the voltage rise with the load, the excitation must be compound. The shunt-winding will still be such that it, alone, gives the no-load E.M.F. ε, and the series-winding will be such that it will produce the additional ampere-turns necessary to give the E.M.F. ε' required with the load I_w. As already stated, in a case like this, it is desirable to decrease the reactance X, and the lag θ, to a minimum.

Regulation of Voltage by Varying the Reactance X in the Circuit. The two windings, in this case, will be determined in the same way as in the preceding case, since the voltage of the alternating current supplied to the converter is still regulated by outside means without causing lag in the current inside the machine. The excitation will be that produced by a simple shunt-winding when constant voltage is wanted, and that produced by a compound-winding, when a rising voltage is wanted. The shunt-winding will be determined by the voltage required with no load, and the series-winding will be determined by the voltage required with full load.

Possibility of Suppressing the Shunt-Winding. It is not absolutely necessary to use either a separate exciting winding, or else a shunt winding, as has been supposed hitherto; because, by making the number of armature-winding turns greater and decreasing the air-gap, it is always possible to obtain the necessary no-load excitation, merely by the effects of the armature-reaction. Eqs. (21) and (22), in that case, reduce to the following forms:

$$\frac{\pi kn}{\sqrt{2}}(I_m - j_0) = A_m. \quad \ldots\ldots\ldots \quad (21a)$$

$$\frac{KN'}{\sqrt{2}}\, i_0 = A_0. \quad \ldots\ldots\ldots \quad (22a)$$

It is not possible to obtain constant voltage or rising voltage, in the direct-current circuit with this method any more than when a shunt-winding is used, *without series-winding*. This method has, to some extent, the self-regulating properties of transformers, but to a degree which is inadequate. owing to the greater reluctance of the magnetic circuit.

On the other hand, the high values which the reactive currents must then be allowed to attain have the great objection of causing increased heating of the armature by increasing the total current in

the armature; and also of increasing the lag of the current in the supply-circuit, thereby reducing the output-capacity of the generator, increasing the voltage drop in the line, etc. In a word, this method has all the faults and objections due to heavy reactive currents, in addition to which the commutation of the machine is much less satisfactory, there being more tendency to sparking, owing to the shortened air-gap.

This method, which was used by certain concerns, has now been abandoned altogether. It is referred to here merely to show the ease with which it is possible to calculate the excitation for the conditions. obtaining in this case as well as in the preceding cases. This method has one advantage, however, that of making the satisfactory performance of the converter more independent of the form of the E.M.F. curve of the generator.

Conclusion. It has been shown, in what precedes, that the vector-diagram for synchronous motors, when referred to two axes, lends itself, as the result of natural simple extensions, to the study of the rotary converter and enables its conditions of operation to be analyzed for the most varied cases. This method of analysis serves to render more intelligible the very complex problem involved in the voltage regulation of the rotary converter, which is, perhaps, the most complicated problem presented in connection with alternating-current machinery.

The methods set forth in the preceding pages not only enable the phenomena to be foreseen qualitatively, but it also enables them to be calculated numerically, by combining the diagrams, or the equations which are derived from them, with the data obtained from experiments or tests.

CHAPTER III

STABILITY OF OPERATION OF ROTARY CONVERTERS

When in operation, rotary converters sometimes produce variations of voltage above and below their normal voltage; and they may also, at times spark, or even flash, at the commutator. These phenomena might be attributed to changes of frequency due either to variations of angular velocity of the prime mover driving the alternator which supplies current to the converters, or else due to oscillations of its speed-governor.

To eliminate these two causes of irregular operation, the following experiment was tried: the rotary converter was driven as a motor by current taken from a storage battery, which also excited the shunt-field winding. Three-phase currents taken from the A.C. side of the machine were passed through a raising transformer to raise their voltage. The voltage was then lowered by step-down transformers and the energy transmitted through them was absorbed by means of three identical rheostats. The arrangement was equivalent, practically, to connecting a three-phase inductive resistance to the secondary of the first transformer (Fig. 26). It was then observed that, on increasing gradually the load on the converter, a critical point was reached where its speed began to oscillate between limits which were all the farther apart the more the load was increased.

To explain this phenomenon, let us note that the converter was sending reactive currents into the circuit, and that these reactive currents tended to weaken the magnetic field. Now in the case of a load which is below a certain value(which we will term the limiting or critical load) the strength of this reactive current should decrease when the speed of the rotary converter increases, and reciprocally, it should increase when the speed decreases. The contrary effect should be produced when the load exceeds this critical load.

To demonstrate this, let us refer to a transformer diagram of the kind which takes magnetic leakage into consideration. It is known (Fig. 27) that if OY is the axis of E.M.F.'s the end of the vector (OM)

representing the strength of the primary current describes a semi-circle which cuts the axis of x at two points, A and B, such that $OA = i_{dv}$ represents the zero-load magnetizing current, and $OB = \frac{i_{dv}}{\sigma}$, where $\sigma = \frac{1}{v_1 v_2}$, in which v_1 and v_2 are the leakage coefficients for the two windings of the transformer.

By hypothesis, the voltage of the source of current-supply is constant. Therefore, an increase of speed of the converter, by causing an increase in the frequency, will bring about a decrease of the magnetizing current corresponding to zero-load. We will have a new circle,

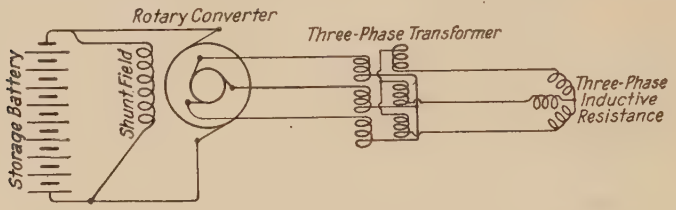


FIG. 26.

$A'B'$, whose diameter will be $i'_{dv}\left(\frac{1}{\sigma} - 1\right)$. The two circles will intersect each other at a point M, which is on the line drawn, from O, tangent to one of the circles.

It is seen, at once, on the diagram, that the right line ON, which represents the active current in relation to the primary current OM, indicates the critical load already mentioned. In fact, for the value OH, corresponding to a load which is lighter than the critical load, i.e., which makes $OK < ON$, it is seen that the reactive current has decreased from the value OL to the value OL_1, as the result of a speed-increment. On the contrary, for the current OP, corresponding to a load which is heavier than the critical load, i.e., which makes $OQ > ON$, the reactive current has increased from OR to OR_1. It is now easy to analyze the phenomena observed.

In the first place, when the load was lighter than the critical load of the machine, an increment of speed produced a decrease of the reactive current, and, consequently a decrease in demagnetizing action; and, since the converter was running as a direct current motor, with constant shunt-field-excitation, its total excitation-flux was increased. Therefore, the E.M.F. on the D.C. side being constant, the speed of

the converter had to decrease, in order to maintain the same induced E.M.F. The converter therefore varied in speed in order to maintain the induced E.M.F. constant.

In the second place, when the load was heavier than the critical load of the machine, an increase of reactive current was produced by a speed increment. Hence, there was a decrease in the total excitation flux. The speed of the motor therefore tended to increase until the current supplied from the battery was limited by the resistance of the circuits. As the armature of the converter was then in a state of

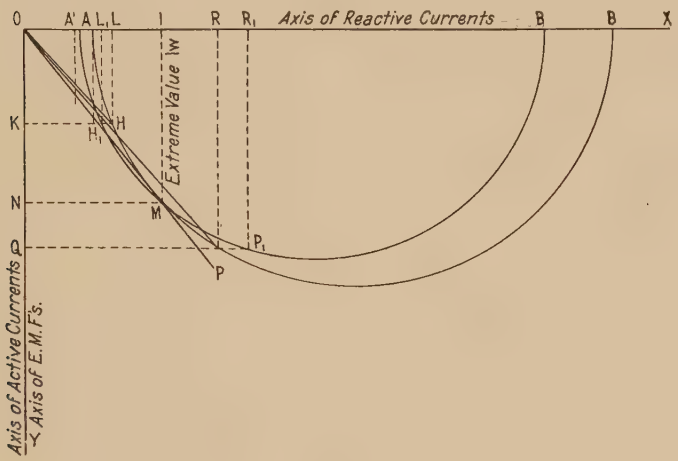


FIG. 27.

unstable dynamic equilibrium its speed soon began to decrease; and thus speed-oscillations were produced.

In practice, therefore, it is desirable that the load put on the rotary converter should not exceed the critical load above mentioned.

Let us see, now, what can be done to raise this load-limit. We will use the same diagram as before (Fig. 27).

We see readily that what is necessary is to increase the vector *ON*. This may be accomplished in different ways:

First, if the frequency is decreased, then, for a given voltage, other things being equal, the magnetizing current will have to be increased. Therefore the diameter of the circle corresponding to $i_{dv}\left(\frac{1}{\sigma}-1\right)$ will be increased, and, consequently, the point of contact M, of the line

which is tangent to this circle, is farther away from the axis Ox; hence ON is increased.

Second, if the coefficient of self-induction of the transformers is decreased, while the other quantities remain unchanged, the magnetizing current corresponding to zero-load will increase; hence ON is increased.

Third, if the magnetic leakage is decreased, then $\frac{1}{\sigma}$ increases, and, consequently, the diameter of the circle also increases, causing as before the point M to move farther away from Ox, and increasing ON.

To sum up, then, the limiting load on a rotary converter can be increased: 1, by diminishing the frequency of the A.C. source; 2, by reducing the self-induction of the transformers; 3, by reducing the magnetic leakage.

These facts have been all verified experimentally.

II. Let us now consider the rotary converter as a transformer of alternating into direct current.

The rotary converter used in the previous experiments was operated regularly at full load with the transformer connected to a suitable source of alternating current supply.

In order to regulate the voltage of the direct current produced by the converter, an (A.C) induction-regulator having a high coefficient of magnetic leakage was included in the primary circuit of the transformer. The rotary converter then became subject to speed oscillations of long period as soon as its load was increased to half the load which was carried without the induction-regulator. To explain this fact we shall show that every speed-increment of the rotary converter causes an increase in the difference of potential at its terminals.

Let us refer again to the converter-diagram. Let OA (Fig. 28), represent the difference of A.C. potential at the terminals of the converter; OC, the supply-voltage which, in this case, is constant; θ, the phase-angle between these two E.M.F.'s; AD, the active current, including the current corresponding to friction, eddy currents, etc., DC, the reactive current, counted as positive on the right side of the axis of currents BY, and counted as negative on the left side thereof.

Every increment of speed of the converter causes a decrease of the angle θ. Consequently the diagram shows that, OC being constant, the active current AD will decrease (to AD') and the reactive current (DC) will increase (to $D'C'$). Moreover, as in the

case of synchronous motors, there appears an E.M.F. which is $\frac{\pi}{2}$ in advance of the E.M.F. ε and which is equal to $\omega L' \Delta(I_w)$. But ΔI_w is itself negative (being equal to $AD' - AD = -DD'$); therefore, this E.M.F. (AF) is $\frac{\pi}{2}$ *behind* the E.M.F. ε. This E.M.F. here tends to *diminish* the increment of I_d, but it *increases* I_w still more.

The ampere-turns by which the E.M.F. ε is induced are equal to $A_d I \frac{KN'}{\sqrt{2}} I_d$, in which A_d represents the ampere-turns due to the constant excitation. It is seen at once, that whether the point C be at the right or at the left of the line of reference (BY) for current-

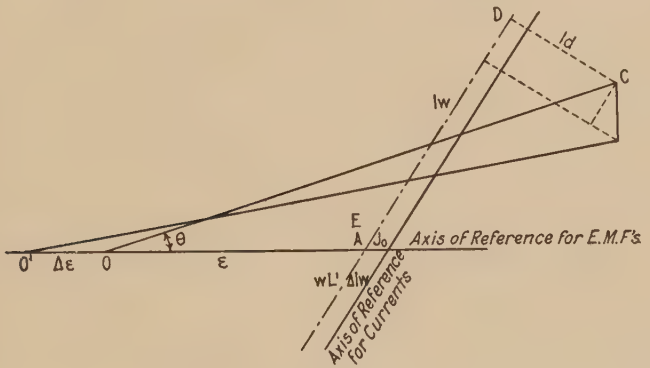


FIG. 28.

values, the value of the excitation for an increment of I_d is always equal to $A_d + \frac{KN'}{\sqrt{2}} \Delta(I_d)$; consequently, the excitation will increase in all cases, and so will the E.M.F. ε.

The power developed at the shaft of the rotary converter, when in normal operation is equal to

$$P = \varepsilon \left(I_w - \frac{\varepsilon}{\xi R} \right).$$

When the increment of speed occurs, the increase of power developed at the shaft is equal to

$$\Delta P = \Delta(\varepsilon I_w) - \frac{I}{\xi R} \Delta(\varepsilon^r) + \omega L' (\Delta I_w) I_d.$$

In order to have stability of operation we should have $\Delta P < 0$.

It is seen, from the diagram, that, at light loads, θ being then small, the increment of I_d is small in comparison with the increment of I_w, whereas with heavy loads, θ being then large, the reverse is the case.

The observed phenomenon can now be explained.

If the voltage at the terminals of the converter remained constant the torque would decrease when the speed increases. But each increase of voltage causes an increase of torque. At light loads the first effect is greater than the second; therefore, since the torque decreases, there is an automatic regulation of the speed.

Beyond a certain load, the second effect preponderates, and there is an increase in the torque. The converter therefore continues to gain in speed until the torque ceases increasing. It is then in a state of unstable equilibrium and it then loses speed until the torque ceases decreasing.

Experiment shows that the amplitude of the speed-oscillations increases rapidly with the load and that it then becomes impossible for the machine to operate. The oscillations can be diminished by taking advantage of the fact that the torque of the converter diminishes with its excitation. If we take a compound excitation comprising a shunt-winding connected to the terminals of the converter and a series-winding connected differentially, it will be seen that whenever there is an increase of voltage at the terminals of the converter, there will be an increase of the load-current. This current, acting upon the series-winding, will cause an increase in the series-excitation which is opposed to the shunt-excitation. Therefore the total excitation will decrease, and the torque will also decrease. These different effects can be calculated and adjusted so as to balance or overbalance the increase of torque. Actual test shows that this can be done, and that the converter can be made to carry full load even when the induction-regulator is used, if the series-winding is reversed. It appears, therefore, that, in the case of a rotary converter which is transforming direct current into alternating current, it is desirable to have a compound winding in which the shunt and series excitations *add* themselves, or act cumulatively.

On the other hand, a rotary converter serving to transform alternating current into direct current should have a compound winding in which the series and shunt excitations act differentially.

CHAPTER IV

OPERATION OF SEVERAL ROTARY CONVERTERS IN PARALLEL

THE parallel working of rotary converters is a much more difficult matter than the parallel working of ordinary generators, owing to the fact that the load has a tendency to distribute itself unequally, without any correcting action in the case of rotary converters.

Any increase of speed occurring in a converter causes the alternating current to lead in phase with respect to the induced E.M.F. This strengthens the magnetic field of the converter, and, consequently, causes the induced E.M.F. to be still further increased. Therefore, there is a tendency to unstable operation. To avoid the unequal distributions of load in rotary converters which are connected in parallel, they not only require careful attendance, but their design and construction demand much care, because the various converters intended for parallel working should, as nearly as possible, have the same armature-resistance and the same magnetic reluctance in the magnetic circuits of the field-magnets; in fact, the magnetic leakage and the magnetization curve should be substantially the same for all the machines.

It is often necessary to give up the attempt to compound the rotary converters and to use only shunt-wound rotary converters, for parallel working. In any case when compound windings are used it is almost always necessary to adjust these windings after the machines are in position, in order to make them run in balance at all loads. It is understood, of course, that an "equalizer" connection is necessary between the different machines as in the case of all other direct-current compound-wound dynamos connected in parallel.

It is more difficult still to make the converters run in balance when, instead of being coupled directly, they are coupled through feeders supplying current to a line which is common to all of them, as, for instance, a section of trolley or of third-rail conductor, in electric railway operation. In such cases it has been found necessary to

reduce the voltage-drop in the feeders to a small proportion, i.e., 2 to 3 per cent of the normal working voltage.

INHERENT OSCILLATIONS OR PUMPING OF CONVERTERS CONNECTED IN PARALLEL

Speed-oscillations can be produced in rotary converters, same as in synchronous motors, by periodic speed-variations in the alternators supplying them with current, when these alternators are driven by steam engines whose angular velocity varies at different portions of each piston-stroke; and they may also be produced by sudden changes or fluctuations of load. These speed-oscillations in rotary converters may even have a much greater amplitude than in other machines, because they are not opposed by a contrary variation of the torque acting upon the armature-shaft. As already stated, when the speed of a converter increases, its current is thrown out of phase and becomes leading with respect to the induced E.M.F. This causes the magnetic field to become stronger and the induced E.M.F., to rise; consequently, the secondary (direct current) output increases. The primary (input) power increases, however, in substantially the same proportion; hence, there is no other variation in the torque acting upon the armature shaft than that which can result from the variations of the losses with the load; and this effect is always quite small in comparison with the inertia of the armature.

The amplitude of the oscillations may, therefore, be very large; sometimes it is sufficient to make the converter "fall out of step." More often, its effects are made evident by fluctuations in the (D.C.) E.M.F. of the converter—which have amounted to as much as 30 per cent—and also by sparking at the commutator.

The E.M.F. fluctations are easily explained by the variations of speed and reactive current which occur simultaneously with them. The sparking is due to a much more complicated action. Owing to the inertia of the armature, every increase in speed, causing additional kinetic energy to be stored in the armature-mass, makes the primary (input) power greater than the secondary (output) power; and every decrease in speed, causing the armature to lose some of the kinetic energy stored in its mass, produces the contrary effect of making the secondary power greater than the primary power.

In both cases the transverse armature-reactions produced by the primary and secondary active currents no longer balance each other, and the axis of the resultant magnetic field oscillates from one side of

the line (diameter) of commutation to the other; and, as the brushes cannot follow these oscillations, sparking occurs, just as it does in the case of a dynamo whose brushes are not at the right position for sparkless running.

To overcome or diminish these difficulties it is desirable to avoid as much as possible using alternators driven by engines whose speed is unsteady or irregular, and to use prime movers giving a speed which is as uniform and constant as possible. However, speed-variations may be produced in the best-regulated steam engines and turbines, by sudden and large changes of load. For this reason it is desirable to apply the remedy to the converter itself, even though, in consequence, a small fraction of the useful energy may be lost.

This remedy, which is quite simple, is well known since the "dampers" of Hutin & Leblanc have been introduced. The principle involved is that the speed-oscillations themselves cause eddy currents to be produced either in the pole-pieces or else in "damping circuits" placed suitably around or about the pole-pieces.

A noticeable damping effect will already be obtained by merely making the pole-pieces of solid annealed wrought iron. The damping effect produced is not always sufficient, however; and as eddy currents are also produced in such pole-pieces in consequence of the irregularity of the revolving field caused by the two-phase or three-phase currents employed, the loss of energy due to these eddy currents is not always utilized in improving the stability of running of the converter.

In attempting to use laminated pole-pieces with the object of eliminating these eddy current losses and of improving the efficiency of the converter, it was soon found that the stability of operation of the converter was, thereby, made altogether insufficient. The proper stability was restored to the converter by placing copper or brass rings around the pole-pieces, or else simply by placing the field-excitation windings on copper spools or forms constituting a closed electric circuit around the field-cores and poles.

These "damper-circuits" are adjusted in such a manner as to reduce as much as possible the energy-loss produced by the currents induced in them, while at the same time obtaining the damping effect required. For example, Mr. Parshall has, in this way, adjusted the damping of several converters of 900 KW. capacity by connecting them in parallel on an artificial line. Even when supplied with currents by an alternator whose speed was irregular they operated satisfactorily with a loss of 3 per cent in the reactances interposed between them.

The loss in the damping circuits was then gradually decreased from 3 per cent to 0.6 per cent only, without the converters falling out of step. Experience shows that, as a general rule, the stability of the converter is better when the supply-current is lagging than when it is leading, probably because the induced E.M.F. is higher with leading currents. Much better results will be obtained, both in respect to the damping of speed-oscillations and in respect to ease of starting, by using the Leblanc dampers in their most perfect form, namely, in the form of grids consisting of copper bars or rods imbedded in the pole-pieces (parallel to the shaft) and connected to copper plates at both ends. (See part I, Fig. 62.) In most cases, however, to reduce the cost, a simpler form of damper is used.

USE OF ROTARY CONVERTERS FOR TRANSFORMING DIRECT INTO ALTERNATING CURRENT

Since rotary converters are reversible machines they can be employed, in certain cases, for producing alternating currents from direct current.

There may be cases where a small portion of the output of a direct-current generating plant must be transmitted to a considerable distance. In such cases the power to be transmitted can be converted into high-tension three-phase alternating currents which, after being transmitted to the receiving end, can be utilized there as alternating currents or else can be reconverted into direct currents. Again, in certain direct-current installations, it may be desirable to convert a portion of the power into three-phase alternating currents for operating certain apparatus, such as, for instance, for supplying induction-motors, in cases where commutator-motors would not be allowable. It is also possible and it may be desirable in certain cases to produce poly-phase alternating currents by means of a converter supplied with direct current from a storage battery.

In such cases the characteristics and the performance of the rotary converter are materially different from what they are in the cases previously considered, being similar in all respects to those of direct current motors.

In the first place the speed is not constant, but it depends upon the field-excitation, owing to the fact that the E.M.F. induced in the direct-current armature-winding must always be substantially equal to the E.M.F. of the source of (D.C.) current, less the ohmic drop. Hence, when the field-excitation is made stronger the speed decreases, and vice versa.

Moreover, the armature-reactions produced by the reactive current produce the same effect on the resultant total field-excitation. The alternating current produced by the converter, in such a case, generally lags behind the (A.C.) E.M.F., hence it weakens the magnetic field, and this, instead of causing a falling off in *voltage*, causes an increase in *speed*. If the A.C. circuit happens to contain a considerable amount of inductance, the reactance will increase with the frequency (due to the increase of speed); and the lag will increase with the reactance. The demagnetizing effect will therefore be increased. This will cause a further increase of speed, a further increase of frequency, and again a further increase of reactance; and the consequence of this cumulative process may be that the converter will "run away" or attain an excessive speed. It is therefore necessary, in such cases, to take special precautions, even including the use of safety-devices such as automatic safety stops, set so as to operate to open the (D.C.) supply-circuit whenever the converter speed exceeds a certain limit.

This want of stability can be avoided, and the converter can be made to operate under synchronous conditions, as before, by simply connecting to it (in parallel with its A.C. circuits), an alternator which is driven at constant speed by an engine or motor, and which has sufficient power to withstand any tendency to be itself driven as a motor by the current produced by the converter. In that case, the alternator regulates the frequency, and it "sets the step" for the converter, as it were, also furnishing to the converter the reactive current necessary to bring back the magnetic field to the normal value, whatever may be the excitation due to the field-windings.

Other Special Applications of Converters. As already stated, rotary converters may be used as motors for developing mechanical power equal to a poriton of the electric power supplied to them either on the A.C. or the D.C. side. In such a case, however, the acttve currents entering and leaving the converter will no longer balance each other, the difference between the energy-values which they represent being converted into mechanical energy. The "transverse" reactions no longer offset each other, and the heating effects are increased.

This case can also be studied by the graphical methods previously presented, because the diagram representing the conditions will be simply intermediate between the two diagrams shown in Figs. 27 and 28, corresponding, respectively, to a synchronous motor and to a converter. All that is needed is to change the lengths of the segment OB, making it equal to $\omega L'(I_w - I_w')$ instead of $\omega L' I_w$, in

which I_w' will represent the reactive current transformed into direct current.

It is also possible to use a rotary converter as a partial generator, by applying mechanical power to its shaft. This mechanical power is added to the electric power applied to the converter from the electrical source (A.C. or D.C.) supplying it. The power-output of the converter, in such a case, will be equal to the sum of the two power-inputs less the conversion-losses. This case can also be discussed easily by means of the preceding diagram.

There is still another case, namely, where a rotary converter is used entirely as a *generator*, for generating both alternating and direct currents. In such a case the machine is not operating as a rotary converter at all, but as a " double-current " or " omnibus " dynamo. The machine is then a particular kind of *alternator* and its characteristics should be studied as such. The reactions due to the two kinds of current no longer compensate, but add, themselves; hence there will be sparking at the brushes unless their position be shifted with the load; and the heating effects will be increased instead of decreased, as compared with the operation of the same machine as a rotary converter.

Phase-Converters. Instead of making a rotary converter produce direct current when supplied with three-phase current, it could be made to supply two-phase alternating current. The reactions occurring in such a machine would be very interesting, because they would be much more complicated than in the case of ordinary rotary converters. In fact, the (two-phase) currents generated by the machine would no longer have a definite, fixed, phase-relation with the E.M.F., as in the case of the direct current, but they would have a lag which would vary with the reactance in the external circuit.

It could again be seen easily, by the same reasoning as before, that since the power applied to the machine, and the power delivered by it have nearly the same values, the transverse reactions due to the active current must practically offset or balance each other, leaving only the direct reactions due to the reactive currents. The latter can be evaluated separately and we can thus obtain, for each value of the power output and of the lag in the secondary circuit, the resultant magnetomotive force produced by the reaction of the armature on the magnetic circuit.

CHAPTER V

VOLTAGE RATIO IN SYNCHRONOUS CONVERTERS WITH SPECIAL REFERENCE TO THE SPLIT-POLE CONVERTER

Résumé of paper by C. A. Adams, Proc. A.I.E.E., June, 1908

The author of this paper shows how the ratio of the D.C. to the A.C. voltage of a synchronous converter may be varied by means of the split pole, without introducing seriously large harmonics into the counter E.M.F. and current. Incidentally a new method is employed for calculating the E.M.F. between adjacent commutator brushes—the *brush E.M.F.*—and the corresponding E.M.F. between collector rings—the *tap E.M.F.*—for any given flux distribution.

The elementary E.M.F. induced in a single conductor cutting through the gap-flux at constant speed is proportional to the density of the flux, and when plotted, will obviously have the same shape as that of the gap-flux curve. It may be expressed thus:

$$e' = a_1 \left| \begin{array}{l} \sin \omega t + q_{a3} \sin 3\omega t + q_{a5} \sin 5\omega t + \ldots \\ \qquad + q_{am} \sin m\omega t + \text{etc.} \\ q_{b1} \cos \omega t + q_{b3} \cos 3\omega t + q_{b5} \cos 5\omega t + \ldots \\ \qquad + q_{bm} \cos m\omega t + \text{etc.} \end{array} \right. \quad . \quad . \quad . \quad (1)$$

where $\omega = 2\pi n$ and n is the fundamental frequency.

The brush voltage which is proportional to the area of the e' curve between brushes, is

$$E_{dc} = N\frac{2}{\pi}a_1[1 + \tfrac{1}{3}q_{a3} + \tfrac{1}{5}q_{a5} + \tfrac{1}{7}q_{a7} + \tfrac{1}{9}q_{a9} + \text{etc.}] \quad . \quad . \quad (2)$$

or

$$E_{dc} = E_1[1 + \tfrac{1}{3}q_{a3} + \tfrac{1}{5}q_{a5} + \tfrac{1}{7}q_{a7} + \text{etc.}] \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where N is the number of conductors in series between brushes.

The tap E.M.F. is the geometrical sum of the E.M.F.'s of the several coils connected in series between taps. The E.M.F. of each

coil is in turn the geometrical sum of the E.M.F.'s of its two sides, which E.M.F.'s may or may not be in phase according to whether or not the coil pitch is equal to the pole pitch. The fundamental E.M.F.'s of the two sides of each coil will differ in phase by the deficiency in coil pitch measured in electrical degrees, and the fundamental E.M.F.'s of the coil sides in two adjacent slots will differ in phase by the electrical angular pitch of the slots. In each case the phase difference between the mth harmonics is m times that between the fundamentals.

In calculating the tap E.M.F. the author introduces the "differential factor," the ratio of the resultant or geometrical sum to the arithmetical sum of the various E.M.F.'s.

Let N_{sp} = slots per pole, and p' the number of belt spans per electrical circumference ($p' = 2$ for diametral connection and 3 for three-phase delta or six-phase double delta); then $\frac{\pi}{N_{sp}}$ = the phase difference between the fundamental E.M.F.'s of two adjacent slots, $\frac{m\pi}{N_{sp}}$ = the phase difference between the corresponding mth harmonics and $\frac{2N_{sp}}{p'}$ = slots per belt. The total phase rotation (or combined phase displacement) of the mth harmonic in all the slots of the belt will be

$$\frac{m\pi}{N_{sp}} \times \frac{2N_{sp}}{p'} = \frac{2m\pi}{p'},$$

and the resultant of the $\frac{2N_{sp}}{p'}$ mth harmonics is proportional to $2 \sin \frac{m\pi}{p'}$. By the same constant a single mth harmonic is proportional to $2 \sin \frac{m\pi}{2N_{sp}}$, and the arithmetical sum of the $\frac{2N_{sp}}{p'}$ mth harmonics is

$$\frac{2N_{sp}}{p'} \times 2 \sin \frac{m\pi}{2N_{sp}}.$$

The differential factor for the mth harmonic of the belt E.M.F.; or the *belt-differential factor* for the mth harmonic is then

$$k_{dm} = \frac{p'}{2N_{sp}} \frac{\sin \frac{m\pi}{p'}}{\sin \frac{m\pi}{2N_{sp}}}, \qquad \ldots\ldots (4)$$

and that for the fundamental

$$k_{d1} = \frac{p'}{2N_{sp}} \cdot \frac{\sin \frac{\pi}{p'}}{\sin \frac{\pi}{2N_{sp}}}. \quad \ldots \ldots \quad (5)$$

The ratio $k_{dm} \div k_{d1}$ is the relative reduction of the mth harmonic by belt-differential action; it may conveniently be called the *belt-reduction factor*, and is

$$k_{rm} = \frac{k_{dm}}{k_{d1}} = \frac{\sin \frac{m\pi}{p'}}{\sin \frac{\pi}{p'}} \cdot \frac{\sin \frac{\pi}{2N_{sp}}}{\sin \frac{m\pi}{2N_{sp}}}. \quad \ldots \ldots \quad (6)$$

The first part of k_{rm}, namely, $\dfrac{\sin \frac{m\pi}{p'}}{\sin \frac{\pi}{p'}}$, is independent of N_{sp} and is equal to 1, except for $m=3$, 9 or 15, when it is zero for $p'=3$, and 2 for $p'=6$.

The following table gives the value of the second part of k_{rm}, which is equal to k_{rm} except for the cases above cited.

TABLE II

TABLE OF $\sin \frac{\pi}{2N_{sp}} \div \sin \frac{m\pi}{2N_{sp}}$

$N_{sp}=$	8	10	12	14	16	18	20	22	24	26	∞
$m=3$	0.351	0.344	0.340	0.338	0.3368	0.3362	0.3358	0.3355	0.3353	0.3351	$\frac{1}{3}$
$m=5$	0.235	0.222	0.214	0.2101	0.2081	0.2062	0.2051	0.2042	0.2034	0.2029	$\frac{1}{5}$
$m=7$	0.199	0.1755	0.1646	0.158	0.154	0.152	0.150	0.149	0.1477	0.1467	$\frac{1}{7}$
$m=9$	0.198	0.158	0.141	0.132	0.126	0.123	0.1208	0.119	0.1177	0.1166	$\frac{1}{9}$
$m=11$	0.2345	0.1583	0.1316	0.1185	0.1111	0.106	0.103	0.1009	0.0992	0.0979	$\frac{1}{11}$
$m=13$	0.351	0.1755	0.1317	0.1124	0.102	0.961	0.0920	0.08916	0.08702	0.08533	$\frac{1}{13}$
$m=15$	0.995	0.2214	0.1413	0.1126	0.0986	0.0903	0.08492	0.0813	0.07865	0.07665	$\frac{1}{15}$
$m=17$	1.000	0.3444	0.1644	0.1185	0.0985	0.0875	0.0807	0.0761	0.0729	0.0706	$\frac{1}{17}$
$m=19$	0.351	1.000	0.214	0.132	0.100	0.0875	0.0788	0.073	0.069	0.0662	$\frac{1}{19}$

As k_{rm} is in most cases much less than one, it appears that the harmonics are much more reduced by the belt-differential action than is the fundamental.

In the case of a fractional pitch or short-chord winding, there is a phase difference between the E.M.F.'s in the two sides of each coil, or in the ordinary two layer winding, this same phase difference exists between the E.M.F.'s of the two belts or groups of coil sides, which together make up one circuit between taps. This introduces another reduction of the resultant E.M.F. If λ_c is the coil pitch in terms of full pitch, the *pitch differential factor* between the two belts is, for the fundamental (see Fig. 29), $\overline{OB} \div (\overline{OA} + \overline{OB}) = \sin \frac{\pi}{2}\lambda_c$, and for the mth harmonic is $\sin \frac{m\pi}{2}\lambda_c$. The ratio of the latter to the former will be designated the *pitch reduction factor* for the mth harmonic; it is

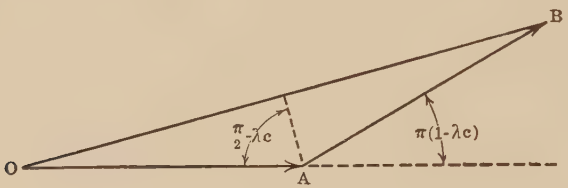


FIG. 29.

$$k_{pm} = \frac{\sin m\frac{\pi}{2}\lambda_c}{\sin \frac{\pi}{2}\lambda_c} = \frac{\text{per cent. of } m\text{th harmonic in fractional pitch winding}}{\text{per cent. of } m\text{th harmonic in full pitch winding}}. \quad (7)$$

This is plotted in Fig. 30 for various pitches and harmonics (signs disregarded).

If the phase belt begins and ends in the middle or within a coil or slot bundle, there is a still farther reduction of the resultant E.M.F. The corresponding *fractional slot reduction factor* is

$$k_{sm} = \left\{ \begin{array}{l} \cos \frac{m\pi}{2N_{sp}} \text{ (for half-slot connection).} \\ \cos \frac{m\pi}{2N_{sp}} \div \cos \frac{m\pi}{6N_{sp}} \text{ (for a one-third or a two-thirds slot connection).} \\ \cos \frac{m\pi}{2N_{sp}} \div \cos \frac{m\pi}{4N_{sp}} \text{ (for a one-fourth or a three-fourths slot connection).} \end{array} \right\} \quad (8)$$

The final ratio of the per cent mth harmonic in the phase E.M.F. to the per cent mth harmonic in the flux distribution curve is

$$k_m = k_{rm} \times k_{pm} \times k_{sm} = \frac{\sin \frac{m\pi}{2}\lambda_c}{\sin \frac{\pi}{2}\lambda_c} \times \frac{\sin \frac{m\pi}{p'}}{\sin \frac{\pi}{p'}} \times \frac{\sin \frac{\pi}{2N_{sp}}}{\sin \frac{m\pi}{2N_{sp}}} \times k_{sm}. \quad . \quad (9)$$

It remains to consider the phases of the harmonics in the resultant E.M.F.

The total phase rotation of the fundamental is $\frac{2\pi}{p'}$, and the phase

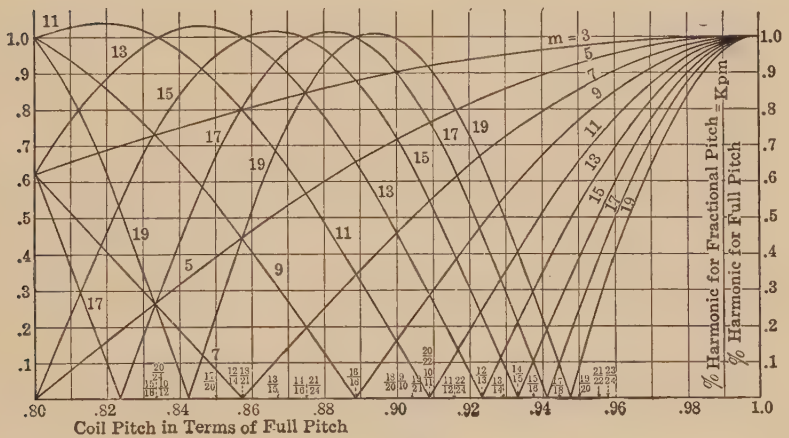


FIG. 30.

of the fundamental of the resultant E.M.F. with respect to the fundamental of slot No. 1 is

$$\alpha_1 = \frac{\pi}{p'} - \frac{\pi}{2N_{sp}}.$$

The total phase rotation of the mth harmonic is $\frac{2m\pi}{p'}$.

Let w = nearest whole number less than $\frac{m}{p'}$ and w_s = nearest whole number less than $\frac{m}{2N_{sp}}$. Then the equivalent phase rotation of the

mth harmonic is $2\pi\left(\frac{m}{p'}-\omega\right)$, and the phase of the mth harmonic of the resultant E.M.F. with respect to the mth harmonic of slot No. 1 is, in mth harmonic radians,

$$\alpha_m=\pi\left(\frac{m}{p'}-\omega\right)-\pi\left(\frac{m}{2N_{sp}}-\omega_s\right).$$

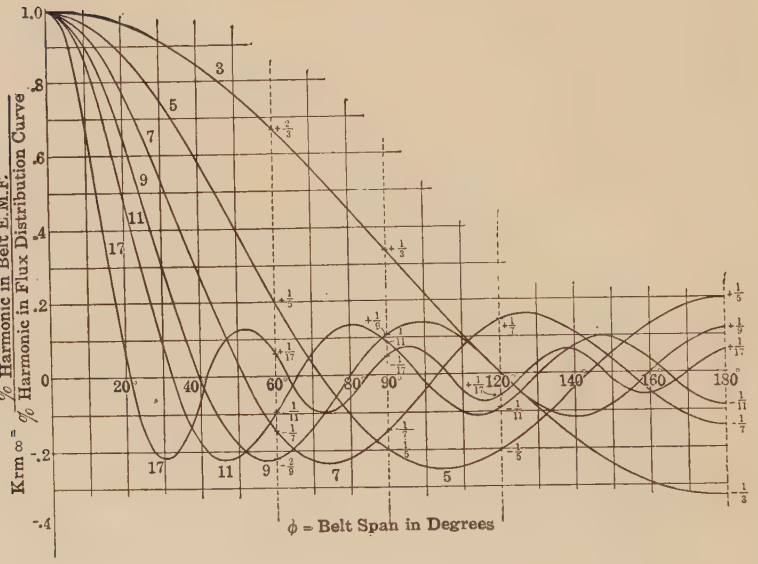


FIG. 31.

Thus in passing from the slot E.M.F. to the resultant E.M.F., the relative phase of the mth harmonic with respect to the fundamental is changed by an amount (in mth harmonic radians),

$$\beta_m=\alpha_m-m\alpha_1=-\pi(\omega-\omega_s).$$

β is therefore always a multiple of π. Moreover, ω_s is practically always zero. Therefore, β_m is zero when ω is even and π when ω is odd; i.e., the mth harmonic can only be reversed and not otherwise changed in phase from the slot E.M.F. In order to see this relation more clearly, consider an infinite number of slots, and

designate the angle of belt span by ϕ. The belt reduction factor is then

$$k_{rm\infty} = \frac{1}{m} \sin \frac{m\phi}{2} \div \sin \frac{\phi}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which is plotted in Fig. 31 for different values of the belt span (ϕ) and for several harmonics. The reversal of the various harmonics with the change in belt span is here shown clearly.

Tap Voltage

For this purpose Eq. (1) may be written:

$$e' = a_1[q_1 \sin(\omega t + \theta_1) + q_3 \sin(3\omega t + \theta_3) + q_5 \sin(5\omega t + \theta_5) + \ldots q_m \sin(m\omega t + \theta_m) + \text{etc.}] \quad . \quad (11)$$

where

$$q_m = \sqrt{q_{am}^2 + q_{bm}^2} \quad \text{and} \quad \theta_m = \tan^{-1} \frac{b_m}{a_m}.$$

The mth harmonic in the tap E.M.F. is then

$$k_m q_m \sin(m\omega t + \theta_m + \pi\omega).$$

The amplitude of the fundamental is

$$a_{t1} = k_{d1} \frac{2N}{p'} q_1 a_1,$$

where k_{d1} may be taken as equal to $\frac{p'}{\pi} \sin \frac{\pi}{p'}$ the limiting value of Eq. (5), as N_{sp} approaches infinity. Then

$$a_{t1} = \frac{2}{\pi} N a_1 q_1 \sin \frac{\pi}{p'} = q_1 E_1 \sin \frac{\pi}{p'}, \quad . \quad . \quad . \quad . \quad (12)$$

and the complete tap voltage is

$$e_t = q_1 E_1 \sin \frac{\pi}{p'} [\sin \omega t + k_3 q_3 \sin (3\omega t + \theta_3 + \pi\omega) + k_5 q_5 \sin (5\omega t + \theta_5 + \pi\omega) + \text{etc.}] \quad . \quad (13)$$

The *root mean square tap voltage* is then

$$E_t = \frac{q_1 E_1}{\sqrt{2}} \sin \frac{\pi}{p'} \sqrt{1 + k_3^2 q_3^2 + k_5^2 q_5^2 + k_7^2 q_7^2 + \text{etc.}}, \quad . \quad . \quad (14)$$

and the *ratio of brush to tap voltage is*

$$K_{bt} = \frac{E_{dc}}{E_t} = \frac{\sqrt{2}}{q_1 \sin \frac{\pi}{p'}} \times \frac{(1 + \frac{1}{3} q_{a3} + \frac{1}{5} q_{a5} + \frac{1}{7} q_{a7} + \text{etc.} \ldots)}{\sqrt{1 + k_3^2 q_3^2 + k_5^2 q_5^2 + k_7^2 q_7^2 + \text{etc.}}}, \quad (15)$$

or

$$K_{bt} = \frac{\sqrt{2}}{q_1 \sin \frac{\pi}{p'}} K_q, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

when

$$K_q = \frac{(1 + \frac{1}{3} q_{a3} + \frac{1}{3} q_{a3} + \frac{1}{5} q_{a5} + \text{etc.})}{\sqrt{1 + k_3^2 q_3^2 + k_5^2 q_5^2 + k_7^2 q_7^2 + \text{etc.}}}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Three-part Pole

With a normal flux distribution, or with a three-part pole symmetrical distortion, the q_b's are zero, $q_1 = 1$ and $q_m = q_{am}$. The denominator of K_q is nearly unity, always a little greater. The numerator may differ considerably from unity on either side, according as the q_s's are positive or negative, slightly less than unity for undistorted flux distribution. Thus K_{bt} will be slightly less than as given by the usual approximate formula. Any variation in the q_a's, due to a symmetrical distortion, causes a variation in the voltage ratio, but also in the harmonics of the tap E.M.F.'s.

The relation between these two changes is shown in Fig. 32 for the 180° E.M.F., and for the third, seventh, eleventh, and seventeenth harmonics, each being taken separately.

In order to change the relative voltage ratio from 1 to .9 by third harmonic control, involves a 10 per cent third harmonic in the diametral tap E.M.F., which corresponds to about 30 per cent third harmonic in flux distribution curve. To make the same change by seventeenth harmonic control, involves a 12 per cent seventeenth

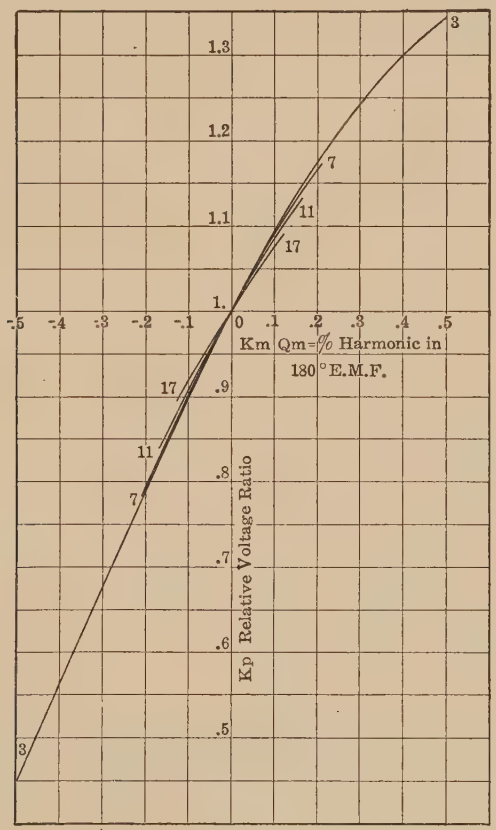


FIG. 32.

harmonic in diametral tap E.M.F., and a seventeenth harmonic in the flux distribution curve of $\frac{12}{k_{r17}}$ per cent. For twenty slots per pole, this would mean a 150 per cent seventeenth flux harmonic. A combination of harmonics produces about the same result as a single one of the same total per cent magnitude.

The use of fractional pitch windings and fractional-slot connections, will reduce some of the harmonics; but as commutation limits the pitch reduction to a comparatively small range, only the higher harmonics are affected.

If the voltage ratio could be under control by a pure third harmonic in the flux distribution curve, the resulting third harmonic in the phase E.M.F. could be eliminated from the line voltage by connecting the transformer primaries in star.

Two-part Pole

This gives unsymmetrical distortion, and a lateral shifting of the flux distribution curve; i.e., the q_b's are no longer zero, particularly

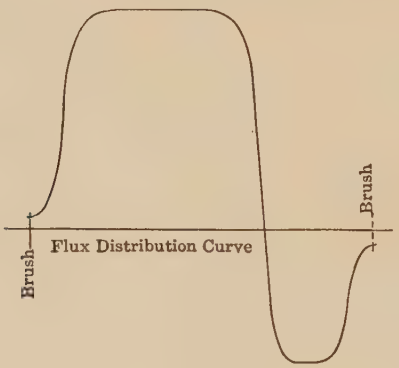


Fig. 33.

q_{b1}. Thus $q_1 = \sqrt{1+q_{b1}^2}$ is no longer unity, and $K_b t$ is changed by q_1 as well as by K_q. If these changes are in the same direction, the harmonics in the tap E.M.F. will be obviously less for a given distortion.

In an actual machine for which the flux distribution curve is shown in Fig. 33, the voltage was

$$\frac{E_{dc}}{E_{t_{180°}}} = \frac{\sqrt{2}}{q_1} \frac{(1+\frac{1}{3}q_{a3}+\frac{1}{5}q_{a5}+\text{etc.})}{\sqrt{1+k_3^2q_3^2+k_5^2q_5^2+\text{etc.}}} = \frac{\sqrt{2}}{1.295}\frac{0.862}{1.016} = \sqrt{2}\times 0.655,$$

less than two-thirds of the ratio for a sinusoidal flux distribution curve, and just about two-thirds of that for a normal undistorted

field. The total harmonics were 21.6 per cent in the 180° E.M.F., and only 7.2 per cent in the 120° E.M.F., which are considerably less than is possible with a three-part pole. Moreover, it is quite possible that a more careful design of the pole faces might result in a farther reduction in the harmonics of the tap E.M.F.'s. Fractional pitch winding and fractional slot connections, would reduce them still slightly farther.

PART III

METHODS OF CALCULATION OF THE ARMATURE REACTIONS (DIRECT AND TRANSVERSE) OF ALTERNATORS

CHAPTER I

METHODS OF CALCULATION OF THE ARMATURE REACTIONS (DIRECT AND TRANSVERSE) OF ALTERNATORS [1]

THE author here proposes to explain and complete the theory of "two armature reactions," which was enunciated by him several years ago,[2] and which has recently been adopted, with slight modifications, by M. Rey,[3] M. R. V. Picou,[4] and M. Guilbert,[5] in France; Professor Arnold [6] in Germany; and Mr. Herdt [7] and Messrs. Hobart and Punga [8] in the United States. The notable authority of all these

[1] A paper by Prof. André Blondel, *Ecole des Ponts et Chausées*, presented before the International Electrical Congress at St. Louis, in 1904. Reprinted from the Transactions, Vol. I, pp. 635–668.

[2] "On the empirical theory of alternators," *L'Industrie Electrique*, Nov. 10 and 25, 1899. This is the first publication in which the reaction in alternators was analyzed, and possesses undisputable priority over all those which are mentioned below on the subject of the two reactions.

[3] M. Rey. *Rapports*, International Congress of Electricians, 1900.

[4] M. R. V. Picou. *Bulletin de la Société Internationale des Electriciens* July, 1902.

[5] C. F. Guilbert. *Eclairage Electrique*, March 7 and 14 and April, 1903, and *La Révue Technique*, June, 1903.

[6] E. Arnold. *Elek. Zeit.*, 1902, page 250. Arnold, as pointed out farther on, has reduced the generality of the method, in contradistinction to the other authors mentioned.

[7] L. A. Herdt. *Trans. Amer. Inst. El. Eng.*, May, 1902, and *Eclairage Electrique*, February 14, 1903.

[8] Hobart and Punga. *Trans. Amer. Inst. El. Eng.*, April 22, 1904.

authors in the matter of dynamo-machine construction has made me read their communications with great interest, and as I have observed that in certain cases my own view has not been well understood, I consider it desirable to present certain supplementary considerations to make this theory still more simple and to complete it finally. At the outset it should be pointed out that my diagram should not be considered as belonging to the category of E.M.F. but rather to that of ampere-turn diagrams. The two classes are often equivalent, because if one commences with E.M.F.'s one proceeds with fluxes, and ends necessarily with ampere-turns. But I desire to reduce to a minimum the complication of considerations relative to the saturation of field-magnets, of which I fear the difficulties have been needlessly exaggerated.

In what follows I will refer first, very briefly, to the essential points of my method of 1899, and I will show in what points it has been improved, or is susceptible of improvement.

Part I. Diagram of Operation.

Principles of the Theory of Two Reactions. I have long been surprised that polyphase alternators and direct-current machines have not been treated from the point of view of reaction, since these phenomena are fundamentally of absolutely the same order, since the dephasing alternating current produces effects of the same order as the displacing of the brushes in a direct-current dynamo. It is known that in the latter case the displacement causes a direct magnetic reaction to be developed, whilst in the neutral position there is only a transverse reaction. By similar reasoning as to the automatic dephasing of alternating currents and of the property which polyphase currents possess of being decomposable into active and reactive components, I have been led to the following proposition:

When an alternator supplies a current dephased by an angle ψ with respect to the internal induced E.M.F., the armature reaction may be considered as the resultant of a direct reaction produced by the reactive current $\sin \psi$ and a transverse reaction due to the active current $I \cos \psi$.

In addition to the above, the stray magnetic fields must be taken into account. These are proportionate to the currents and in phase with them. We will consider them later on.

The second fundamental proposition of this theory is the following:

The two reactions (direct and transverse) and the stray flux take

place in three different magnetic paths; only the direct reaction acts in the main circuit of the field magnets, while the transverse reaction and the stray fields act, in general, upon circuits of low magnetic density.

The conclusion which I have drawn from the above is that, in general, the direct reaction should be expressed as a counter M.M.F.; that is to say, by a number of ampere-turns equivalent to the demagnetizing effect of the armature.[1] The total armature M.M.F. per complete magnetic circuit or per pair of poles is

$$\frac{KN}{2} I\sqrt{2},$$

where K is a coefficient of reduction and N the armature conductors per pair of poles. I have formally given for asynchronous motors a practical value which is approximately the same for alternators, viz.:

$$K = \left(\frac{2}{\pi}\right)^2 k,$$

k being the coefficient of reduction which appears in the formula of E.M.F. written under the form

$$E = \frac{k\omega N\phi \times 10^{-8}}{2\sqrt{2}}.$$

Here N is the number of peripheral conductors for one phase and ω the velocity of pulsation. It is the direct reaction which produces, almost entirely, the variation of terminal voltage. As to the transverse reaction and the reaction of stray fields, with the assumption that the armature is unsaturated, as I assumed and as M. Guilbert also assumes, they may be expressed simply by the coefficients of self-induction l and s.

More recently I have indicated [2] that the transverse reaction could also easily be expressed in ampere-turns.

The analysis of the phenomena taking place in the alternator leads therefore to a new proposition, formulated in my articles of 1899.

[1] No notice is here taken of one of the cases considered by the author in 1899; namely, that in which all the machine is well below saturation, because it is only susceptible of very rare applications; moreover, it has been treated with more detail by M. Jean Rey in a very interesting communication presented to the Congress of 1900, in which the reader will find an interesting example of a calculation of reactive coefficients in a machine actually built by this method, which has since been followed by various authors.

[2] "Theory of Synchronous Motors," Vol. I. Paris, Gauthiers-Villars, 1900.

The dephasing ψ of the current is regulated entirely by the numerical value of the transverse reaction, which, on the contrary, has little effect upon the E.M.F.

This proposition has been demonstrated in the case of unsaturated armatures, as I have just stated, but it is general and remains in effect even in the case where the circuit of the transverse reaction approaches saturation. The demonstration of this will be given below.

Diagram of E.M.F.'s and Currents of an Alternator with Unsaturated Armature and with Saturated Field Magnet. The diagram in Fig. 1 [1] reproduces Fig. 5 of my first paper, supplemented by the

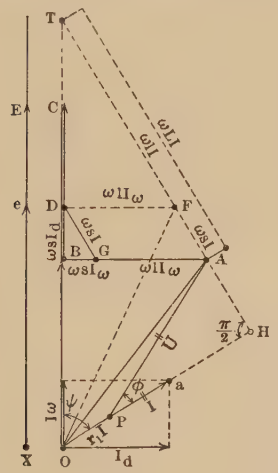


FIG. 1.

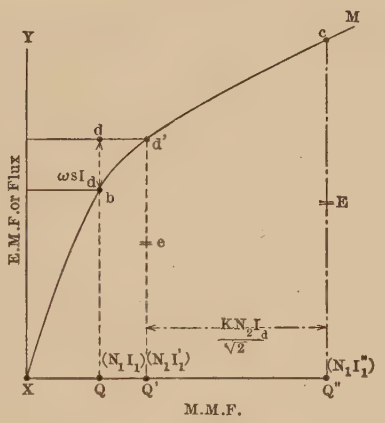


FIG. 2.

definitions of Fig. 2. r' represents the apparent resistance, that is to say, the ohmic resistance augmented by the effects due to Foucault currents; ϕ is the difference of phase in the external circuit, and ψ is the difference of phase with respect to the internal E.M.F. It is proposed to calculate the excitation necessary to develop an E.M.F. U at the terminals under a current-delivery I dephased by ϕ. We have $OP = r'I$ and $PA = U$, with the angle $APa = \phi$.

Let OT be the direction, as yet unknown, of the internal E.M.F. e; the perpendicular AB let fall from A upon OT is the sum of the transverse reaction:

$$AG = \omega l I \cos \psi,$$

[1] The subscript w indicates the *active* or energy component and the subscript d the *reactive* or quadrature reactive component.

(where l is the transverse self-inductance and $\omega(=2\pi f)$ the speed of pulsation); and of a part of the stray field reaction—

$$GB=\omega sI\cos\psi^1=\omega sI_w$$

where s is the self-induction of the stray fields.

The segment GD perpendicular to I represents the E.M.F. of reaction of the stray fields, ωsI, and the segment

$$BD=\omega sI\sin\psi=\omega sI_d$$

represents the second component of the stray-field reaction; thus in OD is obtained the value of the effective E.M.F. e, which should be obtained by the resultant excitation.

The value of the angle ψ is determined by expressing simply the relations between the elements of the figure. Let us analyze the broken line $OPAB$ into components upon OB, and BA; whence

$$e=r'I\cos\psi+Ul\sin(\psi-\phi)$$

$$\omega(l+s)I\sin\psi=r'I\sin\psi+U\sin(\psi-\phi)$$

and

$$tg\,\psi=\frac{U\sin\phi+\omega(l+s)I}{U\cos\phi+r'I}.$$

The angle of real dephasing ψ is thus determined solely by a knowledge of the transverse reaction. This equation, which was given by the author in 1899, is evidently equivalent to the following construction.

From the point A a perpendicular AH is drawn to the direction of the current I, and a segment AF is drawn upon this line equal to ωsI; then a segment $FT=\omega lI$; finally the point O is joined to the point T, and thus is obtained the angle ψ and the position of the required vector OD representing the total effective E.M.F. e. To determine the necessary ampere-turns for the production of this E.M.F., it is only necessary to employ the open circuit characteristic or saturation curve of the alternator.

[1] The segment BT intercepted by OT will evidently be equal to

$$BD+DT=\omega lI\sin\psi+\omega sI\sin\psi.$$

Diagram of Ampere-Turns in the Case of Unsaturated Armature. The consideration of ampere-turns does not need to appear in the method, as is evident in the case of an unsaturated armature, until after having traced the diagram of E.M.F.'s. The excitation ampere-turns are drawn, if desired, along the direction of the vector OC, in order to facilitate certain comparisons; but the calculation of ampere-turns is no longer in this method a vectorial, geometric calculation, but a scalar calculation, and may be made upon the saturation curve as calculated or observed, which represents the induced E.M.F. (or the useful flux traversing the armature) as a function of the total ampere-turns applied on the field magnets.

Their determination is based on the following facts:

(1) The reactive counter-ampere-turns of the armature are proportional to the number of peripheral conductors on the armature per pair of poles, N_2, which supply a number of turns equal to $\frac{N_2}{2}$; but these turns do not act in unison, partly because they belong to different phases, and partly because they are not in the same slots. For this reason it is necessary to apply a reduction-coefficient K. The effective reactive current $I_2 \sin \psi$ thus gives rise to an M.M.F. exactly opposite to that of the field magnets, and having for its magnitude in counter-ampere-turns

$$CAT = \frac{KN_2 I_2 \sin \psi \sqrt{2}}{2} = \frac{KN_2 I_d}{\sqrt{2}}.$$

(2) These counter ampere-turns act, on the one hand, upon the circuit common to the field magnets and the armature; and on the other hand, upon the circuit of the armature and of the stray magnetic fields. This may be represented diagrammatically as in Fig. 3, in which the full lines $ABCDEF$ represent the principal magnetic circuit with its two M.M.F.'s acting in the opposite directions $N_1 I_1$ and $\frac{KN_2 I_d}{\sqrt{2}}$, and the dotted lines AD AD indicate the stray circuits in which the lines of force escape, either between the teeth of the armature f_2 or between the polar horns f_1. In reality the stray fields f_1 of the field magnet are not concentrated along any single path, but

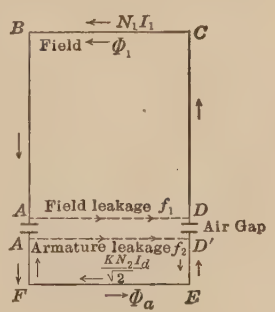


FIG. 3.

are spread out more or less over the entire length of the principal circuit up to its entrance into the armature turns. This fact is, however, unimportant, as has recently been shown by M. Guilbert (see *Eclairage Electrique*, December, 1903).

(3) The self-inductance of the armature is produced by the stray fields f_2, supposedly attributed to the effect of the armature. If we call R_{f_2} the reluctance of the circuit of the stray fields f_2, and R_a the reluctance of the armature, the stray field produced by the armature across itself, is expressed in practical units (N_2 being the number of peripheral conductors per pair of poles),

$$f_2 = 0.4\pi \left\{ \frac{KN_2I_2\sqrt{2}\sin\psi}{2(R_a+R_{f_2})} \right\} = \frac{0.2\pi KN_2I_2\sqrt{2}\sin\psi}{R_{f_2}},$$

assuming that we can neglect R_a with respect to R_{f_2}, and thus produce an E.M.F.

$$E_{f2} = \frac{kN_2\omega f_2}{2\sqrt{2}} = \frac{kN_2\omega \times 0.1\pi KN_2I_d}{R_{f_2}}.$$

It results from this that the E.M.F. of self-induction that we have called ωsI_d can be considered as produced simply by a stray field f_2, which is added to the stray field of the field magnets f_1.

Upon the saturation curve XM (Fig. 2) defined as above, the point b which corresponds to the E.M.F. OB (Fig. 1), represents the NI necessary to force the useful flux through the field magnets into the armature. Adding to the flux Φ_a the stray field of the armature f_2, there is obtained the virtual E.M.F. $OD=e$, corresponding to the total flux emanating from the poles into the air-gap; the corresponding abscissa XQ' represents the necessary field-winding ampere-turns N_1I_1', without taking into account the increase Δf_1 of the stray field f_1 of the field magnet.

(4) The stray field of the field magnets f_1 is inversely proportional to the reluctance of the stray path R_{f_1} between the poles and directly to the difference of magnetic potential between the poles. This latter is formed of two parts; one part is the drop of magnetic potential necessary to force the flux through the armature and air-gap, the other part, the reactive counter-ampere-turns of the armature calculated as above.

(5) Every increase in the ampere-turns of the field magnet increases

the stray flux f_1 of the field magnet, in a manner sensibly proportional to the increase of the ampere-turns of the field. If, therefore, the field-magnet ampere-turns are increased by

$$KN_2I_d\frac{\sqrt{2}}{2},$$

in order to compensate for the counter-ampere-turns, CAT, of the armature, the stray flux f_1 produced by the field magnet following the circuit $BADC$ would be increased by a quantity,

$$\Delta f_1=\frac{0.2\pi KN_2I_d\sqrt{2}}{R_1+R_{f_1}}=\frac{0.2\pi KN_2Id\sqrt{2}}{R_{f_1}}\text{ approximately}=0.4\pi\frac{(CAT)}{R_{f_1}},$$

R_1 being the reluctance of the field magnet.

This increase of the flux through the field magnet increases the magnetic density in the latter and demands consequently a correction, as I pointed out in 1899, without tracing it in detail. At that time I conducted the inquiry simply as follows, supposing the rôle of the field magnets to be sufficiently unimportant to permit approximate correction being applied.

Let B_1 be the flux density in the field magnet, corresponding to the no-load E.M.F. e, that is to say, to the flux ϕ_a+f_1; and let us call v_1 the Hopkinson coefficient $\frac{\phi_a+f_1}{\phi_a}$. The full-load induction will be

$$B'_1=B_1\frac{\phi_a+f_1+\Delta f_1}{\phi_a+f_1}=B_1\left\{1+\frac{0.4\pi(CAT)}{R_{f1}(\phi_a+f_1)}\right\}\text{ approximately}$$

$$=B_1\left\{1+\frac{(CAT)f_1}{N_1I_1(\phi_a+f_1)}\right\}$$

$$=B_1\left\{1+\frac{(CAT)}{N_1I_1}\cdot\frac{v_1-1}{v_1}\right\},$$

and consequently the total ampere-turns will be increased by the quantity by which the change from B_1 to B_1' increases the ampere-turns (which we shall call N_1I_1) specially absorbed by the reluctance of the field magnets in the condition considered. The point d' upon the curve will be in consequence displaced toward the left by the quantity corresponding to this increase of ampere-turns.

Starting from the point Q' duly corrected, it is sufficient to take a length representing the ampere-turns equal to CAT of the armature

in order to obtain the total necessary ampere-turns OQ''. Fig. 4 shows how the diagrams of E.M.F.'s and of ampere-turns may be united upon a single sheet.

The preceding reasoning may be summed up in the following simple equations, employing ordinary language:

The fall of magnetic potential in the armature and air gap is a function of the flux utilized in the armature ϕ_a+the armature stray flux f_2.

The magnetic difference of potential between the pole pieces=the fall of magnetic potential in the armature and air-gap+the reactive counter ampere-turns of the armature.

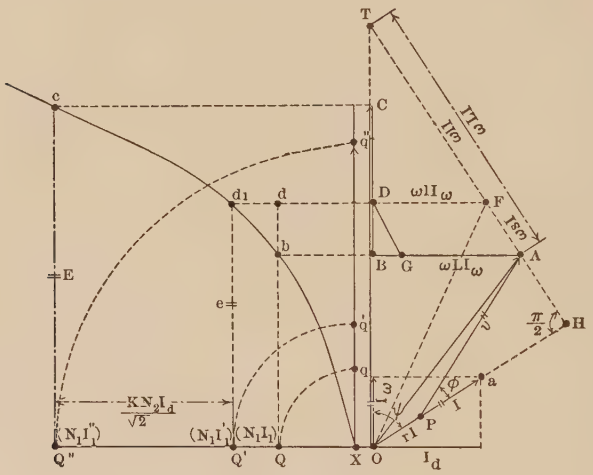


FIG. 4.

The magnetic stray field between the magnets $f_1+\Delta f_1$=a function of the difference of potential of the pole pieces.

The total stray magnetic fluxes=the fluxes $f_1+\Delta f_1$+the stray fields $f_2=f_1\left[1+\frac{(CAT)}{N_1I_1}\frac{v_1-1}{v_1}\right]+f_2$.

The total flux in the field magnet=the useful flux+the flux of the total stray field=$\phi_a+f_1\left[1+\frac{(CAT)}{N_1I_1}\frac{v_1-1}{v_1}\right]+f_2$.

The total ampere-turns of the field magnet=the difference of magnetic potential between the pole pieces+the total drop of potential in the field magnet corresponding to the total flux=the fall of potential

in the armature + the back ampere-turns of the armature + the fall of potential in the air-gap + the fall of potential in the field magnets corresponding to the total flux.

When the alternator is unsaturated or but slightly saturated, this latter fall of potential corresponding to the total flux may be admitted proportional to the flux $\phi_a + f_1 + f_2 + \Delta f_1$

$$= (\phi_a + f_1) + 0.4\pi(\mathrm{CAT})\left(\frac{1}{R_{f_1}} + \frac{1}{R_{f_2}}\right).$$

Thus the flux Δf_1 plays a part entirely similar to the flux f_2, and it may therefore be united with the latter in the coefficient of self-induction of the armature. It must, however, be remarked that the flux Δf_1 only follows the magnetic circuit to the point of emergence of the flux from the field magnets, and only absorbs consequently the fraction $\frac{R_1}{R_{total}}$ of the ampere-turns which would be necessary to make it traverse the entire magnetic circuit. The virtual self-induction s which may be advantageously assumed will then have an approximate expression

$$s = 0.1\pi K k N_2^2 \left(\frac{1}{R_{f_2}} + \frac{R_1}{R_{total}} \frac{1}{R_{f_1}}\right),$$

and this should be employed in the determination of the segment BD, as above.

In order to get the total necessary field-magnet ampere-turns $N_1 I_1''$, it is no longer necessary to add any excitation upon the field magnets except the ampere-turns neutralizing those of the armature $\frac{K N_2 I_d \sqrt{2}}{2}$. The total ampere-turns XQ'' are thus obtained. If the armature current were suddenly suppressed, an E.M.F. $E = Q''c$ would appear in the armature on open circuit, which is that appearing with the same notation in Fig. 2.

The same construction may serve reciprocally to calcuate the fall of potential produced in an alternator having the excitation XQ'' for the reactive or wattless current I_d in the armature.

Remark No. 1, Upon the Case of an Unsaturated Armature. When the armature and the pole pieces are not saturated, the diagram of E.M.F.'s (Fig. 1) is also a diagram of the flux, to a different scale, if care be taken to divide the values of the E.M.F.'s by the coefficient $\frac{k\omega N_2}{2\sqrt{2}}$.

Similarly, having given the magnetic reluctances sensibly constant for the direct flux of the armature and for the transverse flux (excluding the field magnet core), the same diagram may also present to a suitable scale the M.M.F.'s proportional to the flux, multiplied by the reluctance of the armature, of the air-gap, and of the pole pieces respectively. In that case, OA represents the necessary ampere-turns to force the flux through the said reluctances. OB is the part of this M.M.F. furnished by the field magnets; AB the part furnished

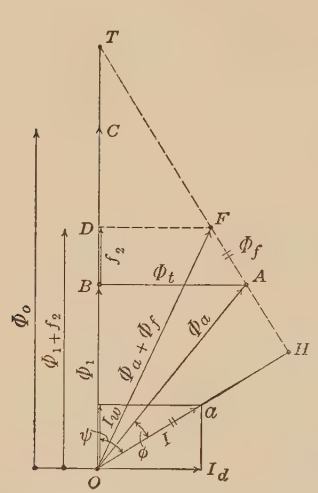


FIG. 5.

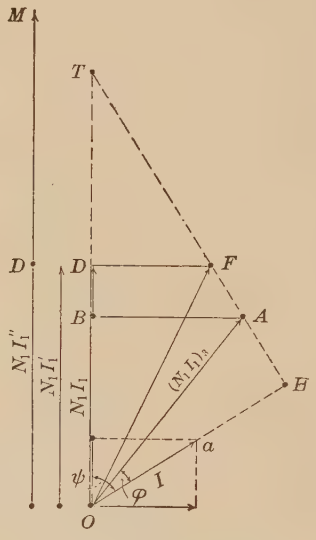


FIG. 6.

by the armature; BD the supplement necessitated by the stray fields of the armature.[1] DF represents the ampere-turns of distortion

$$DF = \text{function of } \frac{K_t N_2 I_w}{\sqrt{2}}.$$

Calling K_t a coefficient of distortion, or of transverse reaction, analogous to the coefficient K of the direct reaction, and f, (the function)

[1] And eventually by the supplementary stray fields Δf_1 in the particular case indicated above, where the effect of the stray field from the field magnets is referred to a supplementary term of the armature stray fields.

the relation which connects the ampere-turns to be produced by the field-magnet with those utilized in the armature, we have similarly

$$FT = \text{function of } \frac{K_t N_2 I}{\sqrt{2}}.$$

The total ampere-turns necessary to the emergence of the flux from the field magnets will be determined as above (Fig. 4): let XQ'' represent the magnitude equal to $N_1 I_1''$, to the scale of the new figure; the distance OQ will evidently represent the ampere-turns absorbed by the field alone; it is this length OQ which would in general be corrected by taking account of the stray field Δf_1. The method of correcting the stray fields indicated above may be employed; we will give further on another more nearly accurate—already suggested, moreover, by MM. Picou and Guilbert.[1]

Remark No. 2, upon the Subject of Diagram No. 1. It is to be observed that following the respective values of the reactive coefficients, both direct and transverse, the point C, the extremity of the available E.M.F. on open circuit, may be either above or below T.

With an alternator of unsaturated field magnets, the two reactions have coefficients nearly equal, and C may then coincide with T, when they are equal. Ordinarily, the coefficient of distortion, K_t, tends to be reduced, as we shall see, by a reduction in the breadth of the poles, while the coefficient K has the opposite tendency. It results from this that C tends to be above T. But the saturation of the field magnets lowers it the more as the saturation is greater; because this latter augments but slightly the supplementary ampere-turns necessary to compensate for those of the armature, but greatly diminishes the variation of the voltage between open circuit and full load. C is, therefore, in general below T, as represented in the diagrams.

The same condition may be found even with saturation, with certain types of alternators, such for example as that which was exhibited in 1900, in Paris, by the firm Sautter-Harlé. This machine, developed along a plan, formerly patented by Professor E. Thomson, of an iron rotor (inductor alternator), has a single armature, two exciting field-windings, and a yoke closing the magnetic circuit through

[1] The diagram of the present figure is analogous to a diagram recently published by M. E. Guilbert (*loc. cit.*); it differs, however, in that the line FE is expressed as a function of the coefficient K_t instead of the coefficient K, and that the expression of dephasing ψ is thus presented as a function of the ampere-turns.

the shaft of the field magnet. It presents a supplementary air-gap of considerable reluctance around the shaft, and this air-gap is traversed only by the direct reaction. The coefficient of direct reaction is therefore rendered smaller than that of the transverse reaction, and if the supplementary stray fields Δf_1, analyzed above, are not exaggerated, C will remain below T.

Those theoretical diagrams are not, therefore, liable to criticism which show C below T. To propose placing C always on T, as

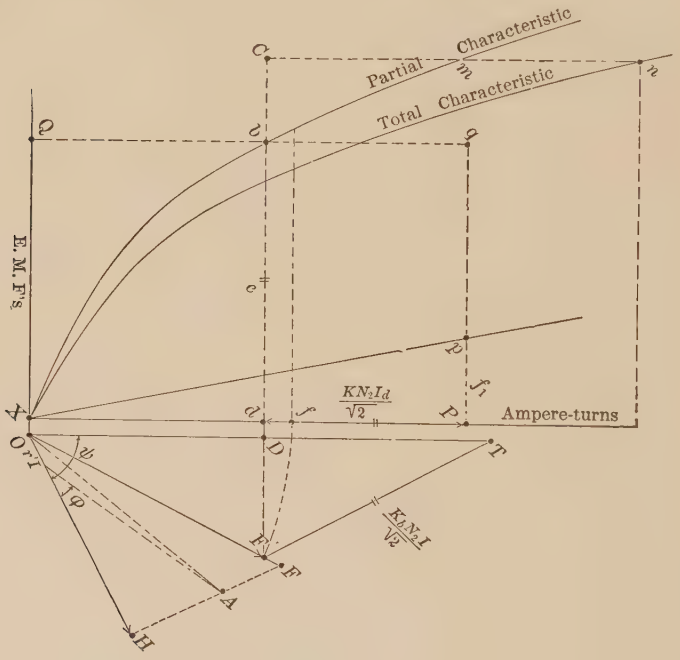


Fig. 7.

Professor Arnold has done,[1] is contrary to the purpose of this method, namely, the calculation of the effects of saturation and of distortion according to rational principles.

The Case of a Saturated Armature. (Fig. 7.) The theory of two reactions permits also of treating the case of a saturated armature, by employing with the total characteristic, the characteristic of the armature alone (comprising the armature, air-gap, and pole pieces),

[1] E. Arnold, *Elek. Zeit.*, 1902, page 250.

already employed moreover by MM. Bauch, Potier, Guilbert, and Picou.

For greater clearness of explanation I shall represent the characteristics upon the same diagram, but, in practice, they would be drawn upon separate figures. The curves are referred to the E.M.F's. as common abscissæ, and the ordinates of the two curves represent respectively the ampere-turns for the passage of a given flux (corresponding to the E.M.F.) in the magnetic circuit, with or without the field magnets. The difference of the ordinates equals then the ampere-turns absorbed by the field magnet alone, in the absence of stray magnetic fields.

This assumed, we shall then construct, according to custom ,the diagram of E.M.F.'s by adding to the voltage U at the terminals the internal drop $r'I$, then the loss by stray magnetic fields AF. We thus obtain the E.M.F. OF produced in the armature. We find upon the characteristic of the armature the ampere-turns OF' corresponding to this E.M.F., and we lay them off on OF' in the direction of OF.

We may then observe that the distortion may be determined by the comparison of the ampere-turns of distortion with the useful ampere-turns in that part of the machine which does not include the field-magnets; because, in the transverse reaction, the reluctance of the pole pieces may be neglected in relation to that of the entrefer and of the armature (especially that of the teeth), and consequently attribute to the path of this reaction the same reluctance as in the path of the useful flux outside the field magnets. The determination of the angle ψ (Fig. 1) will then be transformed into simply replacing the self-inductance by the M.M.F.'s applied to the armature. The ampere-turns $F'T=\frac{K_t N_2 I}{\sqrt{2}}$ will be laid off in the direction perpendicular to I, and OT is joined; then from F' the perpendicular $F'D$ is dropped, which will be equal to the ampere-turns of distortion

$$DF^1=\frac{K_t N_2 I_w}{\sqrt{2}}.$$

The line OD will then represent the ampere-turns to be supplied by the field-magnets at the point of emergence, the remainder DF' being furnished by the armature itself.

It remains to determine the total field-magnet ampere-turns corresponding thereto, taking into account the reluctance of the stray flux

of the field magnets, properly so called. For this purpose it suffices to seek upon the characteristic of the armature the corresponding E.M.F. $db=e$; the ordinate Qb corresponding to this abscissa e upon the curve of the armature measures the necessary ampere-turns to be produced between the poles, the necessary stray-fields f_2 included. We add thereto the ampere-turns equilibrating the direct reaction of the armature, that is to say,

$$bq=\frac{KN_2I_d}{\sqrt{2}}.$$

The ordinate Qq represents the difference of potential (subject to the factor 0.4π) necessary between the poles of the field magnet. From this may be deduced the value of the stray field f_1 between the pole pieces, which may be presented, for example, as a function of the magnetic difference of potential along the curve XP, which is sensibly a straight line; Pp will then represent the stray field f_1.

If, starting from b, a segment bC be drawn representing Pp (measured to the same scale as the flux db corresponding to the E.M.F. e), and through C we draw the straight line Cm parallel to OP, the latter will contain between the two characteristics a segment mn which will represent the fall of magnetic potential in the field magnets under the influence of the total flux dC. The total necessary M.M.F. will thus be equal to $OP+mn$.

The diagram is thus established, taking into account the stray field both of the field magnet and of the armature. It is distinguished from those of Potier,[1] Rothert,[2] and Bauch,[3] because it takes into account the transverse reaction with its real value; it takes account of the difference between the two coefficients of reaction K and K_t and is thus distinguished from the diagram of M. Guilbert[4] for unsaturated field magnets; it finally differs in diagrammatic construction from the very ingenious diagram of the same author for saturated field magnets in the fact that it does not separate the air-gap from the armature, and is also much more simple.

Summing up, the employment of the diagram in Fig. 4 is to be recommended for the case in which there is no appreciable saturation either in the armature or in the field. In all other cases it seems

[1] Potier, *Eclairage Electrique*, July 26, 1902.
[2] A. Rothert, *Elek. Zeit.*, 1899.
[3] Bauch, *Elek. Zeit.*, 1902.
[4] M. Guilbert, *Revue Technique*, April and May, 1904.

preferable to employ the diagram Fig. 7, which lends itself better to determining the different elements without complications.

If it is desired to solve the inverse problem, that is, to determine the fall of potential under constant excitation as a function of the load, the preceding diagrams do not give a direct solution, but it is easy to employ them for an indirect solution, particularly in assuming constant the external dephasing ϕ, and taking successively different values of the reactive current; for each value of I_d the preceding construction will be followed in the opposite direction, and thus will be obtained the voltage at the terminals, the values of ψ and of I_w. Thus may be traced a curve of voltage u as a function of I_d, and of I which is obtained therefrom. It is only necessary to seek upon this curve the point corresponding to the conditions required and the dephasing angle.

The problem is solved no longer for a single point, but along a complete curve, which is also comparatively easy.

Local Corrections of the Air-gap Due to Saturation (Second Approximation). The diagram Fig. 7 is established by supposing that the reactions act *en bloc*, and are represented by coefficients. The same is true of the diagram Fig. 1. But if it be desired to follow the reality somewhat closer, it is well, once the diagram is determined by the aid of the coefficient K (the calculation of which is explained later), to calculate upon the drawing the flux-density of the resulting field at each point along the air-gap, by the aid of magnetic potential curves (to be explained later) and of the local reluctance. In particular, if the teeth of the armature are saturated, they develop marked variations of the reluctance per unit of surface along the air-gap, and the flux calculated according to a mean value of reluctance may be sensibly modified thereby. This is the case not only for the transverse reaction, as has already been remarked by certain authors, but also for the direct reaction, which should not be set aside in this correction. The effect of this latter is to reduce the resultant flux. The curve of the diagram is, in fact, a solution of the first approximation necessary in order to determine the dephasing of the values of the active currents from the reactive currents. After these values have been obtained, a second approximation may be arrived at by tracing the flux-densities from point to point for determining the real flux. In general, however, the precision of the calculations is not sufficiently great to proceed upon this correction unless ample time may be afforded for the study.

Case of Field Magnets with Divided Windings. In certain machines, particularly turbo-alternators, circular field magnets are found in which the windings are carried along the air-gap in slots like those of the armature. The preceding diagrams (Figs. 1 to 7) apply likewise to these machines only on the condition of assuming the two coefficients of reaction equal even when the field magnet is entirely divided into slots. Moreover, the field-magnet winding must be affected by a coefficient of ampere-turns K_1, reducing them to $K_1 N_1 I_1$ (with $K_1 = 0.4$ to 0.5 in completely uniform windings), and by the Hopkinson coefficient v_1, which is calculated like the stray field of a slot in an asynchonous motor. All the other coefficients are calculated as in the ordinary case, by supposing the breadth of the reactive flux equal to that of the field-magnet poles. This method has given me satisfactory results in practice for this type of machines.

Part II. Calculation of Constants.

Practical Calculation of Reactions. In order to apply the diagrams, the coefficients s, l, K_t and K must be determined (l and K_t are of course only two expressions of one and the same coefficient). The stray coefficient is determined by known methods frequently indicated for asynchronous motors, and they need not, therefore, be alluded to here. For K and K_t, I have employed for several years the most direct method, which consists in determining for the same machine curves of distribution of magnetic potential, and of the flux in the air-gap, assuming that the armature is traversed by a known current either active or reactive. By taking into account the position of the pole pieces in these two cases, and their form, as well as that of the slots, the reactions may be determined with sufficient precision.

Let us consider, for example, the case of three-phase currents: the three phases occupy in a double field six slots, or groups of slots, and at the passage of each slot, the magnetic potential along the entrefer undergoes a sudden positive or negative increase equal to 0.4π multiplied by the number of ampere-turns $\frac{N_i}{6}$ contained in the slot. It suffices to mark off on a straight line, representing the development of the circumference of the armature, lengths equal to the distance between the axes of the slots, and on successive ordinates, the variations of the magnetic potential thus calculated. The horizontal

mean line of the curve so obtained is then traced, which indicates the zero of magnetic potential. The fluxes will be at every point proportional to the ordinate of the curve from the zero point, and inversely proportional to the reluctance per unit of surface corresponding to the abscissa considered. For simplicity, the reluctance may first be assumed constant, and account is only taken of its variations in order to arrive at a definite correction for a second approximation. To simplify the calculation, there is attributed to the maximum amplitude of the polyphase currents an arbitrary value I_0, and it is supposed that the currents follow a sinsuoidal law. Since the form of the curves is reproduced for every one-sixth of a period in the three-phase currents (or for one-fourth of a period in two-phase currents), it suffices to study them during such an interval, and then to outline the extreme forms.

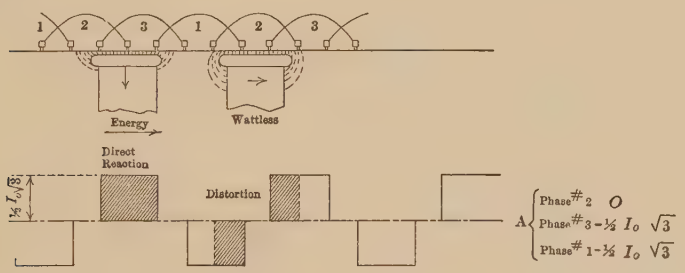


FIG. 8.

Let us take, for example, a three-phase machine with six slots in the field, each containing $N/6$ wires, calling N the total number of peripheral wires per double field (Fig. 8). The potential produced by each is $0.2\pi\,(N/6)i$, calling i the current which traverses the winding, and it suffices to construct the curve of i, to which that of the potentials should be proportional. We take two positions; one for which the current is *nil* in the slots 1 and 4 and equals $\pm\frac{\sqrt{3}}{2}I_0$ in the others; the other position for which the current is equal to I_0 in the slots 1 and 4 and equal to $(1/2)\,I_0$ in the others. The curves proportional to the potential thus obtained are respectively represented in Figs. 8 and 9. On these figures are added in position and in magnitude the outlines of the field-magnet poles in the two positions; in full lines the position corresponding to the watt current, that is to say, a pole axis coinciding with the middle of the curve of potential;

and in dotted lines the position corresponding to the reactive current, that is to say, the axis of the pole facing the zero of the curve.

The reaction is then deduced from the figure by determining the mean useful ordinate of the curve. Theoretically this ordinate would be obtained by evaluating the area of the shaded curve situated in front of the pole, and dividing this by the breadth of the pole; but the result so obtained is not practically useful, because it takes no account of the expansion of the lines of force, which greatly broadens the flux, particularly as the air-gap is made larger and the angles of the pole pieces are more rounded. To determine the direct reaction, one must take instead of the breadth of the pole, the breadth of the field-magnet flux which issues from it; and to determine the mean ordinate in this breadth. A similar determination is made for the transverse reaction. It must be observed that the flux which forms

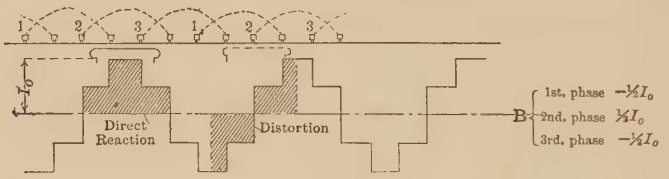


FIG. 9.

it is established not only under the poles, but also around them, although with a lesser density. Consequently, this flux occupies a greater breadth in which the mean reaction should be determined. There is, therefore, a large individual liability to error in the appreciation of these reactions, and this should give preference to the complete method of operation here indicated for the employment of theoretical coefficients, which do not take account of the special conditions in each machine. If the breadth of the flux is equal, for example, to the pitch, Figs. 8 and 9 show the mean ordinates $\frac{2}{3}\left(I_0\frac{\sqrt{3}}{3}\right)$ and $\frac{2}{3}I_0$ for the direct reaction, and similarly for the transverse reaction. The values give those of the coefficients K and K_t themselves, if the ampere-turns obtained are compared with the ampere-turns which would be obtained with the three bobbins united in a single pair of slots and traversed by a current I_0. The curve of potential gives $\frac{2N}{6}\left(\cdot\frac{2}{3}I_0\frac{\sqrt{3}}{2}\right)$ instead of $\frac{N}{2}I_0\times 3$. The ratio gives the coefficient $K=\frac{2}{3}\left(\frac{2}{3}\frac{\sqrt{3}}{2}\right)=0.384$.

Thus the coefficients K and K_t are obtained simply by taking $\frac{2}{3}$ of the mean ordinates. For two-phase currents, one would similarly take $\frac{2}{2}$, that is to say, unity.

If instead of one slot per phase, there were several, n for example, the mean ordinate would be first divided by n.

In this manner, the following figures would be obtained:

TABLE IV

EMPIRICAL COEFFICIENTS

Three-phase winding, with three separate coils per pair of poles.

Ratio $\frac{\delta}{\Delta}$ of the breadth of flux to that of polar pitch.	Coefficient K (Direct).			Coefficient K_t (Transverse).		
	Pos. 1.	Pos. 2.	Mean.	Pos. 1.	Pos. 2.	Mean.
1	0.385	0.444	0.419	0.385	0.444	0.419
$\frac{2}{3}$	0.577	0.500	0.538	0.288	0.333	0.310
$\frac{1}{2}$	0.577	0.555	0.566	0.192	0.333	0.2625

In practice an alternator is rarely found where the flux occupies less than two-thirds of the pitch, and besides in this case a winding of twelve bobbins with six short slots should be taken, in my opinion, instead of the ordinary winding, as will be mentioned further on.

The coefficients of self-induction l and l', corresponding to the two reactions, are deduced from the values of K and K_t by evaluating the corresponding fluxes and the E.M.F.'s which they induce in the windings themselves by means of the ordinary formulas. From this, calling k the winding factor, the mean value of l which takes account of the reduction, by distribution of the wires, in the E.M.F. produced in the winding by a sinusoidal flux,[1]

$$l = \frac{4\pi K_t k \left(\frac{N}{2}\right)^2 10^{-9}}{qR_t}; \quad l' = \frac{4\pi K k \left(\frac{N}{2}\right)^2 10^{-9}}{qR_d},$$

[1] See in particular the coefficients in my above-mentioned analysis of the rotating magnetic fields, *Éclairage Électrique*, 1895.

q being the number of phases (here 3), R_t and R_d the reluctances of the transverse and direct circuits respectively.[1]

These reluctances are determined from the drawing of the machine, taking into account the real path of the lines of force and the saturation of the parts through which they pass, particularly the teeth of the armature, the polar horns, the cores, the yokes of the field-magnets, etc.

If, instead of alternate poles, the machine carries poles of the same name (homopolar inductor), K and K_t may again be determined by the preceding methods, drawing only one inductor-pole for two poles of the armature. It results from this that theoretically the reactions would give rise to coefficients 50 per cent less than in the ordinary case. In practice, however, this is far from being the case, because of the very considerable expansion of the flux of the armature in the large spaces existing between the field-magnet poles. The direct flux reaction and particularly the transverse reaction is, therefore, much larger than if they were produced only by the action of the poles; so that finally the reactions are scarcely reduced more than 25 per cent. The stray fields are, moreover, very large in this type of machine, and every expansion of the field-magnet flux beyond the breadth of the pitch produces a hurtful inverse E.M.F. The induction-density in the air-gap should finally be doubled at least, to produce the same useful flux on open circuit. From all the above it follows that homopolar machines are of little advantage and are almost abandoned.

To completely take into account the practical values of the coefficients, we shall consider again the case of three-phase machines with six coils per field, first concentrated into one pair of slots per coil, and then spread uniformly (or to a large number of slots each) in order to occupy the entire circumference of the armature.

Figs. 11 and 12 represent the curves of magnetic potential obtained in the two hypothetical cases with long bobbins disposed as shown diagrammatically in the figure. The two curves correspond to the same hypothesis as above for currents, and Table II represents the separate mean values obtained for K and K_t from these curves.

[1] In unsaturated alternators, if one calls e the single air-gap, s the polar surface, and d the coefficient of enlargement of the flux, the equation is approximately obtained $\frac{1}{R_d}=\frac{s(1+d)}{2e}$.

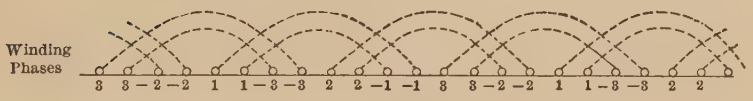


FIG. 10

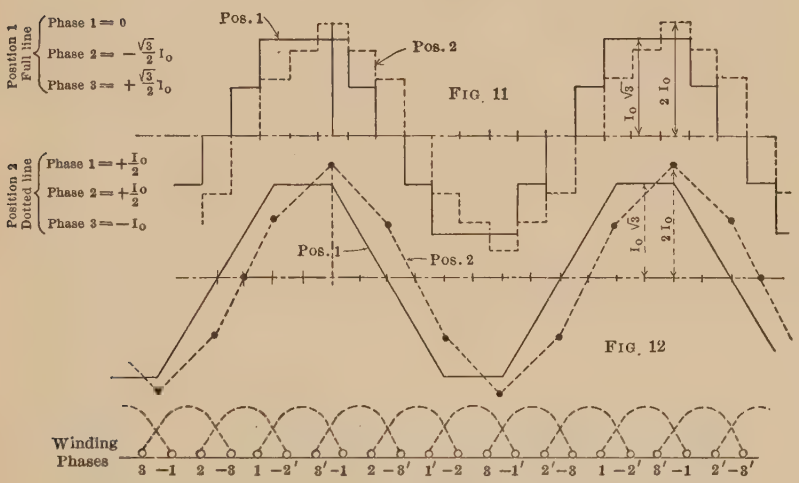
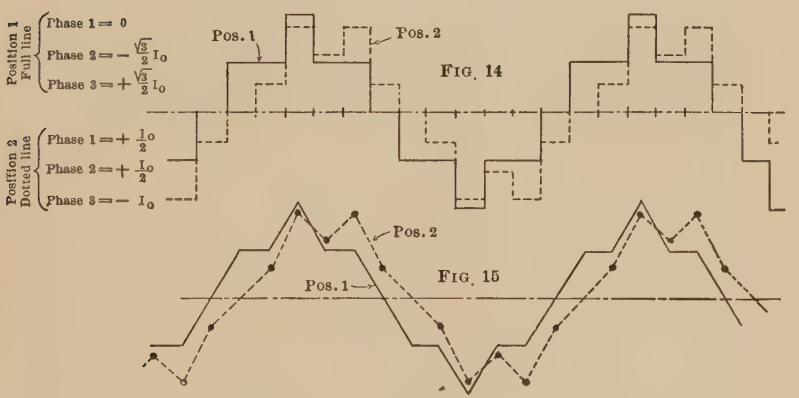


FIG. 13



FIGURES 10 TO 15

But a winding with six coils may also be realized symmetrically following the plan of Fig. 16, from which two new curves 16*a* and 16*b* are obtained. The table also indicates the value of the coefficients thereby deduced.

TABLE V

EMPIRICAL COEFFICIENTS

Three-phase winding with three separate coils per pair of poles.

Ratio $\frac{\delta}{\Delta}$		K.			K_t.		
		Pos. 1.	Pos. 2.	Mean.	Pos. 1.	Pos. 2.	Mean.
Long coils	1	0.385	0.389	0.387	0.385	0.389	0.387
	$\frac{2}{3}$	0.505	0.500	0.5025	0.288	0.291	0.2915
	$\frac{1}{2}$	0.577	0.555	0.566	0.192	0.222	0.207
Short coils	1	0.2886	0.2777	0.283	0.2886	0.2777	0.283
	$\frac{2}{3}$	0.360	0.375	0.3675	0.216	0.208	0.212
	$\frac{1}{2}$	0.385	0.444	0.415	0.192	0.111	0.152

TABLE VI

EMPIRICAL COEFFICIENTS

Three-phase distributed winding, 3 or 6 long coils per pair of poles.

Ratio $\frac{\delta}{\Delta}$ breadth of flux to polar pitch.	K.			K_t.		
	Pos. 1.	Pos. 2.	Mean.	Pos. 1.	Pos. 2.	Mean.
$\frac{1}{1}$	0.385	0.389	0.387	0.385	0.389	0.387
$\frac{2}{3}$	0.505	0.500	0.5025	0.2886	0.2916	0.200
$\frac{1}{2}$	0.553	0.541	0.547	0.216	0.236	0.236

Here again it is seen that the direct reactions increase while the transverse reactions diminish when the breadth of the flux (larger than the pole) diminishes. It is moreover determined that the reac-

TABLE VII

EMPIRICAL COEFFICIENTS

Three-phase winding with 6 short distributed coils per double field.

Ratio $\frac{\delta}{d}$	$K_.$			$K_t.$		
	Pos. 1.	Pos. 2.	Mean.	Pos. 1.	Pos. 2.	Mean.
$\frac{1}{1}$	0.289	0.277	0.283	0.289	0.277	0.283
$\frac{2}{3}$	0.360	0.375	0.3675	0.216	0.208	0.212
$\frac{1}{2}$	0.385	0.444	0.4145	0.192	0.137	0.1645

tions are markedly reduced by the employment of short coils. But it may be readily shown that with a sinsuoidal field-magnet flux of a breadth equal to the pitch, the E.M.F. induced in the winding is reduced approximately in the same ratio. In fact, the mean breadth of the short bobbins, Fig. 16, is $\frac{1}{4}$ of the pitch, while the mean breadth

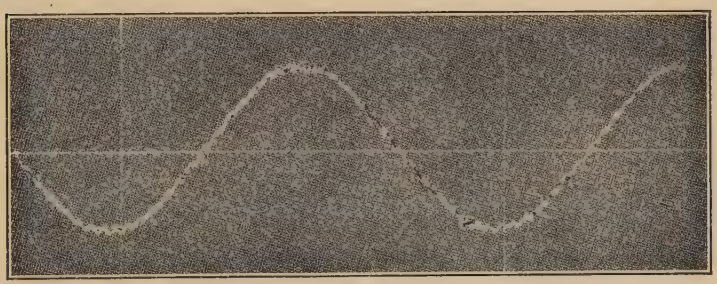


FIG. 16.

of the long bobbins, Fig. 10, is $\frac{1}{2}$ the pitch. The straddling arrangement of coils in Fig. 10 involves only the coefficient of reduction $\frac{2\cos\left(\frac{\pi}{12}\right)}{2}=0.966$, while the arrangement of Fig. 16 reduces the E.M.F. in the same ratio as the flux linkage, that is to say, by the coefficient $k=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$. The two E.M.F.'s are then in the ratio $\frac{E \text{ short bobbins}}{E \text{ long bobbins}}=\frac{0.707}{0.966}=0.73$.

But the ratio of the coefficients K and K_t of the two windings gives approximately the same figure. It is to be observed in this connection

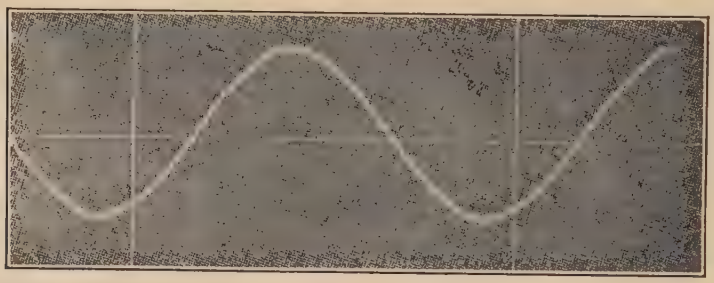


FIG. 16*a*.

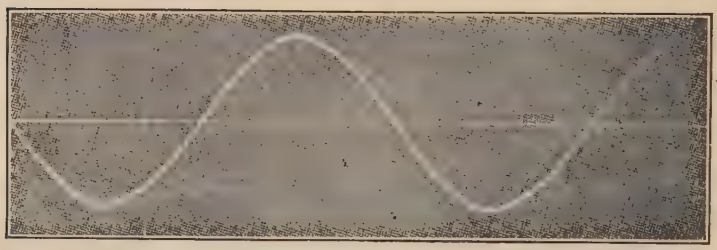


FIG. 16*b*.

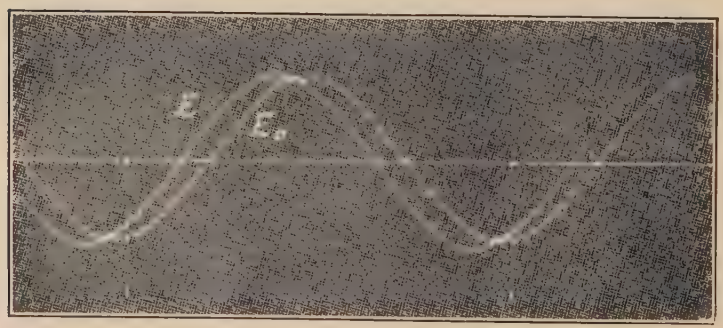


FIG. 16*c*.

that for the case of a flux having a breadth equal to the polar pitch, the coefficient K is substantially equal for these two windings, that is, to k of each winding multiplied by $\left(\frac{2}{\pi}\right)^2$, which verifies the general

law formerly announced and which is alluded to above.[1] Hence it follows that the fluxes produced by multiple-coil windings differ but little from the mean value of the theoretical fluxes, and approach the more nearly as the sections are the more numerous, and also as the local variations or fluctuations of the curves between the extreme forms 1 and 2 of the appended figures are damped out by the Foucault currents of the neighboring pole pieces. The energy expended in these Foucault currents, being supplied by the armature, is represented by an augmentation of its apparent resistance r'.

The reactions of two-phase armatures would be found in a similar manner, and it will not be necessary to reproduce them more in detail. Moreover, two-phase machines are more and more becoming supplanted by three-phase, and the latter present reactions of much smaller fluctuations and a better utilization of materials, just as three-phase motors are superior, from this standpoint, to two-phase motors.

Comparison with Theoretical Coefficients. The theoretical coefficients are easy to establish in the case where a sinusoidal flux is assumed and the harmonics are suppressed.[2] It is then demonstrated that the magnetic potential produced by a polyphase winding of q phases is independent of the number of phases and depends only upon the total number of wires N per double field, and that it is represented by a sinusoid whose amplitude is $2NI_0$. The mean potential in the air-gap is therefore $\frac{2}{\pi} 2NI_0$ and the equivalent mean magnetomotive force producing the reactive flux on closed circuit

$$= \frac{0.4}{\pi} 2NI_0 = \left(\frac{4}{\pi}\right)^2 \times 0.4\pi \frac{N}{2} I_0,$$

that is to say, $(4/\pi)^2$ of the magnetomotive force which will give the same turns if they coincide in position and phase. In this case the sinusoid of potential (Fig. 17) is entirely used and the direct reaction is proportional to the mean ordinate of the area $A3B$, and the transverse reaction to that of the area $3HBH'1O$. If, on the contrary,

[1] In alternators with distributed windings (Figs. 12 and 15), the diagrams of windings 10 and 13 respectively may be employed by assuming that each coil is replaced by a zone of wires occupying along the air-gap a breadth of $\frac{1}{12}$ of the field, of $\frac{1}{6}$ of the polar pitch, and having as median line the old outline of the single bobbin which it replaces.

[2] See my above-mentioned memoir of 1899 upon "Rotating Magnetic Fields," see also Arnold and la Cour's "Vorausberechnung der Ein und Mehrphasenstromgeneratoren." Stuttgart, 1901.

the reactive flux only occupies a part δ of the pitch instead of Δ, as indicated by the dotted intersecting lines in the figure, the reactions will be proportional to the mean ordinates of the areas 12345 and 67B89 respectively, limited to the breadth of the flux (which may be different moreover for the transverse flux from what it is for the direct flux. By integrating the area of the sinusoid from $H3$ on to 54 one finds

$$\text{Ordinate of area } 12345 = \frac{\Delta}{\delta} \sin\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right) \times \frac{2}{\pi} I_0.$$

$$\text{Ordinate of area } 67B89 = \frac{\Delta}{\delta}\left[1 - \cos\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right)\right] \times \frac{2}{\pi} I_0.$$

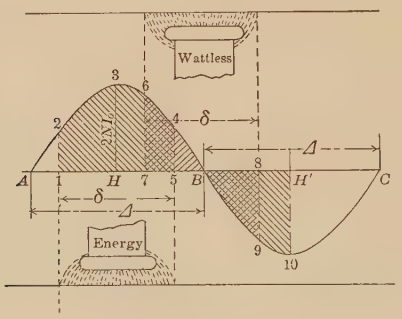


FIG. 17.

The coefficients which apply to the ampere-turns, resulting from a restriction of the flux, are then respectively for K and K_t

$$c = \frac{\Delta}{\delta} \sin\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right) \quad \text{and} \quad c_t = \frac{\Delta}{\delta}\left[1 - \cos\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right)\right],$$

and have the following values for example (not including k)

for $\frac{\delta}{\Delta} = \frac{3}{4}$; $c = 1.232$; $c_t = 0.817$ from which $K = 1.232\left(\frac{2}{\pi}\right)^2 k$

$$K_t = 0.817\left(\frac{2}{\pi}\right)^2 k$$

for $\frac{\delta}{\Delta} = \frac{2}{3}$; $c = 1.299$; $c_t = 0.75$ $\qquad K = 1.299\left(\frac{2}{\pi}\right)^2 k$

$$K_t = 0.75\left(\frac{2}{\pi}\right)^2 k$$

for $\frac{\delta}{\Delta} = \frac{1}{2}$; $c = 1.414$; $c_t = 0.586$ $\qquad K = 1.414\left(\frac{2}{\pi}\right)^2 k$

$$K_t = 0.586\left(\frac{2}{\pi}\right)^2 k$$

TABLE VIII

THEORETICAL COEFFICIENTS

Winding with three coils per pair of poles.

Ratio $\frac{\delta}{\Delta}$	Separate Coils.		Distributed Coils.	
	K.	K_t.	K.	K_t.
$\frac{1}{1}$	0.405	0.405	0.3859	0.3859
$\frac{2}{3}$	0.526	0.304	0.501	0.2897
$\frac{1}{2}$	0.572	0.237	0.544	0.2257

TABLE IX

THEORETICAL COEFFICIENTS

Winding with 6 coils.

	Ratio $\frac{\delta}{\Delta}$	Separate Coils.		Distributed Coils.	
		K.	K_t.	K.	K_t.
Long coils	$\frac{1}{1}$	0.391	0.391	0.3859	0.3859
	$\frac{2}{3}$	0.508	0.293	0.501	0.2897
	$\frac{1}{2}$	0.552	0.228	0.544	0.2257
Short coils	$\frac{1}{1}$	0.286	0.286	0.2825	0.2825
	$\frac{2}{3}$	0.371	0.215	0.3665	0.212
	$\frac{1}{2}$	0.404	0.167	0.399	0.164

Since these figures must also be modified in every case by the coefficient k corresponding to the E.M.F., as has been explained above, it is evident that they will differ but little from those of Table II. But it is not the less necessary to determine, from a drawing of the machine, the breadth of the flux before applying it in these formulæ, and therefore to correct the reactions in order to take account of the saturation of the different parts of the armature and of the pole pieces.

This correction is made by assuming the magnetic conditions which are approximately attained in the machine at full load. The

mean flux-density is then known which must be developed in the air-gap, the teeth, and the pole pieces, and, moreover, one has from the curves of potential the value of the magnetomotive forces acting at all points of the air-gap. From this may be deduced the real flux-density at every point, and consequently the true variation of the total flux produced by the reaction. These expressions of self-inductions given above should be consequently replaced by the integrals of the form

$$l = \frac{N}{q2\sqrt{2}I_0}\int\left(0.4\pi\frac{N}{6}\frac{yb}{R_x}\right)dx,$$

indicating by x the abscissa, and by y the ordinate of the curve, and R the reluctance per unit of surface at this point. b represents the length of the armature, q the number of phases, N the number of peripheral wires per field.

Case of Single-phase Alternators. The problem of the reactions of single-phase alternators is more complex than that of polyphase alternators, as will be seen. It does not appear to have been fully understood by the authors who have previously treated it. It may be analyzed by the same method as that which I have formerly developed for asynchronous motors,[1] but taking into account this important difference, that, in general, the rotating reactions (rotating magnetic field with respect to the armature, which we suppose fixed), are suppressed in motors by the short-circuited windings on the rotor, but not in alternators, except in the case where they are furnished with massive poles, or especially with the damping windings of Leblanc (which, however, only give a partial suppression).

I have shown that each coil of a single-phase armature produces a reactive flux capable of being decomposed in space into sinusoidal harmonics, of which the first, the only one which need be considered in practice, has for amplitude $\frac{0.4\pi nI}{R}$, denoting by nI, the ampere-turns of the coil, and by R, the reluctance of the magnetic circuit traversed by the flux which it produces. In accordance with the theorem of Leblanc, I decompose this pulsating sinusoid into two rotating sinusoids of one-half amplitude: one turning synchronously with the rotor, and not displaced therefore with respect to the field-poles; it provides only a fixed reaction; the other rotate

[1] Blondel, "Properties of Rotating Magnetic Fields," *Éclairage Électrique*, May, 1908.

in the opposite direction with the same speed and is therefore displaced with respect to the field magnets with a speed double that of synchronism; this gives rise to a pulsating flux in the field magnets of frequency double that of the e.m.f. induced in the armature, as I have also shown experimentally.[1] This reaction is somewhat weakened by the currents which it produces in the closed circuit of excitation, but it is not completely extinguished and produces therefore an e.m.f. of normal frequency in the armature which should be taken into consideration. But, while the fixed reaction may be analyzed, exactly as in polyphase alternators, into direct and transverse reactions, the parasitical rotating reaction is effected directly through the field magnets, as well as transversely through the pole pieces, and may be represented, consequently, by a mean coefficient of self-induction similar to the self-induction of the stray field ωs, to which it is added.

It may be directly demonstrated by calculations that this analysis readily lends itself to the interpretation of the facts,[2] and that a single-phase alternator whose armature at rest presents a self-induction

[1] Blondel, "Photographic Record of Periodic Curves," *Lumière Électrique*, August, 1891.

[2] In fact, the self-induction of the armature L, varying between the two values λ and λ' according to position, may be represented by an expression

$$L = A + B\cos 2\omega t + \omega s$$

and abbreviating, $\qquad A + B = \lambda'; \quad A - B = \lambda,$

indicating always by s the inductance of the stray fields in the slots. Let $e = E_0 \sin \omega t$ be the internal E.M.F., and $i = I_0 \sin(\omega t - \psi)$ the strength of the current, and there is immediately obtained as the difference of potential at the terminals of the machine the following expression:

$$u = e - ri - L\frac{di}{dt} - i\frac{d\lambda}{dt} - \omega s\frac{di}{dt}.$$

Replacing e, L, i by their values, and neglecting a term $\frac{B\omega}{2}\cos(3\omega t - \psi)$ which produces an upper harmonic, there remains

$$u = E_0 \sin \omega t - rI_0 \sin(\omega t - \Psi) - \omega\left(A - \frac{B}{2}\right) I_0 \cos\Psi \cos \omega t$$
$$- \omega\left(A + \frac{B}{2}\right) I_0 \sin\Psi \sin \omega t - \omega s I_0 \cos(\omega t - \Psi)$$
$$= E_0 \sin \omega t - rI_0 \sin(\omega t - \Psi) - \frac{(\omega\lambda + \lambda')}{4} I_0 \cos(\omega t - \Psi) - \frac{\omega\lambda}{2} I_0 \cos\Psi \cos \omega t$$
$$- \frac{\omega\lambda'}{2} I_0 \sin\Psi \sin \omega t - \omega s I_0 \cos(\omega - \Psi).$$

The values of $\frac{\lambda}{2}$ and $\frac{\lambda'}{2}$ representing that which we have uniformly denoted by l and l' for all the machines in the construction of the diagrams.

λ when the poles are crossed, and an inductance of λ' when they are coincident, behaves under load, when the real dephasing is ψ for example, as though the active current $I \cos \psi$ traversed an inductance $\frac{\lambda}{2}$, the reactive current $I \sin \psi$ an inductance $\frac{\lambda}{2}$, and the total current I a parasitic inductance $\frac{\lambda+\lambda'}{4}$ equal to the mean of the two preceding.

In order to take into account the Foucault currents of the armature produced by the rotating reaction, it is sufficient to apply a reducing coefficient m to its inductance, which is less than unity, evaluated according to the conditions of construction, and at the same time to increase the apparent resistance of the armature in accordance with the energy lost in these Foucault currents, since it is furnished by the armature. It is for this reason that we attribute to this resistance a value $r' > r$ in all of our diagrams.

If the field magnets or the armature of the alternator are saturated, the direct and transverse self-inductances will be replaced by equivalent back ampere-turns, again calculated as in my theory of rotating field: the sinusoid of the amplitude $0.2\pi nI$ presents the mean ordinate $\frac{2}{\pi}$ times smaller and is consequently equivalent to $\frac{nI}{10}\left(\frac{2}{\pi}\right)^2$ ampere-turns. The value of K and K_t will therefore be $\left(\frac{2}{\pi}\right)^2$ for a single bobbin if it has the same breadth as the flux. If the winding comprises several straddling coils, the coefficients should be multiplied by the straddling factor of the winding k; finally if the flux of the poles is narrower than the pitch, it should be multiplied by the factors $\frac{\Delta}{\delta} \sin\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right)$ and $\frac{\Delta}{\delta}\left[1-\cos\left(\frac{\pi}{2}\frac{\delta}{\Delta}\right)\right]$, respectively the coefficient of direct reaction and the coefficient of distortion for the conditions analyzed above.[1]

[1] It is of course easy to pass from a single-phase machine to a polyphase machine of two phases, observing that each phase gives a fixed reaction and a rotating reaction of the same amplitude. I have shown in my theory of rotating fields, already alluded to, that the fixed reactions unite in space and are added algebraically while the q rotating, parasitic reactions give rise to a resultant zero. The coefficients K and K_t are then themselves theoretically expressions in all the machines independent of the number of phases (N designating always the total number of peripheral wires); but the rotating, parasitic self-inductance disappears in polyphase machines. In this manner the return is made to the theoretical coefficients of the polyphase machines.

To sum up, the case of the single-phase alternator should then be treated exactly by the same general formulæ, the same constructions, and the same diagrams as in the case of a polyphase alternator, but under the condition of considerably increasing the stray flux ωsI, adding thereto a term representing the parasitic rotating inductance. The coefficient ωs is thus replaced by $\omega\left[s+m\left(\frac{l+l'}{2}\right)\right]$ values l and l' are calculated by the theoretical coefficient of polyphase machines (see p. 263) (taking into account the saturation of the circuits by the values given to R and R', as has already been seen above). In this manner a coefficient m of reduction will be determined less than unity, the more or less marked suppression of the parasitic rotating inductance by the Foucault currents induced in the surrounding non-laminated metallic masses, and in some cases in special damping circuits.

Consequences from the Point of View of the Construction of Alternators for Good Regulation. The theory and the calculation for the reactances just as they have been above analyzed, lend themselves to the discussion of the construction of alternators much better than the old methods. We will proceed to give a few examples of such applications.

(*A*) In so far as concerns the employment of short bobbins, (generally abandoned by reason of the disfavor thrown upon them by windings of only three coils per field, which give terrible pulsations), the winding of six short bobbins which I have indicated (Fig. 13) may be compared with the ordinary winding (Fig. 10) by means of the coefficients calculated above. The relations between the K of the two being the same as between the respective coefficients k of the E.M.F., it is evident that the advantage of the short bobbins from the point of view of reactions, involves a loss of E.M.F. in exactly the same proportion. To re-establish the desired value of the latter, the number of turns of the armature must either be increased, consequently re-establishing the same reaction, or the flux density in the air-gap must be increased, and consequently the ampere-turns of the field-magnets as well as the losses by Foucault currents. The two windings are therefore equivalent from the constructive standpoint when the field-magnet flux occupies the entire polar pitch. However, short bobbins may be treated more rationally by reducing the breadth of the field-magnet flux to two-thirds of the field-magnet pitch in such a manner that the flux shall be entirely utilized in the coil. Further

increasing the flux-density, an e.m.f. is obtained equal to that in long bobbins, and the direct reaction remains in the same ratio with respect to the e.m.f. while the transverse reaction is reduced. The winding of Fig. 13, with a flux having the value of two-thirds of the pitch, is then that which for a given ratio of counter-ampere-turns of the armature to the ampere-turns of excitation, produces the smallest transverse reaction. This reduction is of considerable importance.

(*B*) As to the methods of reducing reactions, it results from the preceding that there is no means of reducing the ratio $\frac{K}{k}$. The only means of improving the regulation of alternators are therefore, first, to saturate the field-magnet circuit (which augments the m.m.f. and reduces the variations of e.m.f. at the armature terminals as a function of the reactive current); second, to increase the air-gap, which is less effective; and third, to reduce the transverse reaction, which has the effect of diminishing the dephasing of the diagrams, and consequently the direct reaction, which is proportional to the reactive current. An alternator which would not have transverse reaction, would have nothing to fear from direct reaction, even if it were enormous.

The methods of reducing the transverse reactions alone are: First, the reduction of the breadth of the flux accompanied by an augmentation of the flux-density in the air-gap, re-establishing the same field-magnet flux at the expense of the increase of the flux-density; but in that case the same result would be obtained by a simple increase of the air-gap, without an increase in the loss of energy in the teeth; second, the saturation of the polar horns when the pole pieces possess them; third, the addition of longitudinal slots in the field-magnets, as in direct-current dynamos. This last method is very effective when the field magnets are saturated and the slots occupy their entire length and are continued partially into the yoke; the reduction of distortion thus obtained involves generally a reduction of the total flux, because the mean permeability of the field magnet is reduced by the inequality of the m.m.f.'s established between its two halves; but the augmentation of excitation which results is negligible in comparison with the diminution obtained in the reactive current by the reduction of dephasing.

(*C*) As to the comparison between single-phase and polyphase alternators, it is seen that not only do single-phase alternators utilize less effectively their materials for the production of energy, because

their armature surface is less utilized for e.m.f., but also their armature reaction gives rise to a hurtful, parasitic self-induction which does not exist in polyphase alternators and which reduces their good regulation. This parasitic inductance can only be partially suppressed at a cost of the expenditure of energy equivalent to a considerable augmentation of the apparent resistance of the armature.

Résumé and Conclusion

To sum up, it has been established in this paper that the theory of two reactions of the armature admits of analyzing the phenomena of alternators with greater precision than the old theories, besides having the advantage of referring them to conditions similar to those of direct-current machines. Simple diagrams are given applicable to alternators of saturated field magnets and unsaturated armatures (Fig. 4) and even to saturated armatures (Fig. 7) without involving a complicated correction.

It has been indicated how to calculate the coefficients of reaction, not only theoretical, but also actual values, by means of curves of magnetic potential in the air-gap. Interesting relations have been established between these coefficients and those of the induced e.m.f.

Comparisons have been established between the different types of winding, and the advantages possible for a special winding with short bobbins have been made evident.

It has been shown that single-phase alternators may be treated by the same methods, adding, however, to the inductance of the stray field a parasitic inductance which does not exist in polyphase machines.

Finally the consideration of the transverse reaction has permitted the discussion of a construction in view of good regulation, showing the interest which attaches to reducing the coefficient of distortion in alternators, and indicating the means of such reduction.

The author hopes that, thanks to simplicity of application, much greater than is often believed, and by its relations with the theory of direct-current machines, this method of calculation (in which he has had practical experience for several years) may be of service to designers and satisfy the need of rational precision in this work.

CHAPTER II

METHODS OF TESTING ALTERNATORS ACCORDING TO THE THEORY OF TWO REACTIONS [1]

By Prof. André Blondel, *École des Ponts et Chaussées.*

The author described in the *Bulletin de la Société des Electriciens* in 1892 a method of testing alternators similar to that of Hopkinson for direct-current machines, which permits of studying both their efficiency and their armature reactions, under the same conditions of operation and without a large expenditure of power, under the sole condition that the two alternators shall be similar. This method depends upon the rigid coupling of two similar alternators. Later the author published a variation of the method which does not call for the rigid coupling between the alternators, but which consists in operating the alternator on test as a synchronous motor, by the aid of an auxiliary alternator.[2] The alternator under test revolves on no-load, as a motor, under the normal current. This method has been designed particularly to determine the efficiency with greater precision than by separating the various losses. The object of the present paper is to complete and yet further perfect this method by pointing out how it may be likewise used for measuring armature-reactions (especially by the employment of two reactions, set forth in the preceding chapter).

Method No. 1. When the Rigid Coupling of the two Alternators is Possible. When two similar alternators are available, and when they can be placed side by side so as to be connected rigidly by a short coupling, the following tests may be carried out (Fig. 1):

First a certain difference of phase α is provided between the alternators (for example, a phase-difference of 30°, that is to say, one-sixth of a pole, or 45°, that is to say, one-fourth of

[1] A paper presented by the author before the International Electrical Congress at St. Louis in 1904. Reprinted from the Transactions, Vol. I, pp. 620–634.

[2] See *La Lumière électrique*, 1893.

a pole. The system of two machines is driven by a measuring motor whose duty it is to furnish the power necessary for satisfying the losses. Between the two alternators A_1A_2 (Fig. 18) whose terminals are connected each to each by very short couplings of negligible impedance, a volt-meter V is connected across, an ammeter A and a wattmeter W being inserted in series. The figure is drawn upon the supposition of two single-phase machines, but applies equally well to the case of two similar three-phase machines coupled by their three phases, testing upon a single phase, taking care that the phases remain balanced, in spite of the measuring instruments.

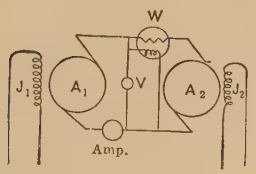


FIG. 18.

Let $U=OB$ (Fig. 19) be the difference of potential observed at the common terminals, OA_1 and OA_2 the direction of the generator e.m.f.'s dephased relatively to each other by the angle α. By symmetry, the vector Ob, which represents the current, will also be directed along OB, and the line A_1A_2 drawn from B perpendicularly to U and I, will represent the double transverse reaction $2\omega L^1 I$. There will be a flow of current between the alternators without the production of any external power. The current will be in phase with the E.M.F. at the terminals U, as if the alternators supplied a conducting system devoid of inductance; the flux density obtained in the armature will be the same in both alternators, since it gives rise to the same E.M.F. at the terminals U. The power furnished by each will be measured by the wattmeter W, and the total loss p will be furnished by the method of double-weighing, by means of the measuring motor which drives both alternators. The efficiency will then be the ratio

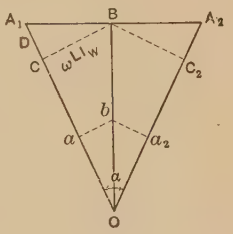


FIG. 19.

$$P_u=\frac{UI}{UI+\frac{p}{2}}.$$

To determine the excitations necessary for the two alternators to produce the condition above described, it is sufficient to apply the graphic method of two reactions, as follows (Fig. 19): from the point

B a perpendicular BC is let fall upon the straight line OA_1, and the condition is such as if the alternator were delivering power to an inductive system according to the total characteristic of excitation (Fig. 20). The current I is formed of two components: one an active component Oa, equal to the projection of I upon OA, and which gives the transverse reaction BC equal to ωLI_w; the other component is a reactive or reactive component ab, which gives the fall of potential to be calculated on the characteristic. Let us lay off on this characteristic (Fig. 20) an ordinate equal to OC, and upon the latter a segment CD equal to the e.m.f. lost by the stray field. From the point D lay off horizontally the counter ampere-turns of the armature, and thus will be obtained on the abscissa $O'P$ the total ampere-turns necessary for the excitation. The ordinate PP' corresponding thereto will represent the e.m.f. E on open circuit necessary for alternator A_1 and which will be, in general, different from the length of OA_1 which was represented in Fig. 19.

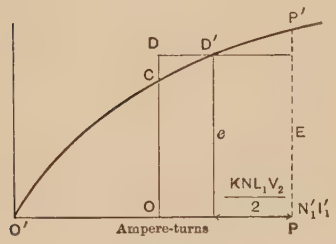


FIG. 20.

In the same way the e.m.f. is determined which is necessary for the alternator OA_2, observing that for the latter the sign of the reactive armature-reaction is changed, as well as the sign of $E=A_2O$, and that, consequently, the armature reaction remains demagnetizing, so that the geometrical construction is identical. It is, therefore, easy to recognize in advance the equal excitations to be given to the two alternators, in order to satisfy the desired conditions. It should also be determined at the time of the test, by means of the wattmeter, that there is no sensible difference of phase between the current and the e.m.f. Inversely, if this condition were directly realized by adjusting the excitations, it would be possible to deduce from an examination of the diagram the total armature-reactions represented by the abscissa OP, indicating the *total* fall of excitation between the open circuit e.m.f. and the e.m.f. under load. In the latter case, the fall due to the stray field is not separated from that due to armature reaction.

The same diagram gives immediately the value of the transverse

reaction, since the angle α is known experimentally, and the values of U and I can be consequently measured. This gives

$$A_1B = \omega L^1 I = U \tan \frac{\alpha}{2},$$

from which L' is known as a function of α, U and I.

The same method permits varying the angle α successively, and repeating the operation, commencing each time with the same voltage at the terminals U, and thus tracing the entire characteristic of an alternator operating upon a dead resistance.

The above method gives immediately the values of the direct and transverse armature reactions. As to the coefficient of self-induction of the stray field ωs, it may be determined for any given alternator by the method indicated later on.

A test may then be made of the two alternators coupled together, without any angular difference of phase between them. The e.m.f.'s. E_1 and E_2 are then in simple opposition of phase, and the difference $E_1 - E_2$ will produce a resultant current which may be regulated in strength by regulating the difference of excitation, and which current is dephased by nearly 90°. The diagram is given in Fig. 21, where OC_1 and OC_2

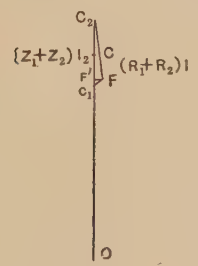


FIG. 21.

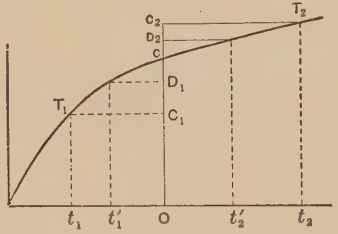


FIG. 22.

are the two internal e.m.f.'s. The difference C_1C_2 represents the fall of potential due to the impedance of the circuit of the armatures, and which can be decomposed into two rectangular straight lines, C_1F representing the total ohmic drop $(R_1+R_2)I$ due to the current, and C_2F the total reactive drop in the armatures. Projecting F upon C_1C_2, a vector C_2F' is obtained, which differs but little from C_2C_1, and which represents the fall of potential of the two machines due to direct reaction. If the characteristic of total excitation be then drawn as in Fig. 22, this drop will represent the sum of the two drops due to the reactive current, of which one CC_1 is positive and the other CC_2

is negative. It is easy to mark these off on the characteristic. Inversely, the drops due to the reactive current about the point C may be deduced, and thus the coefficient K of direct reaction. It is sufficient, starting from the point C, to trace two segments, CC_1, CC_2, representing the two drops, and to trace the horizontal C_1T_1, C_2T_2; thence the abscissas Ot_1, Ot_2, which represent the virtually lost ampere-turns. If the segments CD_1, CD_2 are known, which represent the e.m f.'s. of dispersion, and if the horizontal straight lines are drawn through D_1 and D_2, the corresponding abscissas t_1' and t_2' permit of calculating exactly the back ampere-turns, t_1t_1', t_2t_2', represented by the armature, and which should have equal magnitudes.

Method No. 2. Applicable to a Single Synchronous Machine Operating upon an Actual Conducting System. When only one alternator is available for the test, it is not possible to proceed so conveniently as in the last case, and, in particular, the plan of testing with variable angles of coupling must be given up.

A similar test to that which we have indicated above can, however, be made by driving the alternator on open circuit as a synchronous motor supplied from the conducting system on which it is to be employed (supposing the factory to have other alternators already installed) or by a current furnished from some other alternator of equivalent power. The alternator, or alternators, serving as the source, will then be excited in such a manner as always to maintain the voltage constant at the terminals of the alternator under test which operates as a synchronous motor; this voltage being the normal voltage of operation, the excitation of the motor is to be varied, as if it were desired to obtain the "V-curve" of constant voltage. The latter gives by its minimum ordinate AB (Fig. 23) the value of the ohmic losses (at current I_0), and the indication of the condition of excitation OA corresponding to a power-factor equal to unity ($\cos\phi = 1$) —at least on the hypothesis that the effects of harmonics in the e.m.f. are inconsiderable. For any other excitation Oa, the strength of the reactive current may be obtained by constructing upon ab a triangle of which the angle at b is given by the wattmeter. The side $bd = I_w$, differs little from BA, that is to say, from the active current on

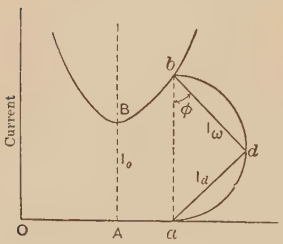


FIG. 23.

open circuit; ad then represents the reactive current I_d. The value of the reactive current delivered or received by the motor may then be deduced from the V-curve for all values of excitation. If reference is made to the characteristic in Fig. 24, on which OC represents the normal e.m.f. at the terminals, the knowledge of Aa gives for each value of the current I_d the corresponding value of the total lost ampere-turns $OF = Aa$. A curve of these ampere-turns may then be drawn as a function of I_d, and the total fall of potential CD thus deduced from the chart for every value of the armature current. This method does not separate the reaction into two parts according to theory, but it gives exactly the required result which makes it possible to calculate the fall of potential for each value of the current and of the phase with respect to the mean e.m.f. CD which the armature designed to supply.

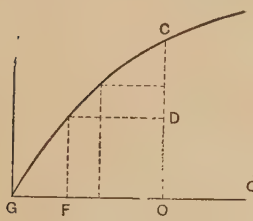


FIG. 24.

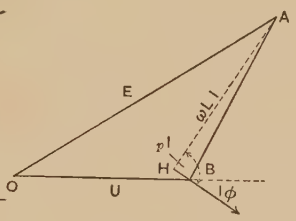


FIG. 25.

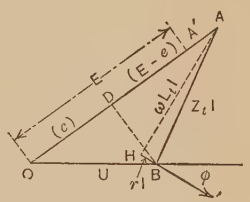


FIG. 25a.

To complete this indication it is sufficient to know the transverse reaction. This may be determined readly enough by the same experiment if care be taken to measure the phase angle, ϕ, of the current with respect to U, the mean e.m.f. at the terminals. Let us, then, draw (Fig. 25) the synchronous motor diagram with OA to represent the internal induced e.m.f., E, and OB the mean voltage, U, at the terminals; then the geometrical difference, AB, will represent the fall of potential due to impedance, which may be resolved into two vectors, one of which (BH) represents the ohmic drop, and the other (HA) the fall of potential $wL'I$ calculated as a function of the transverse reaction. Knowledge of the angle ϕ and the current strength I enables the point H, and also the direction-line HA, to be determined. Knowing, from the experiments set forth below, the angle of lag, ϕ, of the internal e.m.f., E, (whose direction, OA, can thus be drawn) it is only necessary to take the point of intersection of OA with the direction-line HA to locate the point A, and, consequently, to be able to complete the triangle OHA. The length of

HA gives, at once, the value of the transverse inductance L'. The same determination can therefore be made over again with increasing values of ϕ, by simply increasing the excitation each time. The greater the lag-angle, ϕ, the more accurately L' will be determined.

It may be noted, moreover, (Fig. 25*a*), that if a line (BD) is drawn from B perpendicular to OA, the portion intercepted (OD) corresponds to the e.m.f. e obtained with no-load. The difference $DA' = E - e$ therefore gives, in volts, the value of the armature-reaction produced by the reactive component of the output current.

When the angle at A (Fig. 25*a*) is very small, we have substantially,

$$DA = Z_t I \cos A = \omega L_t I_d.$$

Let $$Z_t = \sqrt{r^2 + (\omega o L_t)^2}.$$

If we let L_d = the direct reaction, then, for a very small variation of the excitation we can write

$$DA' = \omega L_d I_d,$$

whence $$AA' = \omega(L_t - L_d) I_d.$$

Instead of evaluating in the preceding test the active current I_w in order to deduce the reactive current I_d, it would be easy, if a measuring motor were at hand, to make this deliver directly to the shaft of the alternator operating as a synchronous motor the necessary power for driving the motor, in such a manner that AB on the V-curve (Fig. 23) becomes *nil*. But the same result may be obtained yet more easily when a steam alternator unit has to be tested, by admitting to the engine just enough steam to satisfy the losses both of the engine and alternator, so that the alternator only receives a reactive current. With this object, the steam admission may be regulated in such a manner that the alternator, excited so as to give on open circuit the normal e.m.f. U of the system, runs idly in synchronism; then no change is made either in steam admission or in the pressure, and the operations are conducted entirely on the electric side of the alternator, connecting this with the system and varying its excitation so as to develop the V-curve. It is possible to measure in advance the electric power necessary to drive the alternator and its steam engine on open circuit, and thus to deduce the total losses on open circuit. The power wasted may thus be measured by the steam-engine indicator-diagram, which permits of determining the constant of the curve of steam consumption as a function of the power produced

(a curve which generally is nearly a simple straight line); it is also possible to separate the mechanical losses, first making this test without field-excitation, and then exciting the field-magnets so as to obtain the normal e.m.f.[1]

Summing up, this test permits of determining, with a fair approximation, the losses on open circuit and then on load, without being obliged to actually develop these losses by full-load in the alternator as well as the corresponding heatings.

From an electrical point of view the same test permits of determining the total values of the direct reaction as a function of the reactive current, and the constant L' of the transverse reaction. If it is desired to analyze these phenomena more completely, the value of ωs may be determined as follows:

Determination of ωs. For this purpose the following considerations are made use of (the basis of which is the method of calculation of the short-circuit current given by Kapp, which does not lend itself to experimental verification):

Let ONM be the characteristic of excitation; that is to say, the curve of the armature e.m.f. as a function of the exciting ampere-turns. Suppose the excitation to be constant and equal to OA ampere-turns, giving an e.m.f. represented by AM. Suppose the reactance ωs sought for to be known and also the short-circuit current I_{cc}. If a point P is taken on the curve whose ordinate is equal to $\omega s I_{cc}$, the corresponding segment of the abscissa AH will measure the back ampere-turns of the armature KNI_{cc}. If the calculated values of ωs and of K are taken, the direction of the straight line AP is known, and the line may be traced from which the value of I_{cc} may be deduced. This is the construction of Kapp. We shall take up, on the contrary, the inverse problem, supposing I_{cc} determined by experiment and seeking to deduce from it the two constants K and ωs.

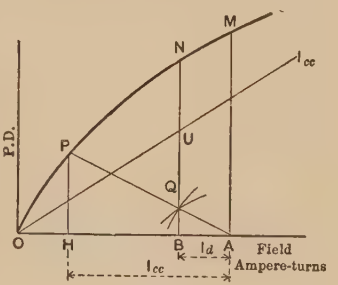


FIG. 26.

The following observation may then be made. If the segment AH be taken as the measure of the short-circuit current I_{cc} and a segment AB as the measure of any other reactive current I_d to the

[1] The total efficiency of the engine and alternator unit may also be obtained later at load by means of the engine diagram.

same scale of volts at the terminals corresponding to this reactive delivery, this segment will be equal to the ordinate $QN = U$, taken between the curve and the straight line AP. In fact, the back ampere-turns will then be equal to AB, and the loss of voltage by dispersion equal to BQ, by reason of similar triangles. Besides, the point Q divides the straight line AP in the ratio of the current I_d to the short-circuit current I_{cc}.

Thus arranged, suppose that any reactive current I_d be taken experimentally at the corresponding e.m.f. U; the point Q will be determined by these two conditions: its vertical distance from the curve of excitation is equal to U and its radial distance from the point A is equal to $\frac{I_d}{I_{cc}} \times AP$.

The point is found, therefore, at the intersection of two new curves that are easily drawn: a curve parallel to ONM traced at a vertical distance U below the former, and a curve homothetic to the curve of excitation with respect to the point A, with homothetic ratio $\frac{I_d}{I_{cc}}$. These two curves are parallel in their rectangular parts, and separate as much one from the other as the point M is selected further beyond the bend in the characteristic. They would coincide if the point M were below the bend. The experiment should therefore be made with an excitation OA sufficient clearly to pass the bend.

The test is made by causing the alternator to operate first on short-circuit, and then upon a reactance-coil having an open magnetic circuit, or upon an under-excited synchronous motor, giving $\cos\phi$ less 0.20, that is to say, a current almost entirely reactive.

It is understood that if not only one, but also several, reactive circuits are tried, the straight line PQ will be still better determined thereby, and consequently ωs will be known with correspondingly greater precision.

Analogies between this Method and that of Potier-Behrend. It is possible also to follow a somewhat different course, by causing the alternator under test to operate upon an inductive circuit with $\cos\phi$ nearly 0 (for example, upon a synchronous motor driven by a motor adjusted in a manner to produce just the power consumed at light load), and by varying the excitation of the alternator; in this manner a *constant reactive-current* curve is obtained, from which may be deduced, by the method of M. Potier, the coefficient of the

back ampere-turns, and the coefficient of the stray field, but which is not directly applicable to the ordinary problem of the calculation of the ampere-turns necessary for constant voltage.

This method is one of those which was devised by Mr. B. A. Behrend, who recently published a number of applications of it. It is wholly different from the preceding, in which the voltage is maintained constant at the terminals instead of the current. The same author employs also another method, which consists in dividing the field magnets into two equal parts to which are given exciting currents of opposite sign and slightly different strength, so as to develop in the armature a certain current, which is necessarily reactive while developing a mean flux-density sensibly equal to that in normal operation. This test may appear equivalent to that which is obtained by means of two alternators mechanically coupled and set at opposite phase (see preceding Method No. 1), with the simplification, however, that only one alternator is employed; but as the current thus obtained is only a reactive current, the conditions are not identical with those of the preceding method, but only equivalent to method No. 2 above described (see the case of the alternator operating as a synchronous motor on no-load). It is not, besides, sufficiently rigorous except for alternators having a large number of poles, because when there are but few poles the presence of two poles of the same sign side by side upon the field magnet at two points on the latter would seem to modify notably the conditions of the magnetic circuit. Consequently, Method No. 1 is preferable when it can be employed.

Method No. 3. For the Determination of Transverse Reaction (coefficient L). Besides the preceding methods, several others may also be pointed out which are very simple for the determination of the transverse reaction, and which avoid the objection so often made to the diagram in which this reaction appears. Moreover, the oscillograph (or ondograph) permits the real angle of dephasing ψ to be measured between the internal e.m.f. E (Fig. 27) and the current, and gives immediately, in consequence, the value of L, when the alternator operates upon a non-inductive resistance $\phi = o$, by the equation

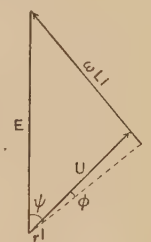


FIG. 27.

$$\tan \psi = \frac{\omega LI + U \sin \phi}{rI + U \cos \phi} = \frac{\omega L}{r + \frac{U}{I}},$$

from which

$$\omega L = \left(r + \frac{U}{I}\right) \tan \phi.$$

With the oscillograph, for example, it suffices to place upon the shaft, or upon one of the pole pieces of the rotating field-magnet (Fig. 60), a contact segment S upon which rub two brushes BB', connected in series with a battery, one of the oscillographs O of a double-oscillograph apparatus, and the electromagnet e which controls the release of the shutter. The contact segment S, insulated from the base, closes the circuit upon its passage below the brushes, and opens it again at the precise moment when the brush B leaves it. A segment is selected large enough in order that the shutter shall have time to open before the rupture of the circuit, in order that the latter may then be photographed upon the plate in the form of a vertical line intersecting the straight-line zero furnished by one of the oscillographs.

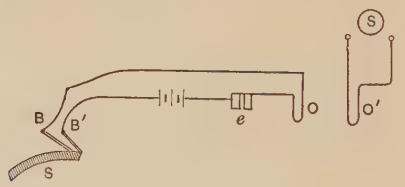


FIG. 28.

The second oscillograph of the system, O', is connected to the terminals of the alternator and serves to register the e.m.f. U at the terminals. Two experiments are made on two different plates, or upon the same plate after having slightly displaced the zero-line produced by the oscillograph O, so as to distinguish the two different records. The e.m.f. at open circuit is then marked upon it, and next the e.m.f. when the circuit is closed upon a dead resistance, at the same time that the current I delivered thereto is measured. In these two tests the distance is measured from the zero of the curve of e.m.f. to the point of intersection by the vertical with the zero-line. The difference between these two lengths thus measured determines the displacement of phase of the e.m.f. at the terminals, that is to say, the angle of dephasing sought (taking for the value of 2π the length of a period measured upon the plate, and taking the ratio of the measured retardation to this length). It is sufficient to take the values of ϕ, of I, and of U in the preceding formula in order to determine the transverse reaction L.

Figs. 13*a*, 14*a*, 15*a* and 16 (Chapter I) represent an example of the

determination of L made by this method in the laboratory of the Société Sautter-Harlé & Company of Paris, which employs with success the methods described in this note. The curves of the first three figures have been obtained by means of the author's oscillograph at the terminals of an alternator of 350 K.W., and represent the periodic curves of the difference of potential between these terminals when the alternator works on open circuit, then when delivering 62 amperes, and finally 102 amperes, its normal load. The effective value of the e.m.f. was 1155 volts in star. The vertical lines represent the tracings produced by the contact of the contact-maker S. Figs. 13*a* and 15*a* are printed upon one and the same sheet of paper, by causing the vertical lines to coincide, from which Fig. 16 is produced, which shows clearly in evidence the dephasing between the internal e.m.f. E, and the pressure at terminals U. The dephasing reaches 24° 30′ in the test with 102 amperes, and 19° in the test with 62 amperes. In applying the preceding formula (with $r=2.5$ ohms) the values are obtained:

$$\omega L=(2.5\times 62+1155)\times 0.344=7.26 \text{ ohms},$$

$$\omega L=(2.5\times 102+1155)\times 0.566=7.81 \text{ ohms},$$

which differ little (and would perhaps be equal if greater precision had been taken in the measurement of the phase-difference between the two curves). In practice one would take the mean value 7.5 ohms.

It is evident that it is easy thus to determine the constant of distortion, and this justifies the employment of the theory of the two reactions rather than the rough method of the " curve of short-circuit," which combines together the two often very different reactions.

The ondograph gives the same result, if one marks off successively upon the same sheet of paper the e.m.f. upon open circuit and then that on closed circuit, and each time the mark obtained when the apparatus is traversed by the current coming from the brushes BB'. It is possible to dispense with making this mark, if the ondograph is driven, not by a synchronous motor, but by a flexible coupling connected mechanically to the shaft of the alternator (with the interposition of gear wheels, as in the recording mechanism of Francke).

Moreover, even in the absence of the preceding analyzing apparatus, one can approximately obtain the transverse reaction, or rather its ratio to the direct reaction, with unsaturated field-magnets, by sending, as M. Herdt has already suggested, into the alternator arma-

ture at rest, alternating currents derived from any available external source, and measuring the apparent self-induction by the method of Joubert for the two characteristic positions of the field-magnet poles. The simplest method applicable to a star-wound alternator consists of sending an alternating current I through one of the phases which divides between the two other phases, starting at the neutral point, and which is received at its emergence by a connection applied to the two terminals A_2A_3. Fig. 29 represents the connections: a is an auxiliary alternator of which the excitation is adjusted at will in order to vary the current I. It is easy to see that an alternator thus traversed by a current is placed in the same conditions as if it were fed by three-phase currents at the moment when one of these currents passed through its maximum. This connection being made, the field-magnet is arranged so that the armature poles have their axes directed in a line with those of the field-magnet poles, and then in such a manner that these axes are directed midway between the field-magnet poles. In each case the self-induction developed by the current I is measured according to the method of Joubert; the ratio of the two measures is that of the direct self-induction to the transverse self-induction, or of the coefficients K and K_t applicable to these two reactions. If the difference of potential u is measured between A and the center of the star O, the ratio $\frac{u}{I}$ represents the impedance of one phase. From this the inductance may be determined, since the resistance of one phase is known. This measure made in the second position indicated above, gives, therefore, the precise and approximate value of the transverse inductance sought. It must be remembered that this inductance also comprises the inductance of dispersion; therefore has the value

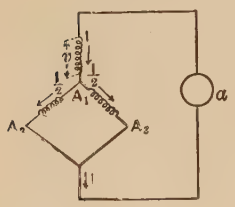


FIG. 29.

$$\omega L = \omega\lambda + \omega s,$$

calling $\omega\lambda$ the transverse reaction properly so called, not including the dispersion ωs.

Repeating the experiment for various values of the current, the constancy of the coefficient L may be determined.

APPENDIX A

If Eq. (*A*) on page 16 be solved completely for i, the solution will include a transient exponential term depending upon the point of the E.M.F. cycle at which the circuit is closed. It is a well-known fact that this transient term becomes negligible a short time after the circuit is closed, and that when the impressed E.M.F. is sinusoidal the current i will settle down to a sinusoidal form.

In the present case both of the active E.M.F.'s (e_1 and e_2) are assumed sinusoidal and of the same frequency; their resultant is therefore sinusoidal as will be the current produced thereby. Then all the variables involved are sinusoidal and may be represented by vectors in such a manner as to indicate clearly their phase-relations.

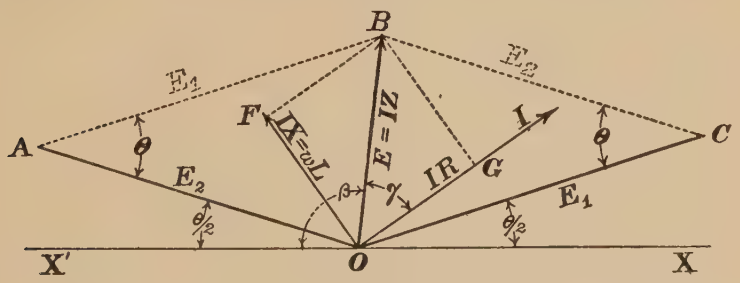


Fig. A

Referring to Fig. *A*, draw $\overline{OC}$, to designate by its length and direction the phase and magnitude of the internal or induced E.M.F. E_1 of the generator, and $\overline{OA}$ to designate in a similar manner the internal or induced E.M.F. E_2 of the motor. Then the resultant or vector-sum of E_1 and E_2 will be E, designated by the line $\overline{OB}$.

The horizontal line OX is taken arbitrarily as the zero of phase,[1]

[1] In Fig. *A* the instantaneous values of the various variables (e.g., e_1 and e_2) are given by the vertical projections of the corresponding vectors (multiplied by $\sqrt{2}$), as the latter revolve counter-clockwise at constant angular velocity, $\omega = 2\pi n$, where n = cycles per second. The diagram is shown at the instant $t = 0$.

and the $\angle COX = \angle AOX = \frac{\theta}{2}$. This corresponds to the equations for e_1 and e_2 on page 16, which show that when $t=0$, e_1 has a phase $+\frac{\theta}{2}$ and e_2 falls short of exact opposition (180°) by the same angle $\frac{\theta}{2}$, where θ is the angle between E_1 and E_2 projected backwards. E_2 may be said to lag behind E_1 by $180°+\theta$, or to lead E_1 by $180°-\theta$.

The current produced in the circuit of the two armatures will be, $I=E/Z$ and will lag behind E by the angle γ whose tangent is $X \div R$; where R, X and Z, $(=\sqrt{R^2+X^2})$, are respectively, the resistance, the reactance, and the impedance of the whole circuit, including the impedance of the connecting line and the synchronous impedance of the two armatures; and $X=2\pi nL=\omega L$, where L is the inductance of the whole circuit.

By trigonometry

$$B=\sqrt{E_1^2+E_2^2-2E_1E_2\cos\theta},$$

and since

$$E=IZ, \quad \text{and} \quad I=\frac{E}{Z},$$

we have

$$I=\frac{E}{Z}=\frac{\sqrt{E_1^2+E_2^2-2E_1E_2\cos\theta}}{Z},$$

and the current-phase (referred to OX) is $\pi-\beta-\gamma$.

These two relations stated algebraically, give as the instantaneous value of i,

$$i=\frac{\sqrt{2}\sqrt{E_1^2+E_2^2-2E_1E_2\cos\theta}}{Z}\sin(\omega t+\pi-\beta-\gamma),$$

which is the same as the value of i given at the top of page 17.

The other forms of equations for i and I given on page 17 are deducible from the diagram of Fig. A.

APPENDIX B

Data Concerning Synchronous Motors of American Manufacture

The Tables A and B, given herein below, contain data characteristic of good American practice in the manufacture of synchronous motors. In comparing these data with those characteristic of good European practice, as given in Part I, on page 5, due allowance should be made for the rating conditions in the two cases. Thus, the normal frequency is 60 for the American motors, and 40 for the European motors; and the kilovolt-amperes per horsepower are 746 in America and 736 in Europe. The power-factor assumed as a basis for rating is equal to unity for American motors. It is presumably the same, though not stated, for European motors.

TABLE A.—SINGLE-PHASE AND THREE-PHASE SYNCHRONOUS MOTORS OF AMERICAN MANUFACTURE

Type.	No. of Poles.	R.P.M. for 60 Cycles.	Pulley.		Approx. net Weight.	Single-phase.				Three-phase.		
			Diam. in Inches.	Face in Inches.		H.P.	Effic'y.	Amp. at 110 V.	K.W. of Trans.	H.P.	Amp. per Line at 110 V.	K.W. of Trans.
AT*	4	1800	8	3.5	715	6.5	79	56	6.5	10	45.6	10
AT*	4	1800	10	4.5	1070	13.	82.5	108	12	20	89.2	17.5
AT*	4	1800	12	5.5	1535	22.5	83	185	20	35	155	30
ATB†	6	1200	15	8	2250	32.5	85	265	30	50	220	45
ATB‡	6	1200	16	11	2580	43.5	87.5	335	37.5	67	287	55
ATB‡	6	1200	16	14	3120	65	89	495	55	100	422	80
ATB‡	8	900	21	17	4380	87	89.5	660	75	134	565	110
ATB‡	10	720	26.5	23	6630	130	90	980	110	200	840	160

* Self-excited.

† Separately excited.

‡ Has direct-connected exciter.

TABLE B.—SINGLE-PHASE AND THREE-PHASE SYNCHRONOUS MOTORS OF AMERICAN MANUFACTURE

Type.	No. of Poles.	Pulley.		Face in Inches.	Approx. net Weight. Lbs.	Single-phase.				Three-phase.		
		R.P.M. for 60 Cycles.	Diam. in Inches.			H.P.	Effic'y.	Amp. at 110 V.	K. W. of Trans.	H.P.	Amp. per Line at 110 V.	Effic'y.
G– 1	6	1200	11	9	1490	23	85	182	.....	36	158	89.3
G– 3	6	1200	15	12	1940	41	87.2	318	.....	61	263	91
G– 5	8	900	20	12	3150	58.5	87.3	445	.....	91.5	394	91
G– 7	8	900	20	15	4150	82.5	88.0	636	.....	123	526	91.4
G– 9	8	900	20	21	6500	120	89.5	910	.....	186	787	92.5
G–11	12	600	30	22	8000	121	90.0	910	.....	186	787	93.0
G–13	12	600	30	28	9600	170	90.5	1272	.....	250	1050	93.7

All these machines have rotating poles and open slot stationary armatures. The field-poles are equipped, in addition to the standard field-coils, with a short-circuited winding similar to an induction-motor, which is utilized to make the machine self-starting as an induction-motor, and also as a "damper" to prevent "hunting" when running in synchronism. When starting, polyphase machines will develop from 20 to 50 per cent of their normal torque with from 100 to 250 per cent of the normal input in kilovolt-amperes. The single-phase machines are not self-starting.

APPENDIX C

USE OF SYNCHRONOUS MOTORS FOR IMPROVING POWER-FACTOR IN AMERICA

It is well known that the power-factor of a system supplying current to rotary converters at various substations can be materially improved by over-exciting the field of the rotary converters. In some cases this benefit was considered of enough importance to warrant the use of rotary converters which are shunt-wound only and consequently can produce no compounding effect.

In a motor-generator set composed of a synchronous motor driving a continuous current generator the compensating effect due the over-excitation of the synchronous motor-element is retained without sacrificing the compounding effect of the combination. This is one reason why motor-generator sets have been preferred in many cases to rotary converters.

Owing to the fact that the compensating effect is the same as that produced by a condenser the synchronous motor producing the "condenser effect" is often called a "synchronous condenser" or a "rotary condenser," and also a "synchronous compensator." The Standards Committee of the American Institute of Electrical Engineers gives the preference to another term, "phase-modifier," as appears from the following definition given in the "Standardization Rules":

"A Synchronous Phase-modifier, sometimes called a Synchronous Condenser, is a synchronous motor, running either idle or under load, whose field-excitation may be varied so as to modify the power-factor of the circuit, or through such modification to influence the voltage of the circuit."

The practice of designing the synchronous motor portion of motor-generator sets with special reference to their use as "synchronous condensers" has become quite general in America. Many articles and papers have appeared on the subject. A partial list of the authors is given at the end of the book.

The following item, taken from *The Electrical World*, refers to an interesting case where synchronous condensers have been used to great advantage:

(*From the Electrical World, March 9, 1912, p. 548.*)

Overrating of Motor-generator Sets to Improve Power-factor. All motor-generator sets installed by the Detroit Edison Company are now specified with their motor-elements sufficiently larger than the generators to insure that, fully loaded, the motors can be operated at a leading power-factor of 80 per cent to improve the power-factor of the system. Operators are instructed to increase the field-excitation of their machines until the motors take full load current at the leading power-factor. As the result of this practice, power-factors approaching within 97 per cent of unity have been observed at the bus of the generating plant.

The Detroit Edison Company, through its Engineering Department, has kindly placed at the disposal of the Translator the following detailed information regarding this case:

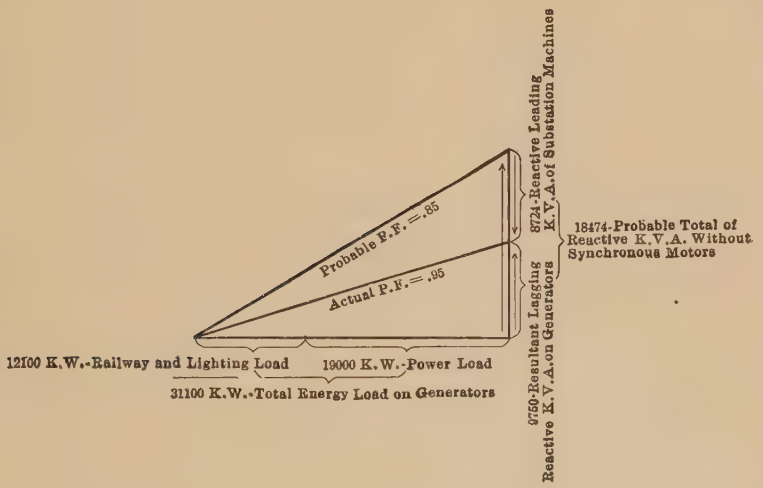
(*From the Engineering Department of the Detroit Edison Co.*)

The motor-generator sets referred to in the above item are synchronous machines installed in the substations. They are supplied with 4600 volt A. C., and deliver 600 volt D. C. for railway, and 250 volt D. C. for the lighting, service. All are over-excited more or less; and the station operators are instructed to adjust the field so that the motors always take the full load current at a leading power-factor. This power-factor varies from 50 to 95 per cent.

The total induction-motor load on the systems is about 19,000 KW. with a power-factor varying from 60 to 85 per cent, lagging. The lagging current of such a load would lower the power-factor at the power-house bus considerably, but by means of the over-excited machines in the substations a leading current is provided to counteract the lagging one, and a very good power-factor is thus maintained.

The extent to which the power-factor at the power-house bus is affected by the over-excitation of the synchronous motors is best shown by a vector diagram.

This was figured from simultaneous readings on the turbo-generators at the power-house, and the synchronous motors at the substations. The average power-factor of the turbo-generators with a load of 31,100 KW., under normal running conditions, is about 95 per cent, lagging. The reactive component of this is 9750 K.V.A., lagging. The total load taken by the synchronous motors is 10,735


Probable P.F. = .85
Actual P.F. = .95
8724-Reactive Leading K.V.A. of Substation Machines
18474-Probable Total of Reactive K.V.A. Without Synchronous Motors
9750-Resultant Lagging Reactive K.V.A. on Generators
12100 K.W.-Railway and Lighting Load
19000 K.W.-Power Load
31100 K.W.-Total Energy Load on Generators

KW. with a power-factor varying from 50 to 95 per cent leading, and the summation of all its reactive components at the various power-factors is 8724 K.W.A. This leading component counteracts an equal lagging one, leaving a resultant reactive lagging component of 9750 K.V.A. on the whole system, corresponding to a power-factor of 95 per cent. Without this leading component the lagging component would be 18,474 K.V.A., which would mean a power-factor of 85 per cent. Hence it is seen that a 10 per cent rise in power-factor is obtained by over-excitation of the synchronous motors in our substations.

Regarding the overrating of the motor-ends of the motor-generator sets, the following two examples of rating might be mentioned:

MACHINE No. 1. (Installed under the old system.)

Motor:	Class, 10–530–720.
	Volts, 4400.
	Amperes, 73 per phase.
Exciter:	Class, 6–7–720.
	Volts, 125.
	Amperes, 56.
Generator:	500 KW. direct-connected.

MACHINE No. 2. (Installed under the new system.)

Motor:	Class, 10–700–720.
	Volts, 4400.
	Amperes, 92.
	Power-factor, 0.80.
Exciter:	Class, 6–8–720.
	Volts, 125.
	Amperes, 64.
Generator:	500 KW. direct-connected.

It will be noticed that the motors of the new machines are larger, and are specified to operate at 80 per cent power-factor. This was necessary to allow for over-excitation in the case of machines of recent design. The old machines, however, will stand considerable over-excitation, because the design of motors was formerly more liberal in overload capacity.

A. A. M.

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